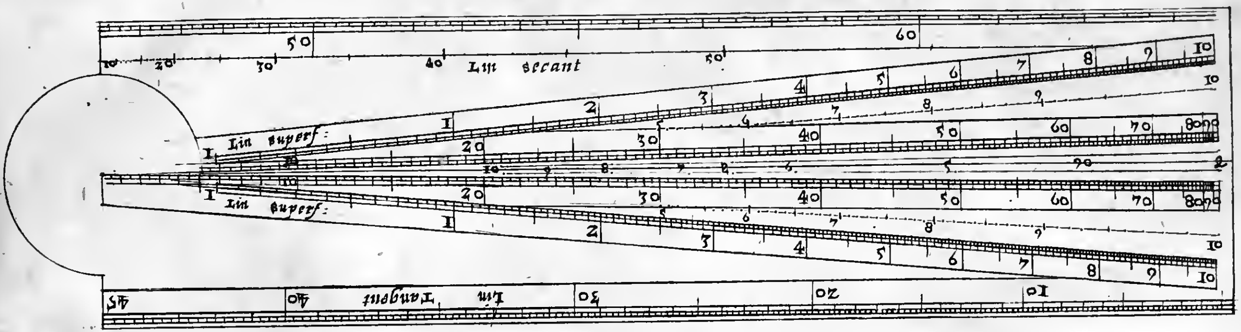
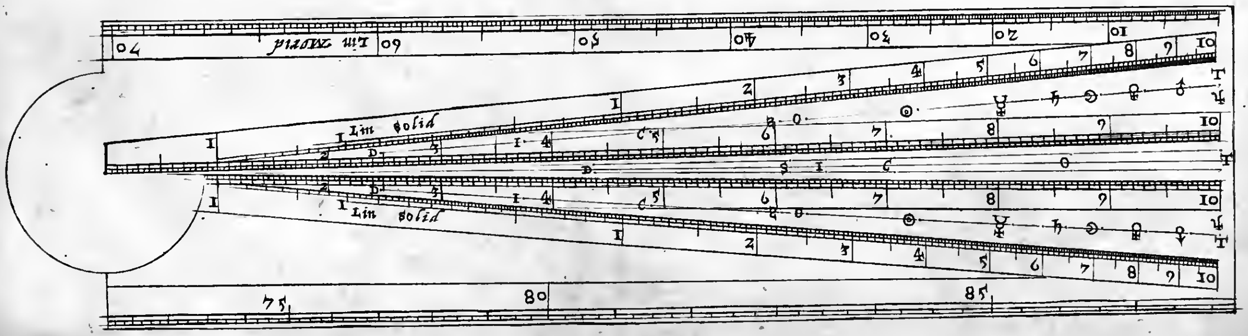
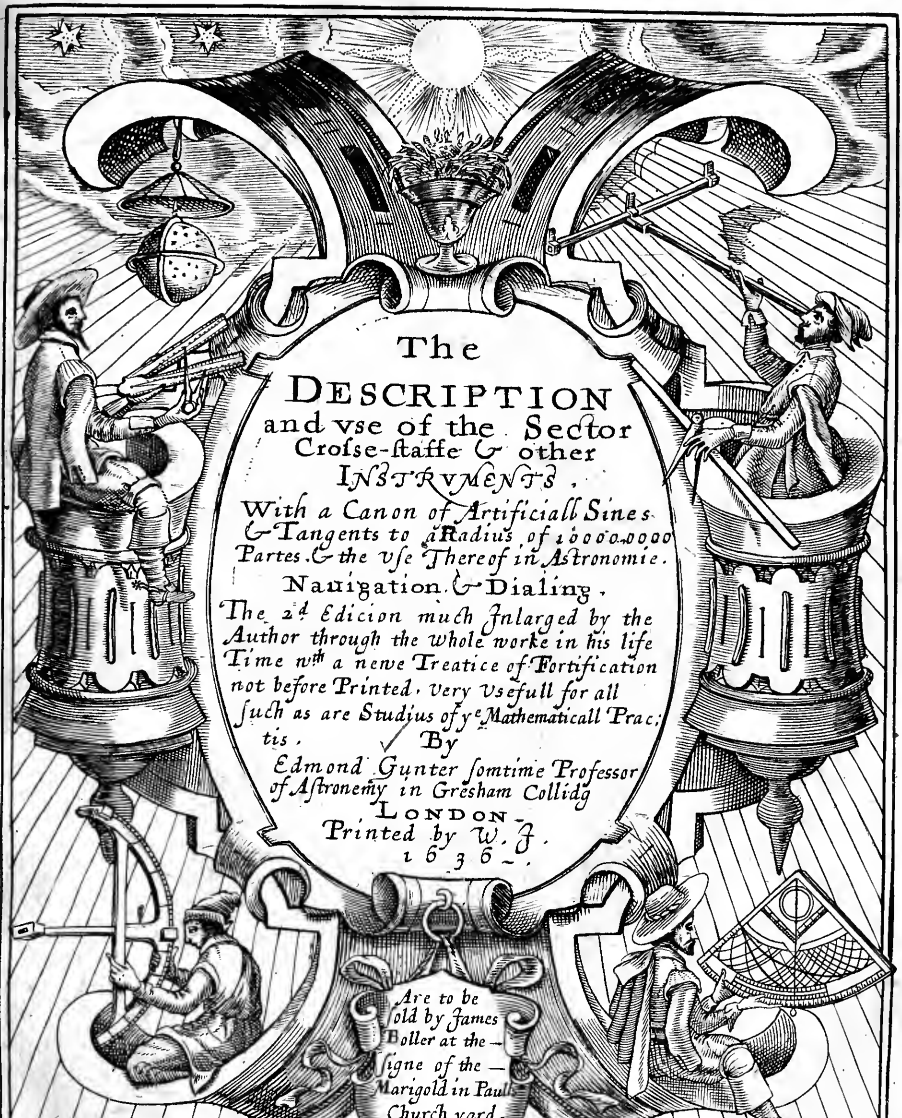


These and all other Mathematicall Instruments are made in Brass by Elias Allen dweling with out Tempel barr a gainst St Clements Church



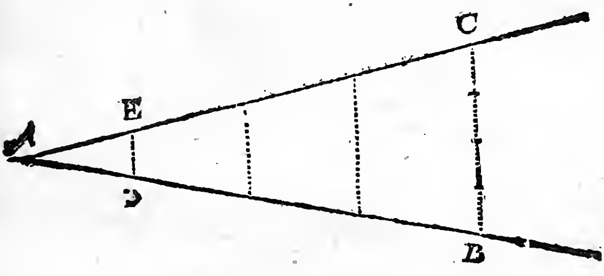

The
DESCRIPTION
and vse of the Sector
Crosse-staffe & other
INSTRVMENTS.
With a Canon of Artificiall Sines
& Tangents to a Radius of 10000.0000
Partes. & the vse Thereof in Astronomie.
Nauigation. & Dialing.
The 2d Edicion much Inlarged by the
Author through the whole worke in his life
Time wth a newe Treatice of Fortification
not before Printed, very Vsefull for all
such as are Studius of ye Mathematicall Prac;
tis. By
Edmond Gunter somtime Professor
of Astronemy in Gresham Collidg
LONDON.
Printed by W. J.
1636.
Are to be
sold by James
Boller at the
Signe of the
Marigold in Pauls
Church yard.
1636

THE DESCRIPTION AND VSE OF THE *SECTOR, CROSSE-STAFFE,* and other INSTRUMENTS:

VVith a Canon of Artificiall Sines and Tangents, to a *Radius* of 10000.0000. parts, and the vse thereof in *Astronomie, Navigation, Dialling,* and *Fortification, &c.*

The second Edition much augmented.

By EDM. GUNTER sometime Professor of Astonomie in *Gresham Colledge* in *London.*



LONDON,
Printed by *William Iones*, for *Iames Bowler*, and are to be sold at the *Marigold* in *Pauls* Church-yard.
1636.

HONORATISSIMO

DOMINO *Dn.* JOHANNI
COMITI de BRIDGEWATER,
VICECOMITI de BRACKLEY,
BARONI de ELLESMERE,
EQVITI ORDINIS HONORATISSIMI QVI DICITVR BALNEI, D° PRÆSIDENTI WALLIÆ LIMITVMQ. NEC NON REGIÆ MAIESTATI A SACRIS CONSILIIS. &c.

Lucubrationes has ſuas Mathematicas

D. D. D.

EDM. GUNTER.

THE CONTENTS Of the first booke of the *Sector*.

Chap. 1.

The contents of the second booke of the *Sector*

The contents of the third booke of the *Sector.*

The Contents of the first Booke of the *Crosse-Staffe.*

The Contents of the second Booke of the *Crosse-Staffe.*

In

The Contents of the third Booke of the *Crosse-Staffe*.

The Contents of the generall vse of the *Canon,* and Table of *Logarithmes.*

First correct, and then reade with practise.

Errata Sector:

PAg. 13. line 23. reade according p. 18. line last r. denomination p. 3. l. 5. r. numbers pa. 42. l. 1. r. were p. 46, li. 15. r. 7. p. 45, l. 2. r. hand p. 5. l. 8. r. arke, p. 54. l. 2. r. his p 55. l. 2. r. 15. p. 65. l. 25. r. signe p. 68. l. 18. r. found by an p. 68 l. 32 .r. generall, l. 33. r. parts p. 84. l. 27. r. tangent, p. 85. l. 20. r. finde it p. 106. l. 5. r. Mercators proiection p. 113. l. 27. r. Rumb. l. 28. r. as in, p. 120. l. 5. p. 115. l. 7. r. by that which l, 15, r. in the out p, 126, l. 4, r, fift, p, 127. l. 22, r. 50. gr. p, 128. l. last r. found to be p. 136. l. lastcolume 7. r. 90.

Errata Crossestaffe:

Pag. 2 l. 5. r. Sorts. p. 15. l 11. r. of latitude p. 18. l. 1. r. to 51. gr. 48 m. p. 27. l 9. r. and to p. 28, l. 12. r. have another p. 37. l. 3. r. sines l. 21. to the sine p. 43 l. 23. r. square p 47. l. 1, r, so in the p. 53, l. r. 19 length p. 97. l. 17. r. of position p. 107. l. 27. r. as are p. 108. l. 30. r, to the day p. 112. l. 30. r. South end in, p 133. l. 5. r: being, p: 144, l. 15, is to be p. 148, l. 12, r. 8 in the, p: 162, l: 21, r: of the p, 167, l: 1, r: here is p, 171. l. 34, r. such the arke of p. 206, l: 12, r. commonly p. 208, l: last r. of the Az, p: 233, l. 31, r. the tenth of p. 235, l. 16. r. belonging to 20. p. 245. l. 12, r. hang.

Errata to the vse of the Canon:

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The Errata by direction to the Crossestaffe for those there.

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THE FIRST BOOKE OF THE SECTOR.

CHAP. I.

The description, the making, and the generall vse of the Sector.

A *Sector* in *Geometrie*, is a figure comprehended of two right lines containing an angle at the center, and of the circumference assumed by them. This *Geometricall instrument* hauing two legs containing all variety of angles, and the distance of the feete, representing the subtenses of the circumference, is therefore called by the same name.

It containeth 12 seuerall lines or scales, of which 7 are generall, the other 5 more particular. The first is the scale of *Lines* diuided into a 100 equall parts, and numbred by 1. 2. 3. 4. 5. 6. 7. 8. 9. 10.

The second, the lines of *Superficies* diuided into 100 vnequall parts, and numbred by 1. 2. 3. 4. 5. 6. 7. 8. 9. 10.

3. The third, the lines of *Solids*, diuided into 1000 vnequall parts, and numbred by 1.1.1.2.3.4.5.6.7.8.9.10.

4. The fourth, the lines of *Sines* and *Chords*, diuided into 90 degrees, and numbred with 10. 20. 30. vnto 90.

These foure, lines of *Lines*, of *Superficies*, of *Solids*, and of *Sines*, are all drawne from the center of the *Sector* almost to the end of the legs. They are drawne on both the legs, that euery line may haue his fellow. All of them are of one length, that they may answere one to the other. And euery one hath his parallells, that the eye may the better distinguish the diuisions. But of the parallells those onely which are inward most containe the true diuisions.

There are three other generall lines, which because they are infinite are placed on the side of the *Sector*.

5. The first a line of *Tangents*, nũbred with 10. 20. 30. 40 50. 60. signifying so many degrees from the beginning of the line, of which 45. are equall to the whole line of *Sines*, the rest follow as the length of the *Sector* will beare.

6. The second, a line of *Secants*, diuided by pricks into 60 degrees, is the same with that of the line of *Tangents*, to which it is joyned.

7. The third, is the *Meridian* line, or line of *Rumbs*, diuided vnequally into degrees, of which the first 70 are almost equall to the whole line of *Sines*, the rest follow vnto 85 according to the length of the *Sector*.

Of the particular lines inserted among the generall, because there was voyd space.

8. The first are the lines of *Quadrature* placed betweene the lines of *Sines*, and noted with 10. 9. 8. 7. *S*. 6. 5. 90. *Q*.

9. The second, the lines of *Segments* placed betweene the lines of *Sines* and *Superficies*, diuided into 50 parts, and numbred with 5. 6. 7. 8. 9. 10.

10. The third, the lines of *Inscribed bodies in the same Sphere*, placed betweene the scales of *Lines*, and noted with *D.S.I.C.O.T.*

11. The fourth, the lines of *Equated bodies*, placed betwen the lines of *Lines* and *Solids*, and noted with *D.I.C.S.O.T.*

12. The fift, are the lines of *Mettalls*, inſerted with the lines of *Equated bodies* (there being roome ſufficient) and noted with theſe Characters ☉.☿♄.☽.♀.♂.♃.

There remaine the edges of the *Sector*, and on the one I haue ſet a line of *Inches*, which are the twelfth parts of a foote Engliſh: on the other a leſſer line of *Tangents*, to which the *Gnomon* is *Radius*.

2 *Of the making of the Sector.*

LEt a *Ruler* be firſt made either of braſſe or of wood, like unto the former figure, which may open and ſhut vpon his center. The head of it may be about the twelfth part of the whole length, that it may beare the moueable foote, and yet the moſt part of the diviſions may fall without it. Then let a moueable *Gnomon* be ſet at the end of the moueable foote, and there turne vpon an *Axis*, ſo as it may ſometime ſtand at a right angle with the feete, and ſometimes be incloſed within the feet. But this is well knowne to the workeman.

For drawing of the lines. Vpon the center of the *Sector*, and ſemidiameter ſomewhat ſhorter then one of the feet, draw an occult arke of a circle, croſſing the cloſure of the inward edges of the *Sector* about the letter *T*.

In this arke, at one degree on either ſide from the edge, draw right lines from the Center fitting them with *Parallells*, and deuide them into an hundred equall parts, with ſubdiuiſions into 2. 5. or 10. as the line will beare, but let the numbers ſet to them, be onely 1.2.3 4.&c. vnto 10. as in the example. Theſe lines ſo diuided, I call the lines or ſcales of *Lines*; and they are the ground of all the reſt.

In this Arke at 5 degrees on either ſide, from the edge neere *T*, draw other right lines from the Center, and fit them with Parallells. Theſe ſhall ſerue for the lines of *Silids*, Then on the other side of the *Sector* in like manner, vpon the Center & equall Semidiameter, drawe another like Arke of a circle: and here againe at one degree neere on either side from the edge neere the letter *Q* draw right lines from the center, and fit them with parallells. These shall serue for the lines of *Sines.*

At 5 Degrees on either side from the edge neere *Q* draw other right lines from the center; and fit them with parallells: these shall serue for the lines of *Superficies.*

These foure principall lines being drawne, and fitted with parallells, wee may draw other lines in the middle betweene the edges and the lines of *Lines*, which shall serue for the lines of *inscribed bodies*, and others betweene the edges and the *Sines* for the lines of *quadrature*. And so the rest as in the example.

3 *To diuide the lines of* Superficies.

SEeing the *Superficies* doe hold in the proportion of their *homologall* sides duplecated, by the 29 Pro. 6. lib. *Euclid.* If you shall find meane proportionalls between the whole side, and each hundred part of the like side, by the 13 Pro. 6 lib. *Euclid.* all of them cutting the same line, that line so cut shall containe the diuisions required. wherefore vpon the center *A* and Semidiameter equall to the line of *Lines*, describe a Semicircle *A C B D*, with *A B* perpendicular to the diameter *C D*. And let the Semidiameter *A D* be divided as the line of *Lines* into an hundred parts, & *A E* the one halfe of *A C* diuided also into an hundred parts so shall the diuisions in *A E* be the centers from whence you shall describe the semicircles *C* 10. *C* 20. *C* 30. &c. diuiding the line *A B* into an hundred vnequall parts: & this line *A B* so diuided shall be the line of *Superficies*, and must be transferred into the *Sector*. But let the numbers set to them bee onely 1. 1. 2. 3. vnto 10. as in the example.

Or these lines of *Superficies* may otherwise be transferred into the *Sector*, out of the line of *Lines*, by a table of square rootes: For the roote taken out of the line of *Lines* shall giue the square in the lines of *Superficies*.

As, to inscribe the diuision of 25 in the lines of *Superficies*; put six ciphers to 25 and make it 25000000 then finde the sq. roote of this number, which will be 5000.

Take therefore 5000, out of the line of *Lines* (supposing the whole line to be 10000) and it will giue the true distance betweene the center, and the points of 25. in the lines of *Superficies*.

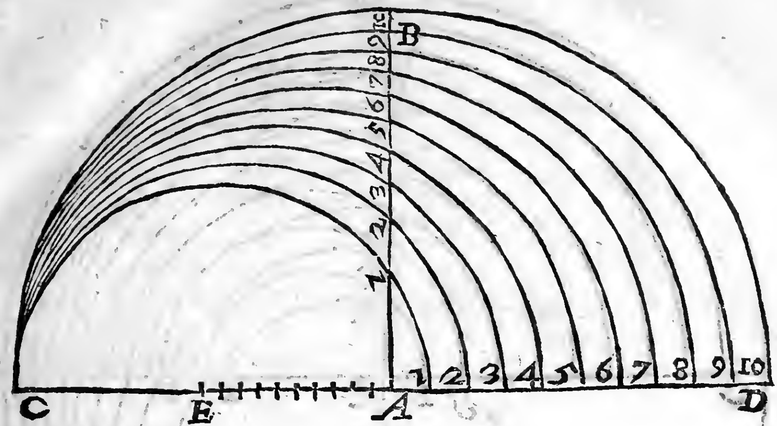
So, for the diuision of 30, put to 6 ciphers, and make it 30000000, whose sq. root is 5477. This (taken out of the line of *Lines*) shall giue the place for the points of 30, in the lines of *Superficies*. And the like reason holdeth for all the rest, according to this following Table.

If any please to make vse of a *Diagonal* Scale, equall to the line of *Lines*, he may put viij ciphers to the number proposed, and make the Table of Roots to v. places: So, his worke will be more exact.

A Table of Square Rootes for the diuision of the Lines of Superficies.

A Table of Square Roots for division of the Line of Superficies.

Sq.	Root.	Sq.	Root.	Sq.	Root.	Sq.	Root.	Sq.	Root.	Sq.	Root.	Sq.	Root.
0	—	15	3873	30	5477	45	6708	60	7746	75	8660	90	9487
	707		3937		5523		6745		7778		8689		9513
1	1000	16	4000	31	5568	46	6782	61	7810	76	8718	91	9539
	1225		4062		5612		6819		7842		8746		9565
2	1414	17	4123	32	5657	47	6856	62	7874	77	8775	92	9592
	1581		4183		5701		6892		7906		8803		9618
3	1732	18	4243	33	5744	48	6928	63	7937	78	8832	93	9644
	1871		4301		5788		6964		7969		8860		9670
4	2000	19	4359	34	5831	49	7000	64	8000	79	8888	94	9695
	2121		4416		5874		7036		8031		8916		9721
5	2236	20	4472	35	5916	50	7071	65	8062	80	8944	95	9747
	2345		4528		5958		7106		8093		8972		9772
6	2449	21	4582	36	6000	51	7141	66	8124	81	9000	96	9798
	2550		4637		6042		7176		8155		9028		9823
7	2646	22	4690	37	6083	52	7211	67	8185	82	9055	97	9849
	2739		4743		6124		7246		8216		9083		9874
8	2828	23	4796	38	6164	53	7280	68	8246	83	9110	98	9899
	2915		4848		6205		7314		8276		9138		9925
9	3000	24	4899	39	6245	54	7348	69	8307	84	9165	99	9950
	3082		4950		6285		7382		8337		9192		9975
10	3162	25	5000	40	6325	55	7416	70	8367	85	9219	100	10000
	3240		5050		6364		7450		8396		9247		
11	3317	26	5099	41	6403	56	7483	71	8426	86	9274		
	3391		5148		6442		7517		8556		9300		
12	3464	27	5196	42	6481	57	7550	72	8485	87	9327		
	3536		5244		6519		7583		8515		9354		
13	3600	28	5291	43	6557	58	7616	73	8544	88	9381		
	3674		5338		6595		7648		8573		9407		
14	3742	29	5385	44	6633	59	7681	74	8602	89	9434		
	3808		5431		6671		7714		8631		9460		
15	3873	30	5477	45	6708	60	7746	75	8660	90	9487		

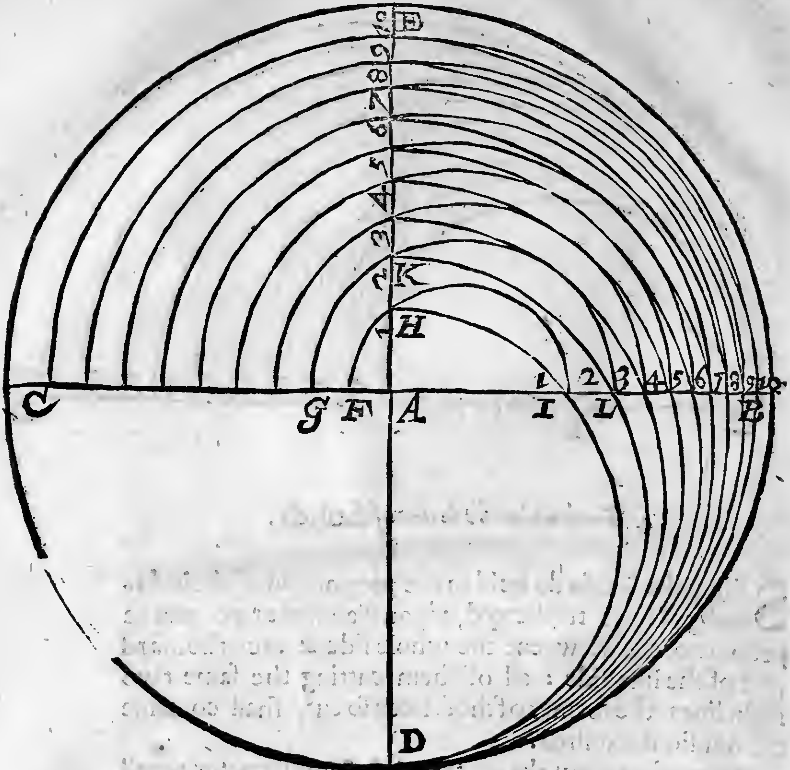


4 *To diuide the lines of* Solids.

SEing like Solids do hold in the proportion of their *homologall* sides triplicated, if you shall finde two meane proportionalls betweene the whole side & each thousand part of the like side: all of them cutting the same two right lines, the former of those lines so cut, shall containe the diuisions required.

Wherefore vpon the center *A*, & Semidiameter equall to the line of *Lines*, describe a circle and diuide it into 4 equall parts *C E B D*, drawing the crosse diameters *C B*, *E D*. Then diuide the semidiameter *A C*, first into 10 equall parts, and between the whole line *A D* & *A F* the tenth part of *A C*, seeke out two meane proportionall lines *A I* and *A H*: againe betweene *A D* and *A G* being two tenth parts of *A C*, seeke out two meane proportionalls *A L* and *A K*, and so forward in the rest. So shall the line *A B*, be diuided into 10 vnequall parts.

Secondly



Secondly, diuide each tenth part of the line *A C* into 10 more, and betweene the whole line *A D*, and each of them, seeke out two meane proportionalls as before: So shall the line *A B* be diuided now into an hundred vnequall parts.

Thirdly, If the length will beare it, subdiuide the line *A C* once againe, each part in ten more: and betweene the whole line *A D* and each subdiuision, seeke two meane

meane proportionalls as before. So ſhould the line *A B* be now diuided into 1000 parts. But the ruler being ſhort, it ſhall ſuffice, if thoſe 10 which are neareſt the center be expreſſed, the reſt be vnderſtood to be diuided, though actually they be diuided into no more then 5 or 2, and this line *A B* ſo diuided ſhall be the line of *Solids*, and muſt be transferred into the *Sector*: But let the numbers ſet to them be onely 1. 1. 1. 2. 3. &c. vnto 10. as in the example.

Or theſe lines of *Solids* may otherwiſe be transferred, into the *Sector*, out of the line of *Lines* (or rather, out of a *Diagonall* ſcale equall to the line of *Lines*) by a table of *Cubique Roots*. For the Root, taken out of the line of *Lines*, ſhall giue the cube in the lines of *Solids*.

As to inſcribe the diuiſion of 125 in the lines of *Solids*; put xij. ciphers to 125, and make it 125000000000000: Then find the cubique Root, of the number, which will be 50000. Take therefore 50000 out of the line of *Lines*, (ſuch as the whole line is 100000) and it will giue the true diſtance betweene the points of 125 in the lines of *Solids*.

So, for the diuiſion of 300, put to xij. ciphers more and make it 300000000000000, whoſe cubique Root is *66943* This, taken out of the line of *Lines*, ſhall giue the place for the points of 300 in the lines of *Solids*. And the like reaſon holdeth for all the reſt, according to the enſuing Table.

A Table of Cubique Rootes.

A Table of Cubique Rootes

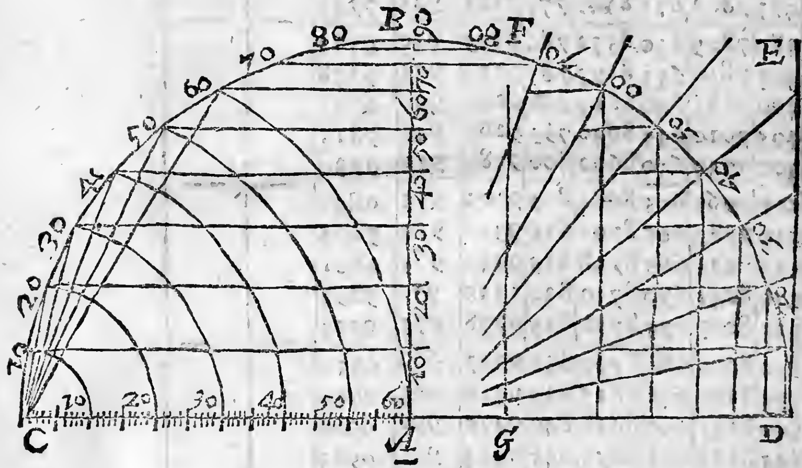
Cube.	Root.	Cube.	Root.	Cube.	Root.	Cube.	Root.	Cube	Root.
0	0	20	2714	50	3684	125	5000	275	6502
	794	21	2758	52	3732	130	5065	280	6542
1	1000	22	2802	54	3779	135	5129	285	6580
	1144	23	2843	56	3825	140	5192	290	6619
2	1259	24	2884	58	3870	145	5253	295	6656
	1357	25	2924	60	3914	150	5313	300	6694
3	14[illegible]2	26	2962	62	3957	155	5371	305	6731
	1518	27	3000	64	4000	160	5428	310	6767
4	1587	28	3036	66	4041	165	5484	315	6804
	1650	29	3072	68	4081	170	5539	320	6839
5	1709	30	3107	70	4121	175	5593	325	6875
	1765	31	3141	72	4160	180	5646	330	6910
6	1817	32	3174	74	4198	185	5698	335	6945
	1866	33	3207	76	4235	190	5748	340	9679
7	1912	34	3239	78	4272	195	5798	345	0713
	1957	35	3271	80	4308	200	5848	350	7047
8	2000	36	3301	82	4344	205	5895	355	7080
	2040	37	3332	84	4379	210	5943	360	7119
9	2080	38	3361	86	4414	115	5990	365	7146
	2117	39	3391	88	4447	220	6036	370	7179
10	2154	40	3419	90	4481	225	6089	375	7211
11	2223	41	3448	92	4514	230	6126	380	7243
12	2289	42	3476	94	4546	235	6171	385	7274
13	2351	43	3503	96	4578	240	6214	390	7306
14	2410	44	3530	98	4610	245	6257	395	7337
15	2466	45	3556	100	4641	250	6299	400	7368
16	2519	46	3583	105	4717	255	6341	405	7398
17	2571	47	3608	110	4791	360	6382	410	7428
18	2620	48	3634	115	4862	265	6423	415	7429
19	2668	49	3659	120	4931	270	6463	420	7488
20	2714	50	3684	125	5000	275	6502	425	7518

For division of the Lines of Solids.

Cube.	Root.	Cube.	Root.	Cube.	Root.	Cube.	Root.
425	7518	575	8315	725	8983	875	9564
430	7547	580	8339	730	9004	880	9582
435	7576	585	8363	735	9024	885	9600
440	7605	590	8387	740	9045	890	9619
445	7634	595	8410	745	9065	895	9636
450	7663	600	8434	750	9085	900	9654
455	7691	605	8457	755	9105	905	9672
460	7719	610	8480	760	9125	910	9690
465	7747	615	8504	765	9145	915	9708
470	7774	620	8527	770	9165	920	9725
475	7802	625	8549	775	9185	925	9743
480	7829	630	8572	780	9205	930	9761
485	7856	635	8595	785	9224	935	9778
490	7883	640	8617	790	9244	940	9795
495	7910	645	8640	795	9263	945	9813
500	7937	650	8662	800	9283	950	9830
505	7963	655	8684	805	9302	955	9847
510	7989	660	8706	810	9321	960	9864
515	8015	665	8728	815	9340	965	9881
520	8041	670	8750	820	5359	970	9898
525	8067	675	8772	825	9378	975	9915
530	8092	680	8793	830	9397	980	9932
535	8118	685	8815	835	9416	985	9949
540	8143	690	8836	840	9435	990	9966
545	8168	695	8857	845	9454	995	9983
550	8193	700	8879	850	9472	1000	10000
555	8217	705	8900	855	9491		
560	8242	710	8921	860	9509		
565	8267	715	8942	865	9529		
570	8291	720	8962	870	9546		
575	8315	725	8983	857	9564		

5 *To diuide the lines of* Sines *and* Tangents *on the ſide of the Sector.*

VPon the center *A*, and ſemidiameter equall to the line of *Lines*, deſcribe a ſemicircle *A B C D*, with *A B*, perpendicular to the diameter *C D*. Then diuide the quadrants *C B*, *B D*, each of them into 90. and ſubdiuide each degree into 2 parts. For ſo, if ſtreight lines be drawne parallell to the diameter *C D*, through theſe 90, and their ſubdiuiſions they ſhall diuide the perpendicular *A B* vnequally into 90.



And this line A B ſo diuided ſhall be the line of *Sines*, and muſt be transferred into the *Sector*. The number ſet to them are to be 10. 20. 30. &c. vnto 90 as in the example.

If now in the point *D*, vnto the diameter *C D*, we ſhall raiſe a perpendicular *D E*. and to it drawe ſtreight lines from the center *A*, through each degree of the quadrant

drant D B these streight lines shalbe secants, and this perpendicular so diuided by them shall be the line of *Tangents*, & must be transferred vnto the side of the *Sector*. The number set to them, are to be 10. 20. 30. &c. as in the example.

If betweene A and D, another streight line *G F*, be drawne parallell to D E, it will be diuided by those lines from the center in like sort as D E is diuided, and it may serue for a lesser line of *Tangents*, to be set on the edge of the *Sector*.

If the compasses shall be extended, from *C*, to each degree of the *Quadrant*, *C B*, and those extents transferred into one line (*C A*) this line *C A* so diuided into 60 (or rather, into 90. *gr*) shall be a line of *Chords*, and may be set on some voyd place of the *Sector*.

These lines of *Sines* and *Tangents*, may yet otherwise be transferred into the *Sector* out of the line of *Lines*, (or rather out of a diagonall Scale equall to the line of *Lines*,) by tables of Sines and Tangents.

If the *Sine* of 90 gr. being equall to the whole *Line* of *lines* of 100000. parts, the *Sine* of 30 gr. will be equall to 50000 (halfe the *Line* of *lines*;) and the *Sine* of 45. gr. equall to 70710 parts of the *line* of *lines*, accord to the vsuall table of *Sines*.

In like manner the Tangent of 45 g. being equall to the whole *Line* of *lines*, the *tang*. of 40 deg. will be equall to 83910 parts of the *Line* of *lines*: and the *tang*. of 50 degr. equall to 119175; that is, to one *Radius* (or whole *Line*) and 19175 parts more of the same line of *lines*, according to the old table of *Tangents*.

And (vpon the same ground) the *Secant* of 40 gr. will be equall to 130540, that is, one *Radius*, and 30540. parts of the *Line* of *lines*: and the *Secant* of 50 degr. equall to 155572, and so the rest, according to the like Table of *Secants*.

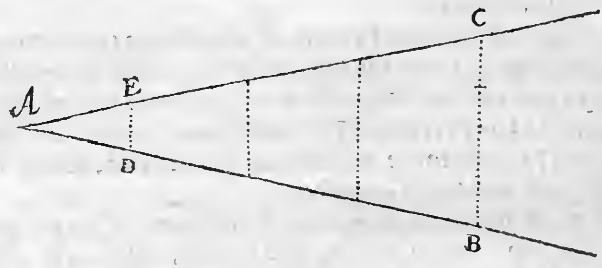
The *Line* of *Chords* may also be divided by help of the Table of *Sines*, and *line* of *lines*. For the double *sine* of halfe the ark, taken out of the *line* of *lines*, will giue the *chord*.

As, if the *Ark* proposed were 60 gr. The halfe of this Ark is 30. degr. and the *sine* thereof 50000, which being doubled make 100000, the whole *line* of *lines*, equall to a *chord* of 60 degr.

So, for the *chord* of 60 degr. The halfe *ark* is 45 degres, and the *sine* thereof 70710. which being doubled, make 1414240. (that is,) one *Radius* and 41420 parts of the *line of lines*, equall to the *chord* of 90 gr. required.

6 To shew the ground of the Sector.

LEt A B, A C, represent the leggs of the *Sector*: then seing these two A B, A C, are equall, and their sections A D, A E, also equall, they shall be cut proportionally: and if we draw the lines B C, D E, they will be parallell by the second Pro. 6 lib. of *Euclid*, and so the Triangles A B C. A D E, shalbe equiangle; by reason of the common angle at A, and the equall angles at the base, and therefore shall haue the sides proportionall about those equall angles, by the 4 Pro. 6 lib. of *Euclid*.



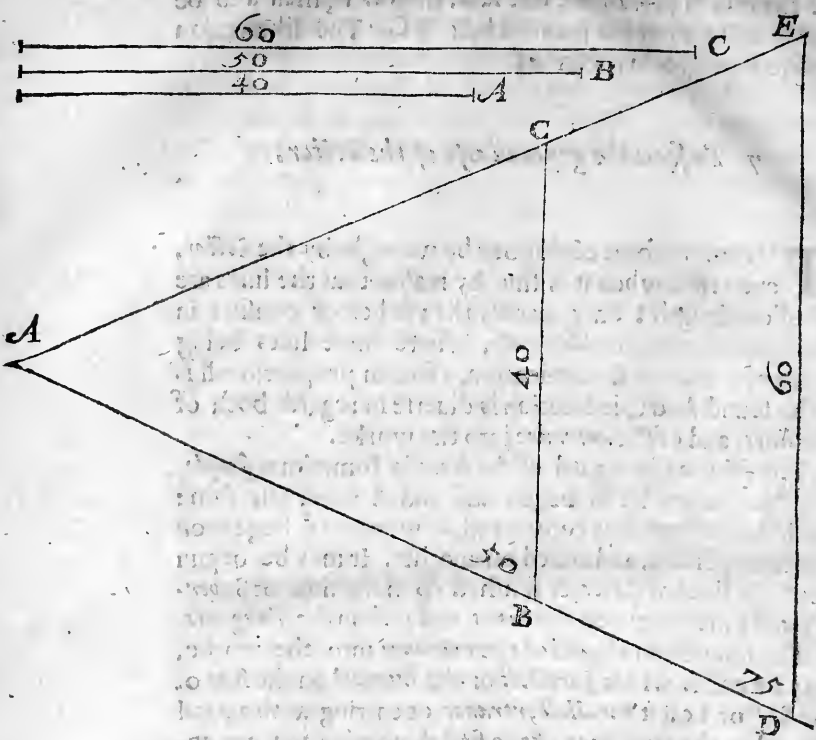
The side A D, shalbe to the side A B, as the basis D E, vnto the parallell basis BC, and by conuersion A B, shall be vnto A D, as B C, vnto D E: and by permutation A D, shall be vnto D E, as A B, to B C. &c. So that if A D, be be the fourth part of the ſide A B, then D E, ſhall alſo be the fourth part of his parallell baſis B C. The like reaſon holdeth in all other ſections.

7 *To ſhew the generall vſe of the Sector.*

THere may ſome cōcluſions be wrought by the *Sector*, euen then when it is ſhut, by reaſon that the lines are all of one length: but generally the vſe hereof conſiſts in the ſolution of the *Golden rule*, where three lines being giuen of a known denomination, a fourth proportionall is to be found. And this ſolution is diuerſe in regard both of the *lines*, and of the *entrance* into the worke.

The ſolution in regard of the *lines* is ſometimes *ſimple*, as when the worke is begun and ended vpon the ſame *lines*. Sometimes it is compound, as when it is begun on one kind of lines, and ended on another. It may be begun vpon the lines of *Lines*; & finiſhed vpon the lines of *Superficies*. It may begin on the *Sines*, and end on the *Tangents*.

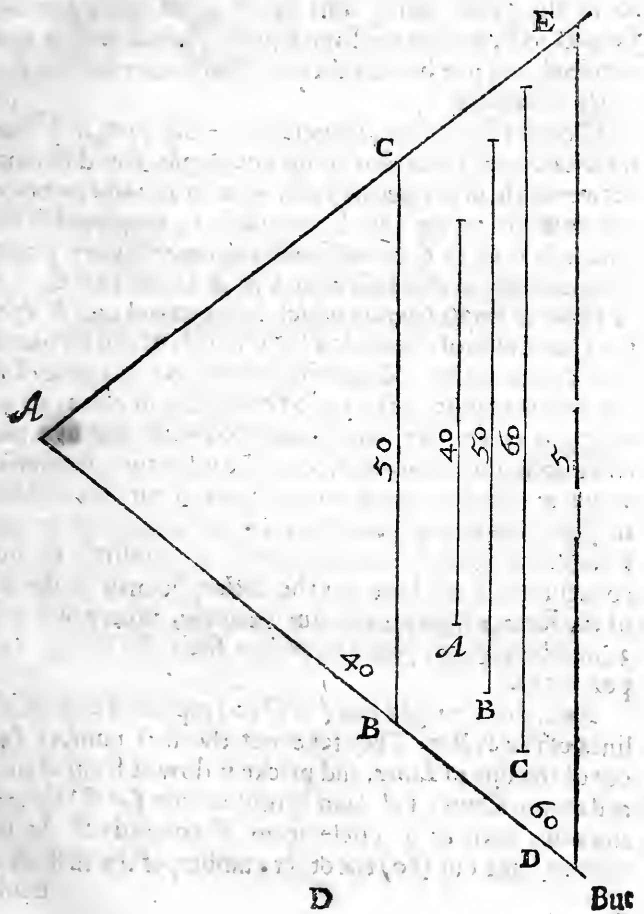
The ſolution in regard of the *entrance* into the worke, may be either with a *parallell* or elſe *laterall* on the ſide of the Sector, I cal it *parallell entrance*, or entring with a parallell, when the two lines of the firſt denomination are applied in the parallells, and the third line, and that which is ſought for, are on the ſide of the *Sector*. I call it *laterall entrance*, or entring on the ſide of the *Sector*, when the two lines of the firſt denomination are on the ſide of the *Sector*, and the third line and that which is to be found out, doe ſtand in the parallells.



As for example, let there be giuen three lines *A,B,C*, to which I am to find a fourth proportionall. let A, measured in the line of *lines*, be 40, B 50, and C 60, and suppose the question be this. If 40 *Monthes* giue 50 *pounds*, what shall 60? Here are lines of two denominatiõs, one of *months* another of *pounds*, and the first with which I am to enter must be that of 40 *monthes*. If then I would *enter with a parallell*, first I take A, the line of 40, and put it ouer as a *parallell* in 50, reckoned in the line of *lines*, on either side of the *Sector* from the center, so as it may be the Base of an *Isoscheles* triangle B A C, whose side A B, A C are equal to B, the line of the second denomination.

Then

Then the *Sector* being thus opened, I take C the line of 60, betweene the feete of the compasses, and carrying them *parallell* to B C, I finde them to crosse the lines A B, A C, on the side of the *Sector* in D and E, numbred with 75, wherefore I conclude the line A D, or A E, is the fourth proportionall and the correspondent number 75 which was required.



But

But if I would *enter on the side* of the *Sector*, then wou'd I dispose the lines of the first denomination A and C, in the line of *Lines*, on both sides of the *Sector*, in A B, A C, & in A D, A E, so as they should all meete in the center A, and then taking B the line of the second denomnation put it ouer as a *parallell* in B C, that it may be the Basis of the Isoscheles triangle B A C, (whose sides A B, A C, are equall to A, the first line of the first denomination,) for so the *Sector* being thus opened; the other *parallell* from D to E, shall be the fourth proportionall which was required, and if it be measured with the other lines, it shal be 75, as before.

In both this manner of operations, the two first lines do serue to opē the *Sector* to his due angle, the difference betweene them is especially this that in *parallell entrance*, the two lines of the first denomination, are placed in the parallells B C, D E, & in *laterall entrance* they are placed on both sides of the *Sector*, in A B, A D and in A C, A E.

Now in *simple solution* which is begun and ended, vpon the same kinde of lines, it is all one which of the two latter lines be put in the secōd or third places. As in our exāple we may say, as 40 are to 50, so 60 vnto 75, or else as 40 are to 60, so 50 vnto 75. And hence it cometh that we may enter both with a *parallell*, & on the sides two manner of wayes at either entrance, and so the most part of questions may be wrought 4 seuerall wayes, though in the propositions following, I mention onely that which is most conuenient. If any haue not the *Sector*, he may make vse of the former figure, as in our example, where we haue 3 numbers giuen (40. 50. 60.) to finde the fourth Proportionall.

First, draw a right line (*A D*) to represent one of the lines of the *Sector*. Then take out the first number (40) out of the line of *Lines*, and pricke it downe from *A* to *B*; and on the Center (*A*,) and Semidiameter (*A B*) describe an occult arke of a circle from *B* towards *C*. In like manner, take out (60) the other number, of the first denominion

minion) and pricke it downe from A to D. And on the center (A,) and Semidiameter (A D) describe a second arke of a circle, from D toward E. That done, take the third number (50) and inscribe it into the first arke from B to C; and laying the ruler to the center (A) and the point C, draw the right line (A C,) out in length, till it cutt the second arch in the point E. So the distance from D to E (taken and measured in the same scale with the third number) will giue the 75 for the fourth proportionall.

Thus much for the generall vse of the *Sector*, which being considered and well understood, there is nothing hard in that which followeth.

CHAP. II.

The vse of the Scale of Lines

1 *To set downe a Line, resembling any giuen parts or fraction of parts.*

THe lines of *Lines* are diuided actually into 100 parts, but we haue put onely 10 numbers in them. These we would haue to signifie either themselues alone, or ten times themselues, or an hundred times themselues, or a thousand times themselues, as the matter shall require. As if the numbers giuen be no more then 10, then we may thinke the lines onely diuided into 10 parts according to the number set to them. If they be more then 10, and not more then 100, then either line shall containe 100 parts, and the numbers set by them shall be in value 10. 20. 30. &c. as they are diuided actvally. If yet they be more then 100, then euery part must be thought to be diuided into 10, and either line shall be 1000 parts, and the numbers set to them shall be in value 100. 200. 300, and so forward still increasing themselues by 10.

 This

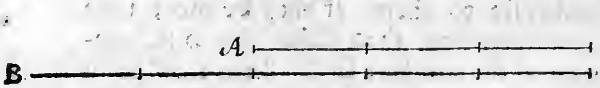
This being preſuppoſed, we may number the parts and fraction of parts giuen in the line of *lines*; and taking out the diſtance with a paire of compaſſes, ſet it by, for the line ſo taken ſhall reſemble the number giuen.

In this manner may we ſet downe a line reſembling 75, if either we take 75 out of the hundred parts, into which one of the line of *lines* is actually diuided, and note it in A, or $7\frac{1}{2}$ of the firſt 10 parts, and note it in B, or onely $\frac{3}{4}$ of one of thoſe hundred parts, and note it in C. Or if this be either to great or to ſmall, we may run a Scale at pleaſure, by opening the compaſſe to ſome ſmall diſtance, and running it ten times ouer, then opening the compaſſe to theſe ten, run them ouer nine times more, & ſet figures to them as in this example, and out of this we may take what parts we will as before.

To this end I haue diuided the line of inches on the edge of the *Sector*, ſo as one inch containeth 8 parts, another 9, another 10, &c. according as they are figured, and as they are diſtant from the other end of the *Sector*, that ſo we might haue the better eſtimate.

2 *To encreaſe a line in a giuen proportion.*
3 *To diminiſh a line in a giuen proportion.*

TAke the line giuen with a paire of compaſſes, and open the *Sector*, ſo as the feete of the compaſſes may ſtand in the points of the number giuen, then keeping the *Sector* at this angle, the parallell diſtance of the points of the number required, ſhall giue the line required.



Let *A* be a line giuen to be increaſed in the proportion of 3 to 5. Firſt I take the line *A*, with the compaſſes, and open the *Sector* till I may put it ouer in the poynts of of 3 and 3, so the parallell betweene the poynts of 5 & 5, doth giue me the line *B*, which was required.

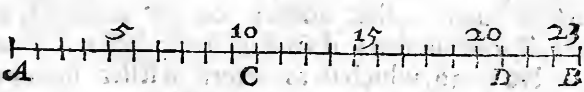
In like manner, if *B*, be a line giuen to be diminished in the proportiō of 5 to 3, I take the line *B* & to it open the *Sector* in the poynts of 5, so the parallell betweene the poynts of 3, doth giue me the line *A* which was required.

If this manner of worke doth not suffice, we may multiplie or diuide the numbers giuen by 2, or 3, or 4. &c. And so worke by their numbers *equimultiplices*, as for 3 and 5, we may open the *Sector* in 6 and 10, or else in 9 and 15, or else in 12 and 20, or in 15 and 25, or in 18 and 30. &c.

4 *To diuide a line into parts giuen.*

TAke the line giuen, and open the *Sector* according to the length of the said line in the points of the parts, whereinto the line should be diuided, then keeping the *Sector* at this angle. the parallell distance betweene the points of 1 and 1 shall diuide the line giuen into the parts required.

Let AB, be the line given to be diuided into fiue parts, first I take this line AB, and to it open the *Sector* in the point of 5 and 5, so the parallell betweene the points of 1 and 1, doth giue me the line AC, which doth diuide it into the parts required.



Or let the like line AB, be to be diuided into twenty three parts. First I take out the line and put it vpon the

Sector in the points of 23, then may I by the former propoſition diminiſh it in A C, C D, in the proportion of 23, to 10, and after that diuide the line A C into 10, &c, As before.

5 *To finde a proportion betweene two or more right lines giuen.*

TAke the greater line giuen, and according to it open the *Sector* in the points of 100 and 100, then take the leſſer lines ſeuerally, & carry them parallell to the greater, till they ſtay in like points, ſo the number of points wherein they ſtay, ſhall ſhew their proportion vnto 100.

Let the lines giuen be *AB*, *CD*, firſt I take the line *CD*, and to it open the *Sector* in the points of 100, and 100, then keeping the *Sector* at this angle, I enter the leſſer line *AB*, parallell to the former, and finde it to croſſe the lines of *Lines* in the points of 60. Wherefore the proportion of *AB* to *CD*, is as 60 to 100.

Or if the line *CD*, be greater then can be put ouer in the poynts of 100, then I admit the leſſer line *AB*, to be 100, and cutting of *CE* equall to *AB*, I finde the proportion of *CE*, vnto *ED* to be as 100, almoſt to 67; wherefore this way ỹ proportiō of *AB* vnto *CD*, is as 100 vnto almoſt 167.

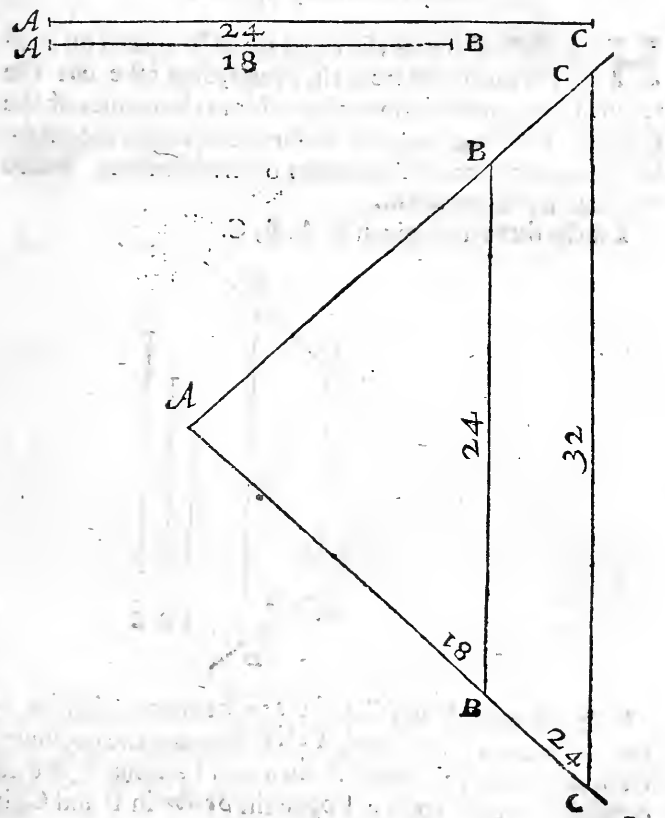
this propoſition may alſo not vnfitly be wrought by any other number, that admits ſeuerall diuiſions, and namely, by the numbers of 60. And ſo the leſſer line will be found to be 36, which is as before in leſſer numbers, as 3 vnto 5. It may alſo be wrought without opening the *Sector*. For if the lines betweene which we ſeek a proportion, be applyed to the lines of *Lines*, (or any other Scale of equall parts) there will be ſuch proportion found between

them

them, as betweene the lines to which they are equall.

6 *Two lines being giuen to finde a third in continuall proportion.*

First place both the lines giuen, on both sides of the *Sector* from the Center, and marke the termes of of their extension, then take out the second line againe, and to it open the *Sector*, in the termes of the first line, so keeping the *Sector* at this angle, the parallell distance betweene the termes of the second line, shall be the third proportionall.



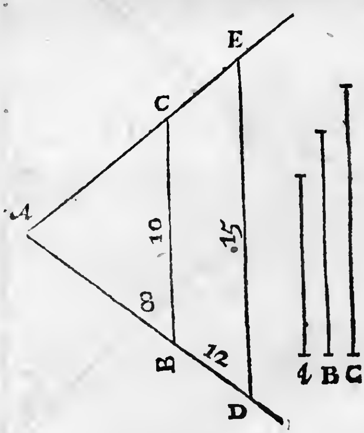
Let

Let the two lines giuen be A B, A C, which I take out and place on both sides of the *Sector*, so as they all meete in the center A, let the termes of the first line be B and B, the termes of the second C and C. Then doe I take out AC the second line againe, and to it open the *Sector* in the termes B B So the parallell betweene C and C doth giue me the third line in continuall proportion. For as *A B* is vnto *A C*, so B B, equall to *AC*, is vnto *CC*.

7 *Three lines being giuen to finde the fourth in discontinuall proportion.*

HEre the first line & the third are to be placed on both sides of the *Sector* from the center, then take out the seccond line, and to it open the *Sector* in the termes of the first line. For so keeping the *Sector* at this angle, the parallell distance betweene the termes of the third line, shalbe the fourth proportionall.

Let the three lines giuen be A, B, C.



First I take out A and C, and place them on both sides of the *Sector*, in A B, A C, and A D, A E, laying the beginning of both lines at the center A, then do I take out B the second line, according to it I open the *Sector* in B and C, the

termes

termes of the first line: so the parallell betweene D and E, doth giue me the fourth proportionall which was required.

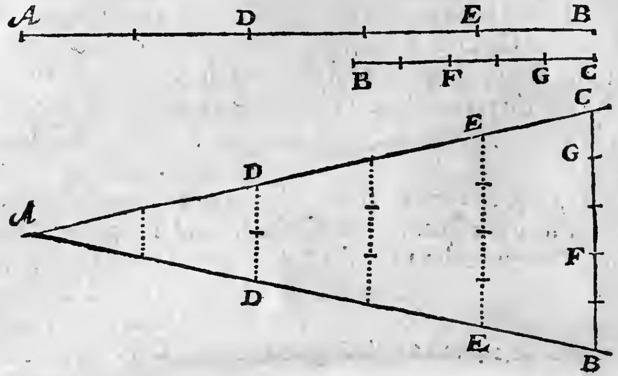
As in *Arithmetique*, it sufficeth if the first and third number giuen be of one denomination, the second & the fourth which is required be of another. For one and the same denomination is not required necessarily in them all. So in *Geometrie*, it sufficeth if the sides A B, A D, resembling the first and third lines giuen be measured in one Scale, and the parallells B C, D E be measured in another. Wherefore knowing the proportion of A the first line, and C the third line, by the fift *prop.* before. Which is here as 8 to 12, & descẽding in lesser nũbers is as 4 to 6, or as 2 to 3, or ascending into greater numbers, as 16 vnto 24 or 18 to 27, or 20 to 30, or 30 to 45, or 40 to 60 &c. If the *Sector* be opened in the points of 8 and 8, to the quantity of *B*, the second line giuen, then a parallell betweeene 12 and 12, shall giue D E, the fourth line required. So likewise if it be opened in 4 and 4, then a parallell betweene 6 and 6, or if in 16 and 16, then a parallell betweene 24 and 24 shall giue the same D E. And so in the rest.

8 *To deuide a line in such sort as another line is before diuided.*

First take out the line giuen, which is already diuided, and laying it on both sids of the *Sector* from the center; mark how farre it extendeth. Then take out the second line which is to be diuided, and to it open the *Sector* in the termes of the first line. This done, take out the parts of the first line, and place them also on the same side of the *Sector* from the center. For the parallells taken in the termes of these parts, shal be the correspondent parts in the line which is to be diuided

Let *A B*, be a line diuided in *D* and *E*, and *B C*, the line which I am to diuide in such sort, as *A B* is diuided.

First I take out the line *A B*, and place it on the line of *Lines* in *A B*, *A C*, both from the center *A*, then take I out the second *B C*, and to it open the *Sector* in *B* and *C*, the termes of the first line. The *Sector* thus opened to his due angle, I take out *A D* and *A E*, the parts of the first line *A B*, and place them also on both the sides of the *Sector* *A D*, *A E*, so the parallell *D D*, giueth me *B F*, and the parallell *E*, *E*, giueth me B *G*, and now the line B *C*, is diuided in F & G as is the other line *A* B, in D and *E*, which is that which was



required

If the line *A* B, were longer then one of the sides of the Ruler, then should I finde what proportion it hath to his parts *A* D, *A E*, and that knowne I may worke as before in the former proposition.

9 *Two numbers being giuen to finde a third in continuall proportion.*

First reckon the two numbers giuen on both sides of the lines of *Lines* from the center, and marke the termes to which either of them extendeth, then take out a line resembling the second number againe, and to it open the *Sector* in the termes of the first number, for so keeping the *Sector* at this angle, the parallell distance betweene the termes of the second laterall number, being measured in the same Scale

Scale, from whence his parallell was taken, ſhall giue the third number proportionall.

Let the two numbers giuen be 18, 24, theſe being reſembled in lines, the worke will be in a manner all one, with that in the ſixt *Prop.* and ſo the third proportionall number will be found to be 32.

10. *Three numbers being giuen to find a fourth in diſcontinuall proportion.*

THe ſolution of this propoſition, is in a manner all one with that before in the ſeuenth *Prop.* onely there may be ſome difficulty in placing of the numbers. To avoyd this, we muſt remember that three numbers being giuen, the queſtion is annexed but to one, and this muſt allwayes be placed in the third place, that which agrees with this third number in denomination, ſhalbe the firſt number, and that which remaineth the ſecond number. This being conſidered, reckon the firſt, and third numbers, which are of the firſt denomination on both ſides of the lines of *Lines* from the center, and marke the termes to which either of them extendeth, then take out a line reſembling the ſecond number, and to it open the *Sector* in the termes of the firſt number, for ſo keeping the *Sector* at this angle, the parallell diſtance btweene the termes of the third laterall number, being meaſured in the ſame Scale from whence his parallell was taken, ſhall giue the fourth number proportionall.

As if a queſtion were propoſed in this manner 10 yards coſt 8 li, how many yards may we buy for 12 li? heere the queſtion is annexed to 12; and therefore it ſhall be the third number, and becauſe 8 is of the ſame denomination, it ſhall be the firſt number, then 10 remaining, it muſt be the ſecond number, ſo will they ſtand in this order, 8, 10, 12. Theſe being reſembled in lines, the worke will be in a manner the ſame, with that in the ſeventh *Prop.* and the fourth proportionall number will be found to be 15. For as 8 are to 10, ſo 12 unto 15.

And this holdeth in direct proportion, where, as the first number is to the second, so the third to the fourth. So that if the third number be greater then the first, the fourth will be greater then the second, or if the third number be lesse then the first, the fourth will be lesse then the second; but in *reciprocall* proportion, commonly called the *Backe rule*, where by how much the first number is greater then the third, so much the second will be lesse then the fourth, or by how much the first number is lesse then the third, so much the second will be greater then the fourth. The manner of working must be contrary, that is; the *Sector* is to be opened in the termes of the third number, and the parallell resembling the number required, is to be found betweene the termes of the first number, the rest may be obserued as before, as for example.

If twelue men would raise a frame in ten dayes, in how many dayes would eight men raise the same frame? Here, because the fewer men would require longer time, though the numbers be 12, 10, 8, *yet the fourth proportionall will be found to be* 15.

So if 60 *yards, of three quarters of a yard in bredth, would hang round about a roome, & it were required to know how many yards of halfe a yard in bredth, would serue for the same roome. The fourth proportionall would be found to be* 90.

So if to make a foote superficiall, 12 *inches in bredth doe require* 12 *inches in length, & the bredth being* 16 *inches, it were required to know the length. Here, because the more breadth, the lesse length, the fourth proportionall will be found to be* 9.

So if to make a Solid foote, a base of 144 *inches require* 12 *inches in hight, and a base giuen being* 216 *inches, it were required to know how many inches it shall haue in hight. The fourth proportionall would be found to be* 8.

This last proposition of findingt a fourth proportionall number

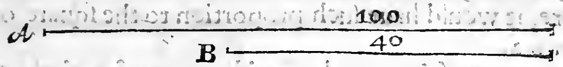
number, may be wrought alſo by the lines of *Superficies*, and by the lines of *Solids*

CHAP. III.

The vſe of the lines of Superficies.

1 *To finde a proportion betweene two or more like* Superficies.

TAke one of the ſides of the greater *Superficies* giuen, and according to it open the *Sector* in the points of 100 and 100, in the lines of *Superficies*, then take the like ſides of the leſſer *Superficies* ſeuerally, and carry them parallell to the former, till they ſtay in like points, ſo the number of points, wherein they ſtay, ſhall ſhew their proportion vnto 100.

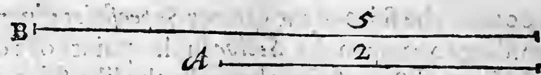


Let *A* and B, be the ſides of like *Superficies*, as the ſides of two ſquares, or the diameters of two circles, firſt I take the ſide *A*, and to it open the *Sector* in the points of 100, then keeping the *Sector* to this angle, I enter the leſſer ſide B, parallell to the former, and finde it to croſſe the lines of *Superficies* in the points of 40, wherefore the proportion of the *Superficies*, whoſe ſide is *A*, to that whoſe ſide is B, is as 100 vnto 40, which is in leſſer number, as 5 vnto 2.

This propoſition might haue beene wrought by 60, or any other number that admits ſeuerall diuiſions. It may alſo be wrought without opening the *Sector*, for if the ſides of the *Superficies* giuen, be applied to the lines of *Superficies* beginning alwayes at the center of the *Sector*, there will be ſuch proportion found betweene them, as betweene the number of parts whereon they fall.

2 To augment a Superficies in a giuen Proportion.
3 To diminish a Superficies in a giuen Proportion.

TAke the side of the *Superficies*, and to it open the *Sector* in the points of the numbers giuen; then keeping the *Sector* at that angle, the parallell distance between the points of the number required, shall giue the like side of the *Superficies* required.



Let *A* be the side of a Square to be augmented in the proportion of 2 to 5. First I take the side *A*, and put it ouer in the lines of *Superficies*, in 2 and 2; so the parallell between 5 and 5, doth giue me the side *B*, on which if I should make a Square, it would haue such proportion to the square of *A*, as 5 vnto 2.

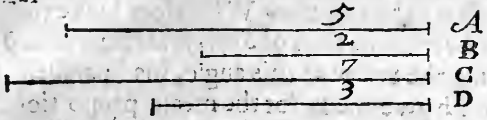
In like manner if *B* were the semidiameter of a circle to be diminished in the proportion of 5 unto 2, I would take out *B*, and put it ouer in the lines of *Superficies*, in 5 and 5; so the parallell betweene 2 and 2 would giue me *A*; on which Semidiameter if I should make a circle, it would be lesse then the circle made upon the Semidiameter *B*, in such proportion as 2 is lesse then 5.

For varietie of worke the like caution may be here obserued to that which we gaue in the third *Proportion of Lines*.

4 To adde one like Superficies to another.
5 To subtract one like Superficies from another.

FIrst, the proportion betweene like sides of the *Superfisies* giuen, is to be found by the first *Prop.* of *Superficies*, then adde or subtract the numbers of those proportions, and

and accordingly augment or diminiſh by the former *Propoſition.*

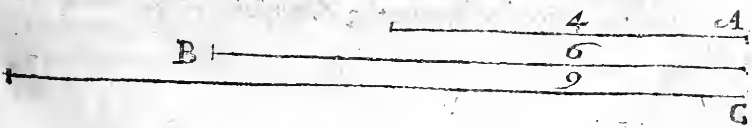


As if *A* and *B* were the ſide of two Squares, and it were required to make a third Square equall to them both. Firſt the proportion betweene the Squares of *A* and *B*, would be found to be as 100 unto 40, or in the leſſer numbers as 5 to 2; then becauſe 5 and 2 added doe make 7, I augment the ſide *A* in the proportion of 5 to 7, and produce the ſide *C*, on which if I make a ſquare, it will be equall to both the ſquares of *A* and *B*, which was required.

In like manner *A* and *B* being the ſides of two Squares, if it were required to ſubtract the ſquare of *B* out of the ſquare of *A*, and to make a ſquare equall to the remainder, here the proportion being as 5 to 2, becauſe 2 taken out of 5, the remainder is 3, I would diminiſh the ſide *A* in the proportion of 5 to 3, and ſo I ſhould produce the ſide *D*, on which if I make a ſquare, it will be equal to the remainder when the ſquare of B is taken out of the ſquare of *A*, that is, the two ſquares made vpon *B* & *D*, ſhall be equall to the firſt ſquare made vpon the ſide *A*.

6 *To finde a meane proportionall betweene two lines given.*

First find what proportion is betweene the lines giuen, as they are lines, by the fifth *Prop.* of *Lines*, then open the *Sector* in the lines of *Superficies*, according to his number, to the quantitie of the one, and a parallell taken betweene the points of the number belonging to the other line ſhall be the meane proportionall.



Let the lines given be *A* and C. The proportion betweene them as they are lines will be found by the fifth *propofit.* of *lines* to be as 4 to 9. Wherefore I take the line C, and put it over to the lines of *Superficies* betweene 9 and 9, and keeping the *Sector* at this angle, his parallell between 4 and 4 doth give me B for the meane proportionall. Then for proofe of the operation I may take this line B, and put it over betweene 9 and 9: so his parallell betweene 4 & 4, shall give me the first line A. Whereby it is plaine that these three lines doe hold in continuall proportion; and therefore B is a meane proportionall betweeene A and C the extremes given.

Vpon the finding out of this meane proportion depend many Corollaries, as

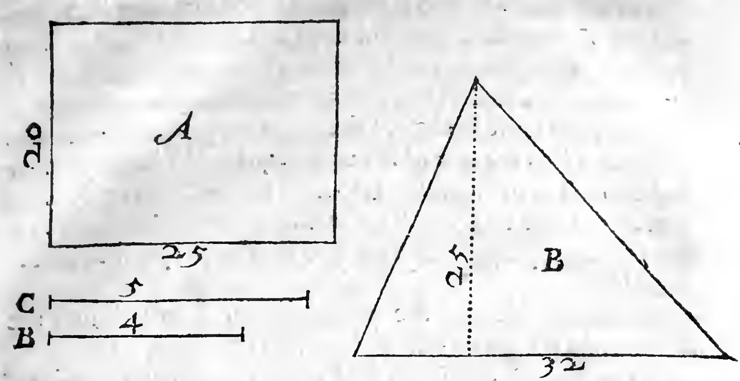
to make a Square equall to a Superficies giuen.

IF the *Superficies* giuen be a rectangle porallellogram, a meane proportionall betweene the two vnequall sides shall be the side of his equall square.

If it shall be a triangle, a meane proportion betweene the perpendicular and halfe the base shall be the side of his equall square. If it shall be any other right-lined figure, it may be resolued into triangles, and so a side of a square found equall to euery triangle; and these being reduced into one equall square, it shall be equall to the whole right-lined figure giuen.

To finde a proportion betweene Superficies, though they be unlike one to the other.

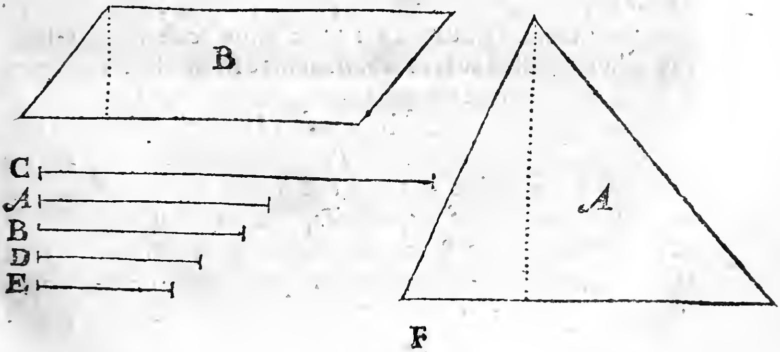
IF to every *Superficies* we find the side of his equall square, the proportion betweene these squares, shall be the proportion betweene the *Superficies* given.



Let the *Superficies* given, be the oblonge *A*, and the triangle *B*. First between the vnequall sides of *A*, I finde a meane proportionall, and note it in *C*: this is the side of a square equall unto *A*. Then betweene the perpendicular of B, and halfe his base, I finde a meane proportionall, and note it in B: this is the side of a Square equall to *B*: but the proportion between the squares of *C* and B, will be found by the first *Prop.* of *Superficies* to be as 5 to 4: and therefore this is the proportion between those given *Superficies.*

To make a Superficies like to one Superficies and equall to another.

LEt the one *Superficies* given be the *triangle* A, and the other the *Rhomboides* B; and let it be required to make an-



other *Rhomboides* like to B, and equall to the *triangle* A.

First betweene the perpendicular and the base of *B*, I find a meane proportionall, and note it in B, as the side of his equall square: then betweene the perpendicular of the triangle *A*, and halfe his base, I find a meane proportionall, and note it in *A*, as the side of his equall square. Wherefore now as the side B is to the side *A*, so shall the sides of the Rhomboides giuen be to *C* and *D*, the sides of the Rhomboides required, & his perpendicular also to *E*, the perpendicular required.

Hauing the sides and the perpendicular, I may frame the *Rhomboides* up, and it will be equall to the triangle *A*.

If the *Superficies* giuen had been any other right-lined figures, they might haue been resolved into triangles, and then brought into squares as before.

Many such Corollaries might haue been annexed, but the meanes of finding a meane proportionall being knowne, they all follow of themselues.

7 *To finde a meane proportionall betweene two numbers giuen.*

First reckon the two numbers giuen on both sides of the Lines of *Superficies*, from the center, and mark the termes whereunto they extend; then take a line out of the Line of *Lines*, or any other scale of equall parts resembling one of those numbers giuen, and put it ouer in the termes of his like number in the lines of *Superficies*; for so keeping the *Sector* at this angle, the parallell taken from the termes of the other number and measured in the same scale from which the other parallell was taken, shall here shew the meane proportionall which was required.

Let the numbers giuen be 4 and 9. If I shall take the line *A*, in the *diagram* of the sixt *Prop.* resembling 4 in a scale of equall parts, and to it open the *Sector* in the termes of 4 and 4. in the lines of *Superficies*, his parallell betweene 9 and 9 doth giue me B for the meane proportionall. And this measured in the scale of equall parts doth extend to 6, which

which is the meane proportionall number between 4 and 9.

For as 4 to 6, so 6 to 9.

In like manner if I take the line *C*, resembling 9 in a scale of equall parts, and to it open the *Sector* in the termes of 9 and 9, in the lines of *Superficies*, his parallell between 4 and 4 doth giue me the same line *B*, which will proue to be 6, as before, if it be measured in the same scale whence *C* was taken.

For, the figures 1, 2, 3, 4, &c. heere set downe upon the line, do sometime signifie themselues alone: sometime, 10, 20, 30, 40 &c. sometime 100, 200, 300, 400 &c. and so forward as the matter shall require. The first figure of euery number is alway that which is here set down: the rest must be supplied according to the nature of the question.

If you suppose pricks under the number given (as in *arithmeticall* extraction) and the last prick to the left hand shall fall under the last *fig.* (which will be as oft as there be odd figures) the unite will be best placed at 1, in the middle of the line; so the root, & the square will both fall forward, toward the end of the line. But, if the last pricke shall fal under the last figure but one (which will be as oft as there be euen Figures) then, the unite may be placed at 1 in the beginning of the line, and the square in the second length: or the unite may be placed at 10, in the end of the line, so the root and the square will both fall backward, toward the middle of the line.

8 *To find the square roote of a number.*

9 *The roote being giuen to find the square number of that roote.*

IN the extraction of a square roote it is usuall to set pricks under the first figure, the third, the fifth, the seventh, and so forward, beginning from the right hand toward the left, and as many pricks as fall to be under the square number given, so many figures shall be in the roote: so that if the number given be lesse then 100, the roote shall be onely of one figure; if lesse then 10000, it shall be but two figures; if lesse then 1000000, it shall be three figures, &c.

Thereupon the lines of *Superficies* are divided first into an hundred parts, and if the number given be greater then 100, the first division (which before did signifie only one) must signifie 100, and the whole line shall be 10000 parts: if yet the number given be greater then 10000, the first division must now signifie 10000, and the whole line be esteemed at 1000000 parts: and if this be too little to expresse the number given, as oft as we have recourse to the beginning, the whole line shall increase it selfe an hundred times.

By these meanes if the last pricke to the left hand shall fall under the last figure, which will be as oft as there be odde figures, the number given shall fall out betweene the center of the *Sector* and the tenth division: but if the last prick shall fall under the last figure but one, which will be as oft as there be even figures, then the number given shall fall out betweene the tenth division and the end of the *Sector*.

This being considered, when a number is given and the square roote is required, take a paire of compasses and setting one foote in the cenrer, extend the other to the terme of the number given in one of the lines of *Superficies*; for this distance applied to one of the Lines of *Lines*, shall shew what the Square root is, without opening the *Sector*.

Thus 36 doth give a root of 6 and 360, a root of (almost) 19: and 3600, a root of 60: and 36000, a root of 189 &c.

In like manner, the neerest root of 725 is here found to be (about) 27: the neerest root of 7250, about 85: the neerest of 72500, about 269: and the neerest root of 725000, about 851: And so in the rest.

On the contrary, a number giuen may be squared, if first we extend the compasses to the number given in the lines of *Lines*, and then apply the distance to the *Lines* of *Superficies*, as may appeare by the former examples.

10 *Three numbers being given to find the fourth in a duplicated proportion.*

IT is plaine by the 19 and 20 *Prop. 6 Lib.* of *Euclid.* that like *Superficies* do hold in a duplicated proportion of their homologall sides, whereupon a question being moved concerning *Superficies* and their sides. It is usuall in Arithmeticke that the proportion be first duplicated before the question be resolved, which is not necessarie in the use of the *Sector*, onely the numbers which doe signifie *Superficies* must be reckoned in the lines of *Superficies*, and they which signifie the sides of *Superficies*, in the lines of *Lines*, after this manner.

If a question be made concerning a *Superficies*, the two numbers of the first denomination must be reckoned in the lines of *Lines*, and the *Sector* opened in the termes of the first number to the quantitie of a line out of the scale of *Superficies* resembling the second number; so his parallells taken betweene the termes of the third number, being measured in the same scale of *Superficies*, shall give the Superficiall number which was required.

As if a Square, whose side is fortie perches in length, shall containe ten acres in the *Superficies*, and it be required to know how many acres the Square should containe, whose side is sixtie perches.

Here If I tooke 10 out of the line of *Superficis*, and put it over in 40 in the lines of *Lines*, his parallel between 60 and

60

60 measured in the line of *Superficies*, would be $22\frac{1}{2}$; and such is the number of acrees required. For Squares doe hold in a duplicated proportion of their sides; wherefore when the proportion of their sides is as 4 to 6, and 4 multiplied into 4 become 16, and 6 multiplied into 6 become 36, the proportion of their squares shall be as 16 to 36; and such is the proportion of 10 to $22\frac{1}{2}$.

If a field measured with a statute perch of $16\frac{1}{2}$ foote, shall containe 288 acres, and it be required to know how many acres it would containe if it were measured with a woodland perch of 18 foote.

Here because the proportion is reciprocall, if I tooke 288 out of the line of *Superficies*, and put it ouer in 18, in the lines of *Lines*, his parallell betweene $16\frac{1}{2}$ and $16\frac{1}{2}$ measured in the line of *Superficies*, would be 242; and such is the number of acres required.

For seeing the proportion of the sides is as $16\frac{1}{2}$ to 18, or in lesser numbers as 11 to 12, and that 11 multiplied into 11 become 121, and 12 into 12 become 144, the proportion of these *Superficies* shall be as 121 to 144, and so have 288 to 242, in *reciprocall* proportion.

On the contrary, if a question be proposed concerning the side of a *Superficies*, the two numbers of the first denomination must be reckoned in the lines of *Superficies*, and the *Sector* opened in the termes of the first number, to the quantitie of a line, out of the line of *Lines* or some Scale of equall parts, resembling the second number; so his parallell taken betweene the termes of the third number being measured in the same scale with the second number, shall giue the fourth number required.

As if a field contained 288 acres when it was measured with a statute perch of $16\frac{1}{2}$, and being measured with another perch, was found to containe 242 acres, it were required to know what was the length of the perch with which it was so measured.

Here because the proportion is reciprocall, if I tooke $16\frac{1}{2}$ out of the line of *Lines*, and put it ouer in 242 in the lines

of

of *Superficies*, his parallell betweene 288 and 288, being measured in the line of *Lines*, would be 18, & ſuch is the length of the perch in feete whereby the field was laſt meaſured.

For ſeeing the proportion of the acres is as 288 unto 242, or in the leaſt number as 144 to 121, and that the roote of 144 is 12, and the root of 121 is 11, the proportion of roots and conſequently of the perches ſhall be as 12 to 11, and ſo are $16\frac{1}{2}$ to 18, in *reciprocall* proportion.

If 360 men were to be ſet in forme of a long ſquare, whoſe ſides ſhall haue the proportion of 5 to 8; and it were required to know the number of men to be placed in front and file: if the ſides were only 5 and 8, there ſhould be but 40 men; but there are 360: therefore, working as before, I finde that.

As 40 to the ſquare of 5,
ſo 360 to the ſquare of 15.

As 40 to the ſquare of 8,
ſo 360 to the ſquare of 24.

and ſo 15 and 24 are the ſides required.

If 1000 men were lodged in a ſquare ground, whoſe ſide were 60 paces, and it were required to know the ſide of the ſquare wherein 5000 might be ſo lodged, here working as before, I ſhould finde that

As 1000 are to the ſquare of 60:
ſo 5000 to the ſquare of 134.

And ſuch very neare is the number of paces required.

CHAP. IV.

The vſe of the lines of Solids.

To finde a proportion betweene two or more like Solids.

IN the Sphere, in regular, parallell, and other like bodies, whoſe ſides next the equall angles are proportionall, the worke

worke is in a manner the same, with that in the first *Prop.* of *Superficies*, but that it is wrought on other lines.

Take one of the sides of the greater *Solid*, & according to it open the *Sector* in the points of a 1000 & 1000, in the lines of *Solids*, then take the like sides of the lesser *Solids* severally, and carry them parallell to the former, till they stay in like points, so the number of points wherein they stay, shall shew their proportion to 1000.

Let *A* and *B*, be the like sides of like Solids, either the diameters, or semidiameters of two spheres, or the sides of two cubes, or other like. First I take the side *A*, and to it open the *Sector* in the points of 1000, then keeping the *Sector* at this angle, I enter the lesser side *B*, parallell to the former, and finde it to crosse the line of *Solids* in the points of 400, and such is the proportion betweene the Solids required, which in lesser number is as 5 to 2.

This proposition might have been wrought by 60, or any other number that admits severall divisions.

It may also be wrought without opening the *Sector*, for if the sides of the Solids given, be applied to the lines of *Solids*, begining all wayes at the center of the *Sector*, there will be such proportion betweene them, as betweene the numbers of parts whereon they fall.

2 To augment a Solid in a given proportion.
3 To diminish a Solid in a given proportion.

TAke the side of the Solid given, and to it open the *Sector*, in the points of the number given: then keeping the Sector at that angle, the parallell distance betweene the points of the number required, shall giue the like side of the Solid requyred.

If it be a *parallell opipedon*, or some irregular Solid, the other like sides may be found out in the same manner, and with them the Solids required, may be made up with the same angles.

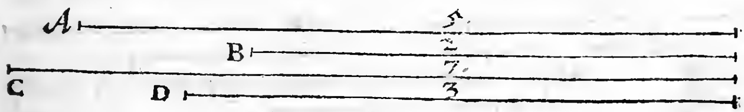
Let *A* be the side of a cube, to be augmented in the proportion of 2 to 3. First I take the side *A*, and put it over in the lines of *Solids* in 2 and 2, so the parallell betweene 3 and 3, doth give me the side B, on which if I make a cube, it will have such proportion to the cube of *A*, as 3 to 2.

In like manner, if B were the diameter of a Sphere, to be diminished in the proportion of 3 to 2. I would take out B, and put it over in the lines of *Solids*, in 3 and 3, so the parallell betweene 2 and 2, would give me *A*: to which diameter if I should make a Sphere, it would be lesse then the Sphere, whose diameter is B, in such proportion as 2 is lesse then 3.

Here also for variety of worke, may the like caution be observed to that which we gave in the third *Prop.* of *Lines*.

4 *To adde one like Solid to another.*
5 *To subtract one like Solid from another.*

First the proportion betweene the sides of the like Solids given, is to be found by the first *Prop.* of *Solids*: then adde or subtract those proportions, and accordingly augment or diminish by the former *Prop.*

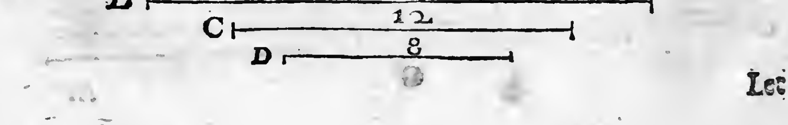


As if *A* and *B* where the sides of two cubes, and it were required to make a third cube equall to them both: first the proportion betweene the sides *A* and *B*, would be found to be as 100 to 40, or in lesser termes as 5 to 2. Then because 5 and 2 being added do make 7, I augment the side *A* in the proportion of 5 to 7, and produce the side *C*, on which if I make a cube, it will be equall to both the cubes of *A* and *B*, which was required.

In like maner *A* and *B* being the sides of two cubes, if it were required to subtract the cube of B out of the cube of A, and to make a cube equall to the remainder. Here the proportion being as 5 to 2, because 2 taken out of 5; the remainder is 3, I should diminish the side A in the proportion of 5 to 2, and so I should have the side D, on which if I make a cube, it will be equall to the remainder when the cube of B is taken out of the cube of A, that is the two cubes made upon B and D, shall be equall to the first cube made upon the side A.

6 *To find two meane proportionall lines betweene two extreme lines given.*

First I find what proportion is betweene the two extreme lines given as they are lines, by the fifth *Prop:* of *Lines*, then open the *Sector* in the lines of *Solids*, to the quantitie of the former extreme, and a parallell betweene the points of the number belonging to the other extreame, shall be that meane proportionall which is next the former extreme. This done, open the *Sector* againe to this meane proportionall in the points of the former extreme, and the parallell distance betweene the points of the latter extreme, shall be the other meane proportionall required.



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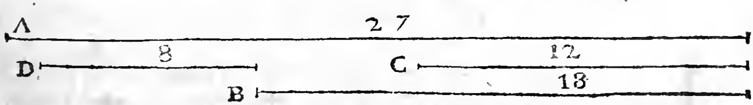
Let the two extreme lines given be A and D, the proportion betweene them, as they are lines, will be found to be as 27 to 8. Wherefore I take the line A, and put it over in the lines of *Solids* betweene 27 and 27, and keeping the *Sector* at this angle, his parallell betweene 8 and 8, doth give me *B*, the meane proportionall next unto A. Then put I over this line B, betweene the aforeſaid 27 and 27, and his parallell betweene 8 and 8 doth give me the line C, the other meane proportionall which was required.

Againe, for proofe of the operation I put over thiis line C in the aforeſaid 27 and 27, and his parallell betweene 8 and 8 doth give me the very line D: whereby it is plaine that theſe foure lines do hold in continuall proportion; and ſo B and C are ſound to be the meane proportionals betweene A and D the extremes given.

7 *To find two meane proportionall numbers betweene two extreme numbers given.*

First reckon the numbers given on both ſides of the lines of *Solids*, beginning from the center, and marking the termes whereto they extend: then take a line out of the line of *Lines*, or any other ſcale of equall parts reſembling the former of thoſe numbers, and put it over in the lines of *Solids*, betweene the points of his like number, and a parallell betweene the points belonging to the other extreme, meaſured in the ſcale from whence the other parallell was taken, ſhall give that meane proportionall number which is next the former extreme. This done open the *Sector* againe to this meane proportionall in the points of the former extreme, and the parallell diſtance betweene the points of the latter extreme, meaſured in the ſame ſcale as before, ſhall there ſhew the other meane proportionall required.



Let the two extreame numbers given be 27 and 8; if I ſhall take the line A, reſembling 27 in a ſcale of equall parts, and to it open the *Sector* in 27 and 27, in the line of *Solids*, his parallell betweene 8 and 8 doth give me B for his next meane proportionall, and this meaſured in the former ſcale doth extend to 18. Then put I over this line B between the aforeſaid 27 and 27, and his parallell between 8 and 8 doth give me C for the other meane proportionall, and this meaſured in the former ſcale doth extend to 12. Againe, for proofe of my worke, I put over this line C betweene 27 and 27, as before, and his parallell betweene 8 and 8 doth give me D, which meaſured in the former ſcale doth extend to 8, which was the latter extreame number given; whereby it is plaine that theſe foure numbers do hold in continuall proportion: and therefore 18 and 12 are meane proportionalls betweene 27 and 8, which was required.

If you ſuppoſe pricks under the number given as in *arithmeticall* extraction and that laſt prick to the left hand ſhall fall under the laſt figure, as in 1728, the unite will be left placed at 1, in the middle of the line and the Root ſquare and cube will all fall forward toward the end of the line.

If the laſt pricke ſhall fall under the laſt figure but one, as in 17280; the unite may be placed at 1, in the beginning of the line, and the cube in the ſecond length: or the unite may be placed at 10, in the end of the line, and the cube in the firſt length.

But if the laſt prick ſhall fall on the laſt figure but two, as in 172800; then, place the unite alwayes at 10, in the end of the line: ſo, the Root ſquare and cube will all fall bacward and be found in the ſecond length.

8 *To find the cubique roote of a number.*

9 *The roote being given to finde the cube number of that roote.*

IN the extraction of a cubique root, it is uſuall to ſet pricks under the firſt figure, the fourth, the ſeventh, and tenth, and

and so forward, omitting two, and pricking the third from the right hane toward the left; and as many pricks as fall to be under the cubique number, so many figures shall be in the roote. So that if the number given be lesse then 1000, the roote shall be only of one figure; if lesse then 1000000, it shall be but of two figures; if above these, and lesse then 100000000, it shall be but three figures; &c. whereupon the lines of *Solids* are divided, first into 1000, parts, and if the numbers given be greater then 1000, the first division (which before did signifie onely one) must signifie 1000, and the whole line shall be 1000000: if yet the number given be greater then 1000000, the first division must now signifie 1000000, and the whole line be esteemed at 1000000000 parts, and if these be to little to expresse the numbers given, as oft as we have recourse to the beginning, the whole line shall encreače it selfe a thousanp times.

By these meanes, if the last pricke, to the left hand, shall fall under the last figure, the number given shall be reckoned at the beginning of the lines of *Solids* from 1 to 10, and the first figure of the roote shall be alwayes either 1, or 2. If the last pricke shall fall under the last figure but one, then the number given shall be reckoned in the middle of the line of *Solids*, between 10 and 100, and the first figure of the roote shall be alwayes either 2, or 3, or 4. But if the last pricke shall fall under the last figure but two, then the number given, shall be reckoned at the end of the line of Solids, betweene 100, and 1000.

This being considered when a number is given, and the cubique roote required: Set one foote of the compasses in the center of the *Sector*, extend the other in the line of *Solids* to the points of the number given: for this distance applied to one of the lines of *Lines*, shall shew what the cubique root is, without opening the *Sector*.

So the neerest roote of 8490000, is about 204.
The neerest roote of 84900000, is about 439.
The neerest roote of 849000000, is about 947.

On the contrary, a number may be cubed, if first we extend the compasses to the number given, in the line of *Lines*, and then apply the distance to the lines of *Solids*; as may appeare by the former examples.

10 *Three numbers being given to finde a fourth in a triplicated proportion.*

AS like *Superficies* doe hold in a duplicated proportion, so like solids in a triplicated proportion of their homologall sides: and therefore the same worke is to be obserued here on the lines of *Solids*, as before in the lines of *Superficies*; as may appeare by these two examples.

If a cube whose side is 4 inches, shall be 7 pound weight, and if it be required to know the weight of a cube whose side is 7 inches; here the proportion would be,

As 4 are to a cube of 70.
so 7 to a cube of $37\frac{1}{2}$

And if I tooke 7 out of the lines of *Solids*, and put it over in 4 and 4, in the lines of *Lines*, his parallell between 7 and 7 measured in the lines of *Solids*, would be $37\frac{1}{2}$; and such is the weight required.

If a bullet of 27 pound weight haue a diamiter of 6 inches, and it be required to know the diamiter of the like bullet, whose weight is 125 pounds; here the proportion would be,

As the cubique root of 27 is unto 6:
So the cubique root of 125 is unto 10.

And

And if I tooke 6 out of the line of *Lines*, and put it over in 27 and 27 of the lines of *Solids*, his parallell betweene 125 and 125 measured in the line of *Lines*, would be 10; and such is the length of the diameter required.

The end of the first Booke.

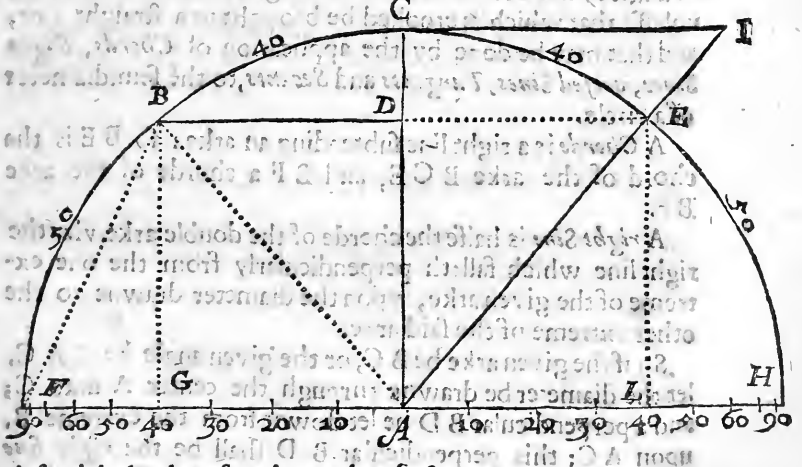
THE SECOND BOOKE OF THE *SECTOR*,

Containing the vse of the Circular *Lines*.

CPAP. I.

Of the nature of Sines, Chords, Tangents *and* Secants, *fit to be knowne before hand in reference to right-line Triangles.*

IN the *Canon of Triangles*, a circle is commonly divided into 360 *degrees*, each *degree* into 60 *minutes*, each *minute* into 60 *seconds*.



A semicircle therefore is an arke of 180 gr.

A quadrant is an arke of 90 *gr*.

The measure of an angle is the arke of a circle, described out of the angular point, intercepted betweene the sides sufficiently produced.

So the measure of a right angle is alwayes an arke of 90 *gr*. and in this example the measure of the angle B A D is the arke B C of 40 *gr*; the measure of the angle B A G, is the arc B F of 50 *gr*.

The complement of an arke or of an angle doth commonly signifie the arke which the given arke doth want of 90 *gr*: and so the arke B F is the complement of the arke B C; & the angle B A F, whose measure is B F is the complement of the angle B A C; and on the contrary.

The complement of an arke or angle in regard of a semicircle, is that arke which the given arke wanteth to made up 180 *gr*: and so the angle E A H is the complement of the angle E A F, as the arke E H is the complement of the arke F E, in which the arke C E is the excesse aboue the quadrant.

The proportions which these arkes (being the measures of angles) have to the sides of a triangle, cannot be certaine, unlesse that which is crooked be brought to a straight line; and that may be done by the application of *Chords*, *Right Sines*, *versed Sines*, *Tangents* and *Secants*, to the semidiameter of a circle.

A *Chorde* is a right line subtending an arke: so B E is the chord of the arke B C E, and B F a chorde of the arke B F.

A *right Sine* is halfe the chorde of the double arke, viz. the right line which falleth perpendicularly from the one extreme of the given arke, vpon the diameter drawne to the other extreme of the said arke.

So if the given arke be B C, or the given angle be B A C, let the diameter be drawne through the center A unto C; and a perpendicular B D be let downe from the extreme B, upon A C; this perpendicular B D shall be the *right sine* both of the arke B C, and also of the angle B A C: and it is also

also the halfe of the chord B E, subtending the arke B C E. which is double to the given arke B C. In like manner, the semidiameter F A, is the *right sine* of the arke F C, and of the right angle F A C; for it falleth perpendicularly upon A C, and it is the halfe of the chord F H,

This whole Sine of 90 *gr*. is hereafter called *Radius*; but the other *Sines* take their denomination from the degrees and minutes of their arks.

Sinus versus, the *versed sine* is a segment of the diameter, intercepted betweene the *right sine* of the same arke, and the circumference of the circle. So D C is the *versed sine* of the arke C B, and G F the *versed sine* of the arke B F, and G H the *versed sine* of the arke B H.

A *Tangent* is a right line perpendicular to the diameter, drawne by the one extreme of the given arke, and terminated by the *secant* drawne from the center through the other extreme of the said arke.

A *Secant* is a right line drawne from the center, through one extreme of the given arke, till it meete with the *tangent* raised from the diameter at the other extreme of the said arke.

So if the given arke be C E, or the given angle be C A E, let the diameter be drawne through the center A to C, and in C to A C, be raised a perpendicular C I. Then let another line be drawne from the center A through E, till it meet with the perpendicular C I in I; the line C I is a *Tangent*, and A I is the *Secant* both of the arke C E, and of the angle C A E.

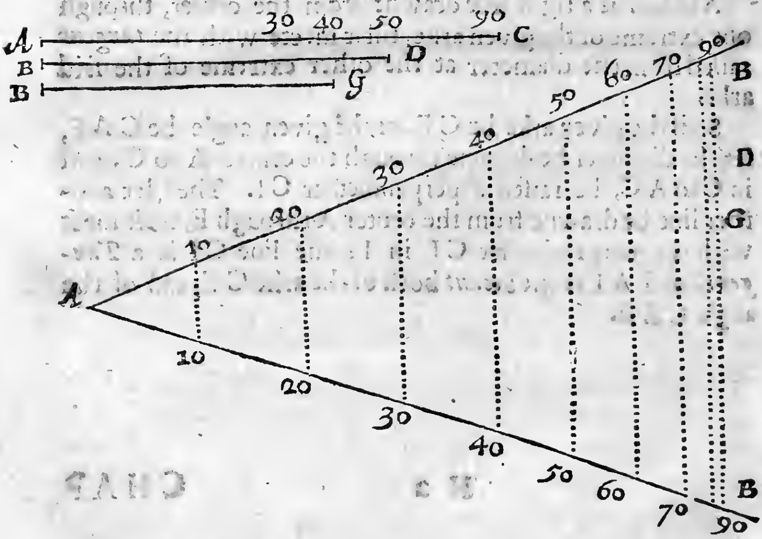
CHAP. II.

Of the generall vse of Sines and Tangents.

1 The Radius being knowne to find the right sine of any arke or angle.

IF the Radius of the circle given be equall to the laterall Radius, that is, to the whole line of *Sines* on the *Sector*, there needs no farther worke, but to take the other sines also out of the side of the *Sector*. But if it be either greater or lesser, then let it be made a parallell Radius, by applying it ouer in the lines of *Sines*, betweene 90 and 90; so the parallell taken from the like laterall sines, shall be the *sine* required.

As if the given Radius be *A C*, and it were required to find the sine of 50 *Gr*. & his complement agreeable to that Radius.



Let *A B*, *A B* represent the lines of *sines* on the *Sector*, and let B B, the distance betweene 90 and 90, be equall to the given

given radius *A C*. Here the lines *A* 40, *A* 50, *A* 90, may be called the *laterall sines* of 40, 50, & 90; in regard of their place on the side of the *Sector*. The lines betweene 40 and 40, betweene 50 and 50, betweene 90 and 90; may be called the *parallell sines* of 40, 50, and 90; in regard they are parallell one to the other. The whole sine of 90 *Gr.* here standing for the semidiameter of the circle, may be called the Radius. And therefore if *A C* be put over in the line of *Sines* in 90 and 90 and so made a *parallell radius*, his parallell sine betweene 50 and 50, shall be B D, the sine of 50 required. And because 50 taken out of 90, the complement is 40; his *parallell sines* betweene 40 and 40, shall be B G, the sine of the complement which was required.

2 *The right sine of any arke being given to find the Radius.*

TVrne the sine given into a parallell sine, and his parallell *Radius* shall be the *Radius* required.

As if B D were the given sine of 50 *Gr.* and it were required to finde the Radius: let B D be made a parallell sine of 50 *Gr.* by applying it over in the lines of *Sines*, betweene 50 and 50; so his parallell Radius betweene 90 and 90 shall be A C, the Radius required.

3 *The Radius of a circle, or the right Sine of any arke being given, and a streight line resembling a Sine, to find the quantitie of that unknowne* Sine.

Let the Radius or right sine given be turned into his parallell; then take the right line given, and carrie it parallell to the former, till it stay in like *Sines*: so the number of degrees and minutes where it stayeth, shall give the quantitie of the Sine required.

As if B D were the given sine of 50 *Gr.* and B G the streight line given: first I make B D a parallell sine of 50 *Gr.*; then keeping the *Sector* at this angle, I carie the line B G parallell, and find it to stay in no other but 40 and 40; and therefore 40 *gr*. is this quantitie required.

4 *The Radius or any right Sine being given, to find the versed sine of any arke*

IF the arke, whose *versed sine* is required, be lesse then the quadrant, take the sine of the complement out of the radius, and the remainder shall be the *sinus versus*, the versed sine of that arke.

As if A B being the laterall *Radius*, it were required to find the versed sine of 40 *gr*; here the sine of the complement is A 50, and therefore B 50 is the *versed sine* required. Or if I reckon from B, at the end of the Sector, toward the center, the distance from 90 to 80, is the versed sine of 10 *gr*; from 90 to 70, the versed sine of 20 *gr*; from 90 to 60, is the versed sine of 30 *gr*: and so in the rest:

If A D be the given *sine* of 50 *gr*. and it be required to find the *versed sine* of 50 *gr*; here because A D is unequall to the laterall sine of 50 *gr*; I make it a parallell. And first I find the radius A C, then the sine of the complement A 40, which being taken out of A C, leaveth C 40 for the versed sine of 50 *gr*. which was required.

But if the arke, whose versed sine is required, be greater then the quadrant, his versed sine also is greater then the *Radius*, by the right sine of his excesse above 90 *gr*.

As if A C being the Radius given, it were required to find the versed sine of 130 *gr*: here the excesse above 90 *gr*. is 40 *gr*: and therefore the versed sine required is equall to the Radius A C and A 40, both being set together.

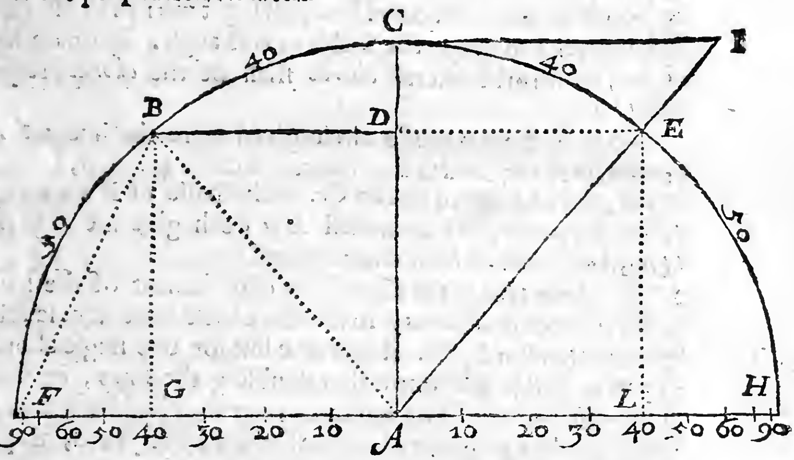
5 *The diameter or* Radius *being given, to finde the Chords of every arke.*

The sines may be fitted many wayes to serue for chords.

1 A *sine* being the halfe of the *chord* of the double arke, if the *sine* be doubled, it giveth the *chord* of the double arke,

a *Sine* of 10 *gr.* doubled giveth a *Chord* of 20 *gr*, and a *Sine* of 25 *gr.* being doubled giveth a *Chord* of 30 *gr.* and so in the rest. As here B D, the sine of B C, an arke of 40 *gr.* being doubled giveth B E the chord of B C E, which is an arke of 80 *gr.* Wherefore if the Radius of the circle given be equall to the laterall Radius, let the *Sector* be opened neare unto his length, so that both the lines of *Sines* may make but one direct line: so the distance on the sines betweene 10 and 10, shall be a chord of 20, the distance betweene 20 and 20, shall be a chord of 40; and the distance betweene 30 and 30, shall shall be a chord of 60; and so in the rest.

2 Because a sine is the halfe of the chord of the double arke, the proportion holdeth.



As the diamiter F H unto the Radius A H, so the chord B E unto the sine D E, or the chord G L unto the sine A L, and then if the Radius A H, be put for the diameter, which is a chord of 180 gr, the sine D E or A L, shall serue for a chord of 80 gr, and the semiradius which is the sine of 30 gr, shall serue for a chord of 60 gr, and go for the semidiameter of a circle, and so in the rest. So that by these meanes we shall not need to double the lines of *Sines* as before, but onely to double the numbers. And to this purpose I have subdivided

each

each degree of the sines into two, that so they might shew how far the halfe degrees do reach in the sines, and yet stand for whole degrees when they are used as chords.

Wherefore if the Radius of the circle given be equall to the laterall semiradius (the sine of 30 *Gr.* and chord of 60 *Gr.*) there needs no farther work then to take the sine of 10 *Gr* for a chord of 20 *Gr.* and a sine of 15 *Gr.* for a chord of 30 *Gr.* &c.

But if the Radius of the circle given be either greater or lesser then the laterall semiradius, take the diameter of it, and make it a parallell chord of 180 *Gr.* by applying it over the lines of *Sines* between 90 and 90 or take the Radius or Semidiameter which is equall to the chord of 60 *Gr.* and make it a parallell Radius of 60 *Gr.* by applying it over in the sines of 30 and 30, and keepe the Sector at this angle. The parallells taken from the laterall *chords* shall be the *chords* required.

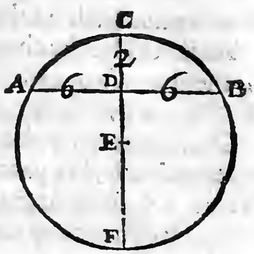
As if the diameter of a circle given were the line *A B*, and it were required to find the chord of 80 gr: first, I make *A* B a parallell chord of 180 *Gr.* or the halfe of it a parallell chord of 60 *Gr*; so his parallell *L G* doth give me *F G* the chord of 80 *Gr.* which was required.

3 Seeing that as the sine of the complement of the halfe arke is unto the *Radius*, so the sine of the same whole arke is unto the chord of it: if we seeke but for one single chord, we may find it without either doubling the sines, or doubling the number. For applying over the Radius given in the sine of the complement of halfe the arke required, his parallell sine shall be the chord required.

As if the semidiameter of the circle given were *A C*, and it were required to find the chord of 40 *Gr*: the halfe of 40 *Gr.* is 20 *Gr.* the complement of 20 *Gr.* is 70 *Gr.* Wherefore I make *A C* a parallell sine of 70 *Gr.* and his parallell sine G *L* doth give me F G the chord of 40 Gr. agreeable to the semidiameter A C.

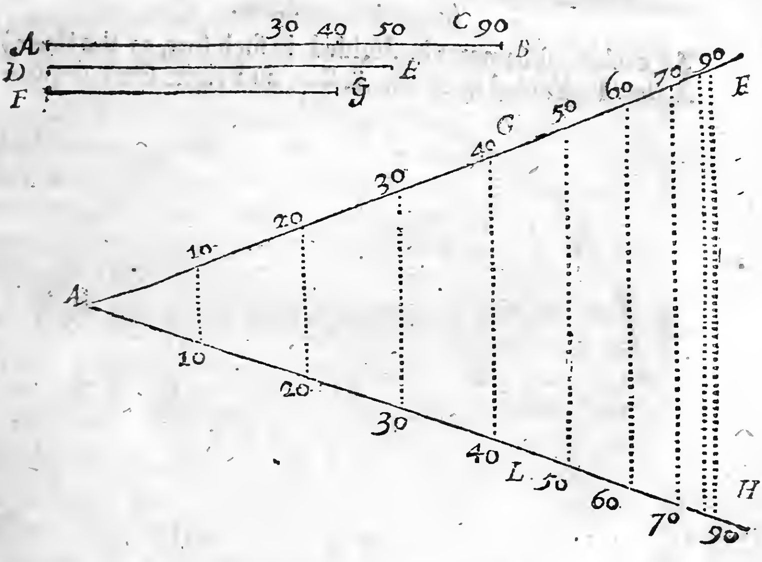
Having

Having two right lines resembling the chord and versed Sine, to find the Diameter and Radius.



Let the two right lines given be A B, resembling the chord, *C D*, the versed sine of a circle, whose arch A C B is unknowne: and let it be required to find the diameter *C F.*

Having 2 lines given, the first C D, the second A D the halfe of A B, we may find a third in continuall proportion (by the 6 or 9 *Prop.* of the lines) and that shall be the line D F (18) the summe whereof and of C D gives the diameter C F (20) and the halfe thereof is the Radius (E C.).



6 *The chord of any arke being given to finde the diameter and* Radius.

TVrne the chord given unto a parallell chord, and his parallell semiradius shall be the semidiameter, and the parallell radius shall be the diameter.

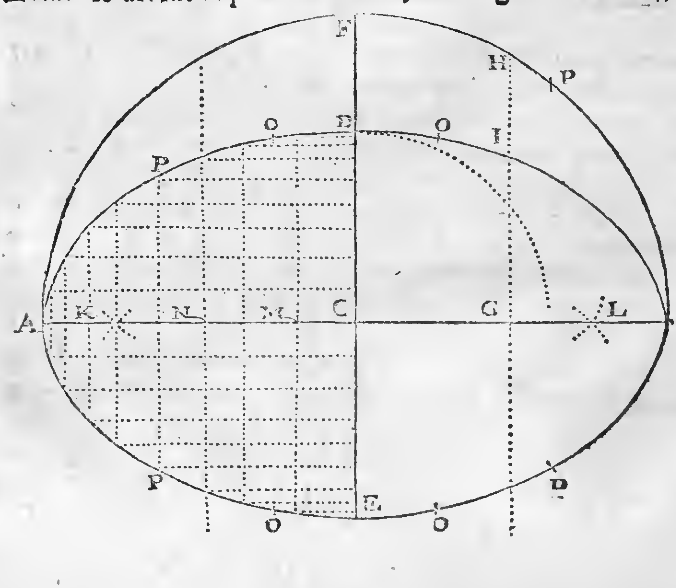
As if F G be the chord of 80 *gr.* I put this over in G and L, the sine of 40, and chord of 80 *gr.* and the parallell chord of 180 *gr:* giveth me A B the diameter required.

Or if I turne the chord given into a parallell sine of the same quantitie, his parallell sine of the complement of halfe the arke, doth give me the semidiameter.

As if F G be the given chord of 40 *gr*; I put it over in G and L, the sines of 40 *gr.* then because the halfe of 40 *gr.* is 20 *gr.* and the complement of 20 gr. is 70 gr. I take out the parallell sine of 70 gr. and it giveth me A B for the semidiameter, agreable to that chord of 40 gr.

Having the Diameter of an Ellipsis, to describe the same upon a plaine.

IF each semidiameter be divided, in such sort, as the line of *Sines* is divided upon the *Sector*, and right lines drawne



through each division perpendicular to those semidiameters like unto sines; The points, where the sines drawne through the one semidiameter do meete the sines of the complement drawne through the other Semidiameter, shall be the points through which the Ellipsis is to be drawne.

Let the diameters be A B, B E, one crossing the middle of the other, in the point *C*. Divide first the semidiameters *C A*, *C B*; then, then the semidiameters C D, C E like unto the lines of *Sines* upon the *Sector*, by the 8 Proposition of *Lines*: So, the Ellipsis shall be drawne through the points at the meeting of the Sines of 10 and 80, of 20 and 70, of 30 and 60 &c.

Or (without the helpe of the line of *Sines*) we may draw the circle A F B upon the center C and semidiameter A C. For so; crossing the diameter A B with severall perpendicular lines continued unto the circumference of the circle, if we divide these perpendiculars on either side of the diameter, in such sort as the greater semidiamete C F is divided, by the lesser, in the point D; and draw a line winding through all those points, the line so drawne shall be the Ellipsis.

Or (without the helpe of the *Sector*) we may with the Radius A C, upon the centers D and E, describe two occult arches meeting in the points K and L. Then taking betweene C and K, any number of points *M N*, we may from the centers K and L, with the semidiameter M B describe foure occult arches; and with the Radius A M, and the same centers K and L, crosse them againe with other 4 arches in the points at *O*. In like manner, from the same centers *K* and *L*, with the Radius *N B*, we may describe other 4 occult arches; and, with the Radius A N, and the former centers crosse them againe, with 4 arches in the points at *P*, and so draw the Ellipsis through the points *O P*. &c.

This is (in effect) as wee should tye a thred about *A* and *L*, and then draw it easily from the point

A, round about the two former centers *K* and *L*, untill it were brought to the point *A* againe; which is also an easy way to describe an *Ellipsis*.

The distance of these former points from either Semidiameter may be set downe in numbers. For, supposing the lesser Semidiameter *C D*, to be 10; the greater (*C B*) to be 16, (or otherwise divided into any number of knowne points,) If we have the proportion betweene *C G* and *C B*, we may find the length of the perpendicular *G I*,

If the proportion be as 1 to 2, the perpendicular will be 8.66.

If the proportion be as 2 to 3, the perpendicular will be about 7.45.

As the greater semidiameter	*C B*
to the part given	*C G*
So 100000, the Radius	*C B*
to the sine of	*C G*
whose complement is	*G H*
As the Radius	*C F*
to the sine of the complement	G *H*
So the lesser semidiameter	*C D*
to the perpendicular	G *I*

The same may also be found without knowing the sines. For the perpendicular G *H*, is a meane proportionall betweene *A* G and G *B*: which being knowne

As *C F* unto *E D*, so is G *H* unto G *I*.

7 *To open the Sector to the quantitie of any angle given.*

8 *The Sector being opened, to find the quantitie of the angle.*

It is one thing to open the edges of the Sector to an angle, and another thing to open the lines on the Sector to the same angle. For the lines of *lines* on the one side, & the lines of *sines* on the other side, do make an angle of 2 *gr.* when the

Sector is close shut, and the edges doe make no angle at all. So likewise the lines of *Superficies* and the lines of *Solids* doe make an angle of 10 *gr*, which are to be allowed to the edges.

The lines of *lines* may be opened to a right angle, if the whole line of 100 parts be applied over in 80 and 60.

The line of *sines* may be opened to a right angle, if the large secant of 45 *gr.* be applied over in the sines of 90 *gr.* or if the sine of 90 *gr.* be applied over in the sines of 45 *gr.* or if the sine of 45 *gr.* be applied over in the sines of 30 *gr.*

If it be required to open those lines to any other angle, take out the chord thereof, and apply it over in the *semiradius*, and those lines shall be opened to that angle.

As if it were required to open the Sector in the lines of *sines* to an angle of 40 *gr.* take out the chord of 40 *gr*, and to it open the Sector in the chord of 60 *gr.* so shall the lines of *sines* be opened to the angle required. Or if the same chord of 40 *Gr.* be applied over betweene 50, and 50, in the lines of *lines*, they shall also be opened to the same angle. If it be applied over in 25 of the lines of *Superficies*, or 125 in the lines of *Solids*, they also shall be opened to the same angle: because the chord of 60 *Gr.* or sine of 30 *Gr.* and 50 in the lines of *lines*, and 25 in the lines of *Supeoficies*, and 125 in the *Solids*, are all of the same length with the semiradius.

Or if the *Semiradius* be applied over betweene the sine of 30 *Gr.* and the sine of the complement of the angle required, it will open the lines of *Sines* to that angle.

As if the semiradius be applied over in the sines of 30 *Gr.* and the sine of 50 *Gr.* it shall open the lines of *Sines* to an angle of 40 *Gr.*

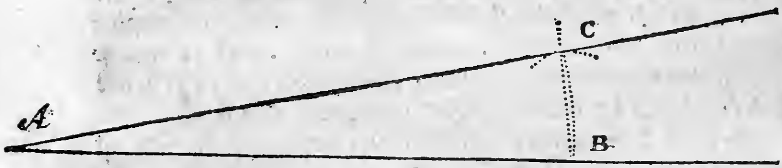
On the contrary, if the *Sector* be opened to an angle, and it be required to know the quantitie thereof, open the compasses to the semiradius, and setting one foote in the sine of 30 *Gr.* turne the other toward the other line of *sines*, and it shall fall there in the complement of the angle; if it fall on 50 *Gr.* the angle is 40 *Gr*, if on 60 *Gr.* the angle is 30 *Gr.* &c.

Or take over the parallell chord of 60 *Gr.* and measure it in the laterall chord, and it shall there shew the quantitie of the angle. As if the *Sector* being opened to an angle, I should take over the parallell of 30 *Gr.* of the sines, and 60 *Gr*; of the chords, and measure it in the laterall chords, find it to be 40 *Gr.* the angle comprehended betweene the lines of *Sines* is 40 *Gr.* but the angle betweene the edges of the Sector is 2 *Gr.* lesse, and therefore but 38 *Gr.*

9. *To finde the quantitie of any angle given.*

IF out of the angular point, to the quantitie of the *Semiradius*, be described an occult arke that may cut both sides of the angle, the chord of this arke measured in the laterall chord, shall give the quantitie of the angle.

Let the angle given be *B A C*: first I take the *Semiradius* with the compasses, and setting one foote in *A*, I cut the sides of the angle in *B* and *C*; then I take the chord *B C*, and measure it in the laterall chord, and I find it to be 11 *Gr.* and 15 *M.* and such is the quantitie of the angle given.



Or if the arke be described out of the angular point at any other distance, let the semidiameter be turned into a parallall chord of 60 *Gr.* then take the chord of this arke, and carrie it parallell till it crosse in like chords: so the place where it stayeth shall give the quantitie of the angle.

As in the former example, if I make the semidiameter *A B* a parallell chord of 60 *Gr.* and then keeping the *Sector* at that angle, carrie the chord *B C* parallell, till it stay in like chords; I shall finde it to stay in no other but 11 Gr. 15 *M.* and such is the angle *B A C.*

10 *Vpon a right line and a point given in it, to make an angle equall to any angle given.*

FIrst out of the point given describe an arke, cutting the same line: then by the 5. *Prop.* afore, find the chord of the angle given agreeable to the semidiameter, and inscribe it into this arke: so a right line drawne through the point given, and the end of this chord, shall be the side that makes vp the angle.

Let the right line given be *AB*, and the point given in it be *A*, and let the angle given be 11 *gr.* 15 *m.* Here I open the compasses to any semidiameter *AB*, (but as oft as I may conveniently to the laterall semiradius) and setting one foote in *A*, I describe an occult arke *BC*; then I seeke out the chord of 11 *gr.* 15 *m.* and taking it with the compasses, I see one foote in *B*, the other crosseth the arke in *C*, by which I draw the line *AC*, and it makes up the angle required

11 *To divide the circumference of a circle into any parts required.*

IF 360 the measure of the whole circumference be divided by the number of parts required, the quotient giveth the chord, which being found will divide the circumference.

So a chord of 120 *gr.* will divide the circumference into 3 equall parts; a chord of 90 *gr.* into 4 parts; a chord of 72 *gr* into 5 parts; a chord of 60 *gr.* into 6 parts; a chord of 51 *gr.* 26 into 7 parts; a chord of 45 *gr.* into 8 parts; a chord of 40 *gr.* into 9 parts; a chord of 36 *gr*, into 10 parts; a chord of 32 *gr.* 44 *m.* into 11 parts; a chord of 30 *gr.* into 12 parts

In like maner if it be required to divide the circumference of the circle whose semidiameter is *AB*, into 32: first I take the semidiamerer *AB* and, make it a parallell chord of 60 *gr*; then because 360 *gr.* being divided by 32 the quotient will be 11 *gr.* 15 *m.* I find the parallell chord of 11 *gr*, 15 *m.* and this will divide the circumference into 32.

But

But here the parts being many, it were better to divide it firſt into fewer, and after to come over it againe. As firſt to divide the circumference into 4, and then each 4 parts into 8, or otherwiſe, as the parts may be divided.

12 *To divide a right line by extreme and meane proportion.*

THe line to be divided by extreme and meane proportion, hath the ſame proportion to his greater ſegment, as in figures inſcribed in the ſame circle, the ſide of an *hexagon* a figure of ſix angles, hath to a ſide of a *decagon* a figure of ten angles: but the ſide of a *hexagon* is a chord of 60 *gr.* and the ſide of a *decagon* is a chord of 36 *gr.*

Let *AB* be the line to be divided: if I make *AB* a parallell chord of 60 *gr.* and to this ſemidiameter find *AC* a chord of 36 *gr.* this *AC* ſhall be the greater ſegment, dividing the whole line in *C*, by extreme and meane proportion. So that,

As *AB* the whole line is unto *AC* the greater ſegment:
ſo *AC* the greater ſegment unto CB the leſſer ſegment.

Or let *AC* be the greater ſegment given: if I make this a parallell chord of 36 *gr.* the correſpondent ſemidiameter ſhall be the whole line AB, and the difference CB the leſſer ſegment.

A *C* *B*

Or let *CB* be the leſſer ſegment given: if I make this a parallell chord of 36 *gr.* the correſpondent ſemidiameter ſhall be the greater ſegment *AC* which added to CB, gives the whole line AB.

To avoid doubling of lines or numbers, you may put over the whole line in the *Sines* of 72 *gr.* and the parallell ſine of 36 *gr.* ſhall be the greater ſegment.

Or if you put over the whole line in the ſines of 54 *gr.* the parallell ſine of 30 gr. ſhall be the greater ſegment, and the parallell ſine of 18 gr. ſhall be the leſſer ſegment.

CHAP

CHAP. III,

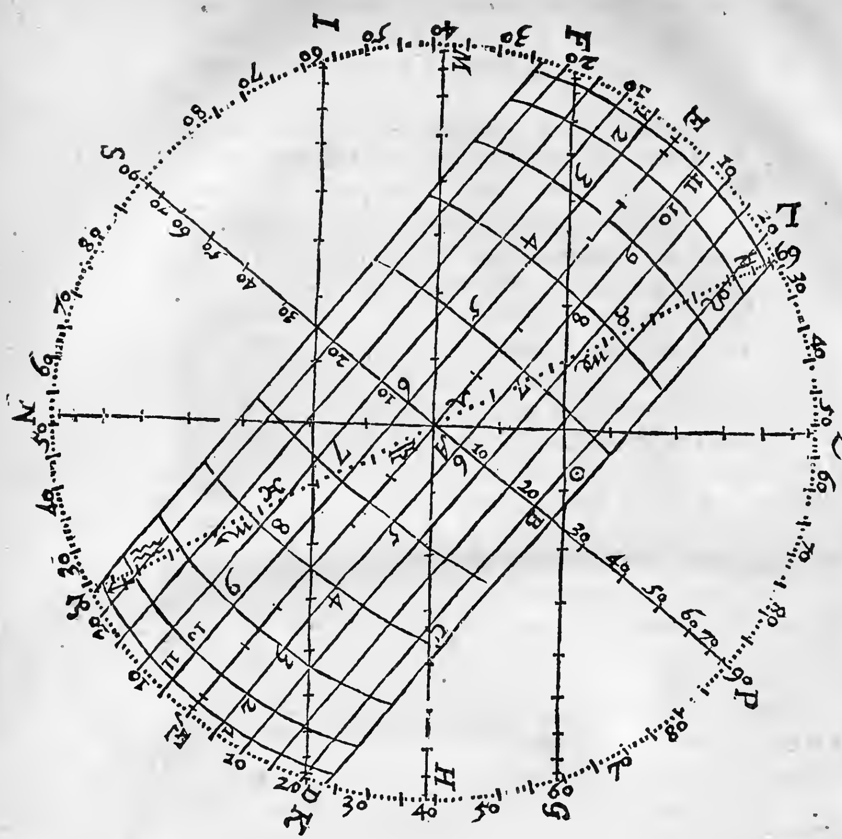
Of the proiection of the Sphere in Plano.

1 THe Sphere may be proiected in *Plano* in ſtreight lines, as in the *Analemma*, if the Semidiamiter of the circle given be divided in ſuch ſort as the line of *Sines* on the Sector.

As if the Radius of the cirle given were *A E*, the circle thereon deſcribed may repreſent the plane of the generall meridian, which divided into foure equal parts in *E*, *P*, *Æ*, *S*, and croſſed at right angles with *E Æ* and *P S*, the diamiter *E Æ*, ſhall repreſent the æquator, and *P S* the circle of the houre of 6. And it is alſo the Axis of the world, wherein *P* ſtands for the North pole, and *S* for the South pole. Then may each quarter of the meridian be divided into 90 *degrees* from the æquator towards the poles. In which if we number 23 *degr.* 30 *min.* the greateſt declination of the Sunne from *E* to ♋ North-wards, from *Æ* to ♑ Southwards, the line drawne from ♋ to ♑ ſhall be the ecliptique, and the lines drawne parallell to the equator through ♋ and ♑ ſhall be the tropiques.

Having theſe common ſections with the plane of the meridian, if we ſhall divide each Semidiameter of the Ecliptique into 90 *degr.* in ſuch ſort as the *Sines* are divided on the *Sector*. The firſt 30 *degr.* from *A* towards ♋, ſhall ſtand for the ſine of ♈. The 30 *degr.* next following for ♉. The reſt for ♊. ♋. ♌ &c. in their order. So that by theſe meanes we have the place of the *Sun* for all times of the yeare.

If againe we divide *A P* *A S*, in the like ſort, and ſet to the numbers 10. 20. 30. &c. unto 90 *degres*, the lines drawne through each of theſe degrees parallell to the equa-



tor, shall shew the declination of the Sunne, and represent the paralells of latitude.

If farther we divide *A E*, *A Æ*, and each of his paralells equally in the like sort, and then carefully draw a line through each 15 *degrees*, so as it makes no angles; the lines so drawne shall be *elipsicall*, and represent the houre-circles.

cles. The meridian *P E S*, the houre of 12 at noone; that next unto it drawne through 75 *degrees* from the Center the houres of 11 and 1, that which is drawne through 60 *degrees* from the center the houres of 10 and 2. &c.

To these wee may adde the monthes of the yeare, and the dayes of each moneth, placing *Ianuarie* about *F*, *March* about *E*, *Iune* about *I*, *Iulie* about *K*, *September*, about *E Æ*, *December*, about the *Tropique* of ♑: and so the rest according to their Declination from the Æquator.

Then having respect unto the latitude, we may number it from *E* Northward unto Z, and there place the Zenith: by which and the center the line drawne Z *A N* shall represent the verticall Circle, passing through the Zenith and Nadir East and West, and the line *M A H* crossing it at right angles, shall represent the horizon.

These two being divided in the same sort as the ecliptique and the æquator, the line drawne through each degree of the Semidiameter *A* Z, parallell to the horizon, shall be the Circles of altitude; and the divisions in the horizon and his parallells shall give the azimuth.

Lastly, if through 18 *gr.* in *A N*, be drawne a right line *I K* parallel to the horizon, it shall shew the time when the day breaketh, and the end of the twilight.

For example of this proiection, let the place of the Sun be the last degree of ♉, the parallell passing through this place is *L D*, and therefore the meridian altitude *M L*, and the depression below the horizon at midnight *H D*: the semidiurnall arke *L C*, the seminoctnruall arke C D, the declination *A B*, the ascentionall difference *B C*, the amplitude of ascention *A C*. The difference betweene the end of twilight and the day breake is very small; for it seemes the paralrell of the Sun doth hardly crosse the line of twilight.

If the altitude of the Sunne begiven, let a line bee drawne from it parallell to the horizon; so it shall crosse the parallell of the *Sunne*, and there shew both the azimuth and the houre of the houre of the day. As if the place of the *Sunne* being given as before, the Altitude in the morning were found to be 20 *degr.* the line *F G*, drawne parallell to the horizon through 20 *degrees* in *A Z*, would crosse the parallell of the Sun in ⊙. Wherefore *F* ⊙ sheweh the azimuth, and *L* ⊙. the quantitie of houres from the meridian. It seemes to be about halfe on houre past 6 in the morning, and yet more then halfe a point short of the East.

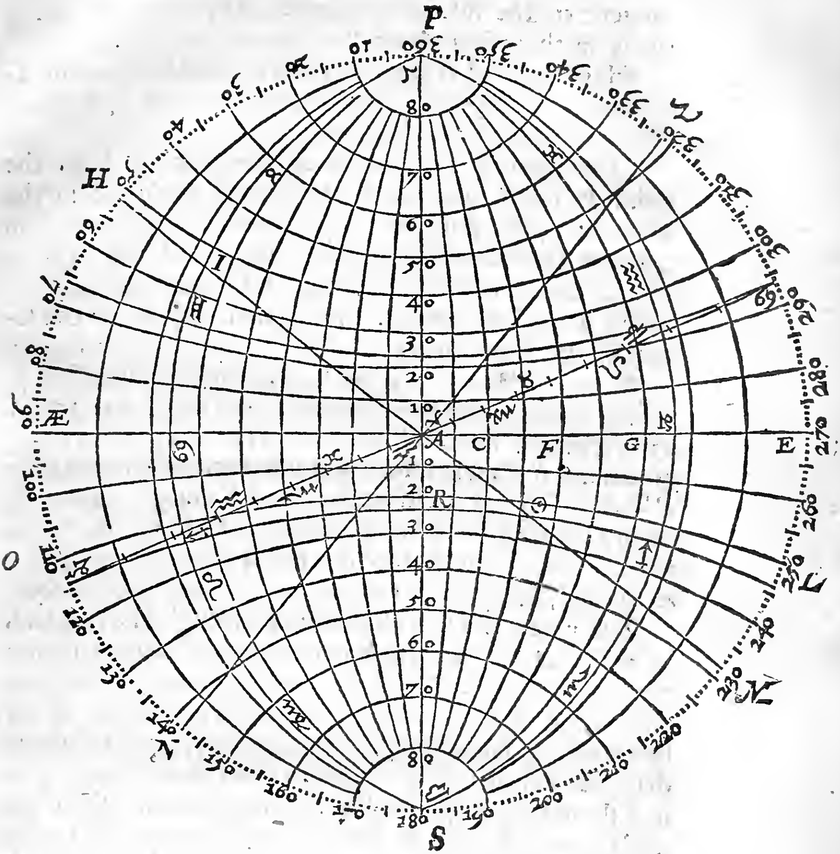
The distance of two places may be also shewed by this proiection, their latitudes being knowne, and their difference of longitude.

For suppose a place in the East of *Arabia*; having 20 *degr.* of North latitude, whose difference of longitude from London, is found to be an Eclipse to be 5 *houres* $\frac{1}{2}$. Let Z be the Zenith of London, the parallell of latitude for that other place must be *L D*, in which the difference of longitude si L ⊙. Wherefore ⊙ representing the site of that place, I drawe through ⊙ a parallell to the horizon *M H*, crossing the verticall A Z neare about 70 *degres* from the zenith, which multiplied by 20, sheweth the distance of London, and that place to be 1400 leagues. Or multiplyed by 60, to be 4200 miles.

2 The Sphere may be proiected in *plano* by circular lines, as in the generall Astrolabe of *Gemma Frisius*, by the help of the tangent on the side of the *Sector*.

For let the circle given represent the plane of the generall meridian as before; let it be divided into foure arts, and crossed at right angles with *E Æ* the equator, and *P S* the circle of the houre of 6, wherein *P* stands for the North pole, and *S* for the South pole. Let each quarter of the meridian be divided into 90. *degres* and so the whole into 360, beginning from *P*, and

and setting to the numbers of 10, 20, 30. &c. 90 at Æ, 180 at *S*, 270 at *E*, 360 at *P*. The semidiameters



A P, *A Æ*, may be divided according to the tangents of halfe their Arkes, that is a tangent of 45 *degrees*, which is alwayes 100000 equall to the Radius, shall give

 the

the semidiamiter of 90 *degrees* a tangent of 40 *degrees* 83910, shall give 80 *degrees*; in the semidiamiter: a tangent of 35 *degrees* 70021 shall give 70. &c. So that the semidiameters may bee divided in such sort as the tangent on the side of the Sector, the difference being onely in their denomination.

Having divided the circumference and the semidiameters, we may easily draw the meridians and the parallels by the help of the Sector.

The meridians are to be drawne through both the poles *P* and *S*, and the *degrees* before graduated in the æquator. The distance of the center of each meridian from *A* the center of the plane, is equall to the tangent of the same meridian, reckoned from the generall meridian *P Æ S E*, and the semidiameter equall to the secant of the same *degree*.

As for example, if I should drawe the meridian *P B S*, which is the tenth from *P Æ S*, the tangent of 10 *gr.* 17633, giveth me *A C*, and the secant of 10 *gr.* 101543, giveth me *S C*, wherefore *C* is the center of the meridian *P B S*, & *C S* his semidiameter: so *A F* a tangent of 20 *gr.* 36397 sheweth *F* to be the center of *P D S*, the twentith meridian from *P Æ S* & *A G* a tangent of 23 *gr.* 30 *m.* 43481, sheweth *G* to be the center of *P 69 S*. &c.

The parallels are to be drawne through the degrees, in *A P*, *A S*, and their correspondent *degrees* in the generall meridian. The distance of the center of each parallell from *A* the center of the plane, is equall to the secant of the same parallell from the pole, and the semidiameter equall to the tangent of the same *degree*. As if I should draw the parallell of 80 *degrees* which is the tenth from the pole *S*, first I open the compasses unto *AC* the tangent of 10 *degrees* 17633, and this giveth me the semidiameter of this parallell, whose center is a little from *S*, in in such distance as 101543 the secants *S C* is longer then 100000, the Radius *S A*.

The meridians and parallels being drawne, if we number

ber the 23 *degr.* 30 *m.* from *E* to ♋ Northwards, from *Æ* to ♑ Southward, the line drawne from ♋ to ♑ shall be the eclíptique: which being divided in such sort as the semidiameter *A P*, the first 30 *degr.* from *A* to ♋ shall stand for the sine of ♈; the 30 *degr.* next following for ♉; the rest for ♊ ♋ ♌. &c. in their order.

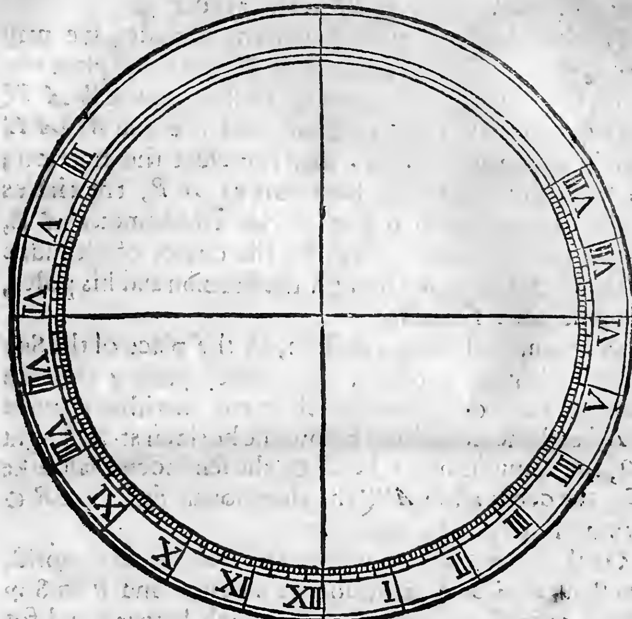
If farther we have respect unto the latitude, we may number it from *E* Northward unto *Z*, and there place the zenith, by which and the center, the line drawne Z *A N* shall represent the verticall circle, and the line *M A H* crossing it at right angles, shall represent the horizon; and these divided in the same sort as *A P*, the circles drawne through each degree of the semidiameter *A Z*, parallell to the horizon, shall be the circles of altitude: and the circles drawne through the horizon and his poles, shall giue the Azimuths.

For example of this proiection, let the place of the Sun be in the beginning of ♒, the parallell passing through this place is ♒ ⊙ *L*, and therefore the meridian altitude *M L*, and the depression below the horizon at midnight *H O*, the semidiurnall arke *L* ⊙, the seminocturnall arke *O* ⊙, the declination *A R*, the ascensionall difference *R* ⊙ the amplitude of ascention *A* ⊙.

Or if *A* be put to represent the pole of the world, then shall *P Æ S E* stand for the æquator, and P ♋ S ♑ for the eclíptique, and the rest which before stood for meridians, may now serue for particular horizons, according to their severall elevations. Then suppose the place of the Sunne given to be 24 *degrees* of ♉, his longitude shall be *P I*, his right ascention *P H*, his declination *H I*. And if the place given be 19 *degr.* of ♌, his longitude shall be *P K*, his right ascention *P N*, his declination *N K*. Againe, the declination brought to the horizon of the place, shall there shew the ascentionall difference, amplitude of ascention, & the like conclusions of the globe. But I intend not here to shew the vse of the Astrolabe, but the vse of the *Sector* in proiection.

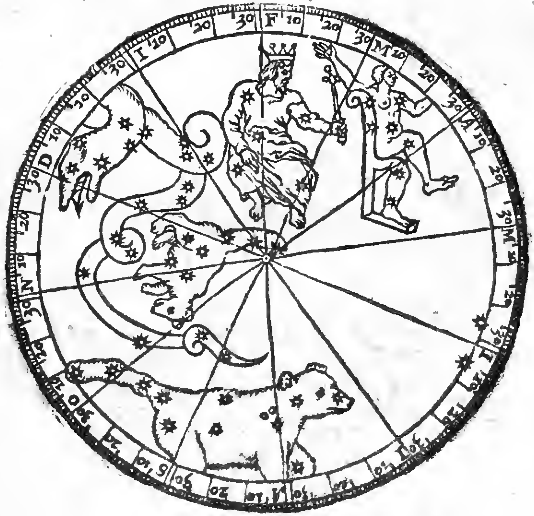
And

And after this manner may a nocturnall be proiected to shew the houre of the night, whereof I will set downe a type for the vse of Sea-men.



It consists as you see of two parts, the one is a plane, divided equally according to the 24 houres of the day, and each houre into quarters or minutes, as the plane will beare: the line from the center to XII, stands for the meridian, and XII, stands for the houre of 12 at midnight. The other part is a rundle for such starres as are neare the North pole, together with the 12 moneths, and the dayes of each moneth fitted to the right ascention of the starres. Those that haue occasion to see the South

The Nocturnall



Pag. 72, of the *Sector*.

South pole, may do the like for the Southerne constellations, and put them in a rundle on the back of this plane, and so it may serve for all the world.

The vse of this nocturnall is easie and ready. For looke vp to the pole, and see what starres are neare the meridian, then place the rundle to the like situation, so the day of the moneth will shew the houre of the night.

3 The Sphære may be proiected in *plano* by circular lines, as in the particular Astrolabe of *Iohn Stophlerin*, by help of the tangent, as before.

For let the circle given represent the tropique of ♑, let it be divided into foure parts, and crossed at right angles with *A C* the equinoctiall coloure, and *M B* the solstitiall coloure, and generall meridian, the center *P* representing the pole of the world. Let each quarter be divided into 90 *degrees*, and so the whole into 360, beginning from *A* towards *B*. The meridian *P M*, or *P B*, may be divided according to the tangent of halfe his arke. So as the aker from the North pole to the tropique ♑, being 90 *degrees* and 23 *degrees* 30 *m.* that is 113 *degrees* 30 *m.* and the halfe arke 56 *degrees* 45 *m.* the meridian shall be divided into 90 *degrees* and 23 *degrees* 30 *m.* in such sort as the tangent of 56 *degrees* 45 *m.* on the side of the *Sector* is divided into *degrees* and halfe *degrees*; of which *P Æ* the arke of the æquator 90 *degrees* from the pole, shall be given by the tangent of 45 *degrees*. And *P* 69 the arke of the Summer tropique 66 *degrees* 30 *m.* from the pole, shall be given by the tangent of 33 *degrees* 15 *m.* And the circles drawne vpon the center *P* through Æ. and ♋, shall be the æquator, and the Summer tropique.

Having the æquator and both the tropiques, the ecliptique ♈ ♋ ♎ ♑ shall be drawne from the one tropique to the other, through the intersection of the æquator and the Equinoctiall colure. And it may be divided first into the twelue signes after this manner: *P E* the arke of the pole of the ecliptique 23 *degrees* 30 *m.*

 from

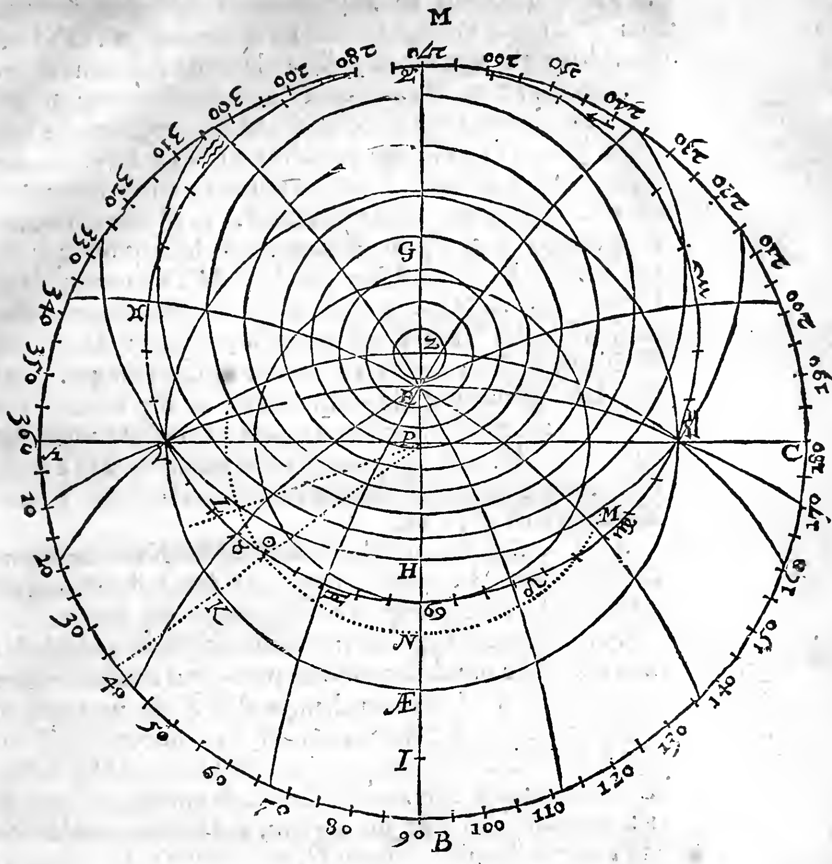
from the pole of the world, ſhall be given by the tangent of 11 *degrees* 45 *m.* The center of the circle of longitude paſſing through this pole E ♈ and ♎, ſhall be found at D (ſomewhat belowe B) by the tangent of 66 *degrees* 30 *m.* Then through D draw an occult line parallell to A C. and divide it on each ſide from D, in ſuch ſort as the tangent is divided on the ſide of the Sector, allowing 45 *degrees* to be equall to D E, So the thirtith *degree* from D toward the right hand, ſhall be the center of the circle of longitude paſſing through E ♉ and ♍. The ſixtith *degree*, the center of ♊ E ♐. The thirtith *degree* from D towards the left hand, the center of ♓ E ♍. The ſixtith, the center of ♒ E ♌. And the other intermediate *degrees* ſhall be the centers to divide each ſigne into 30 *gr.*

If farther we have reſpect unto the latitude, we may (the meridian being before divided) number it from P North-ward unto H, and there place the North interſection of the meridian and horizon: then the complement of the latitude being numbred from P Southward unto Z, ſhall there give the zenith; and 90 *degr.* from Z Southward unto F, ſhall there give the South interſection of the meridian and horizon. The middle betweene F and H ſhall be G the center of the horizon ♈ H ♎ F, paſſing through the beginning of ♈ and ♎. unleſſe there be ſome former errour.

All parallels to the horizon may be found in like ſort by their interſections with the meridian, and the middle betweene thoſe interſections is alwayes the center.

The Azimuths may be drawne as the circles of longitude were before. For the circle of the firſt verticall ♈ Z ♎ will be found at I (ſomewhat neere unto B) by the tangent of the latitude. And if through I we draw an occult line parallell to A C, and divide it on each ſide from I, in ſuch ſort as the tangent is divided

on



on the side of the *Sector*, allowing 45 *degrees* to be equall to *L Z*, these divisions shall be the centers, and the distance from these divisions unto Z, shall be the semidiameters whereon to describe the rest of the Azimuths.

For example of this proiection, let ☉ the place of the Sunne given be 10 *degr.* of ♉: a right line drawne from *P* through this place unto the æquator, shall there shew his right ascention ♈ *K*, and his declination *K* ☉. Then may we on the center *P* and semidiamiter ☉ *P*, draw an occult parallell of declination, crossing the horizon in *L* and *M*, the meridian in *G* and *N*. So the right lines *P L* and *P M* produced, shall shew the time of the Sunnes rising and setting, ♈ *Q* the difference of ascention, ♎ *R* the difference of descention, ♈ *L* the amplitude of his rising, and ♎ *M* the amplitude of his setting. *L* N *M* sheweth the length of the night. Z *G* sheweth his distance from the zenith at noone, *H N* his depression below the horizon at midnight. And then having the altitude of the Sunne at any time of the day, the intersection of the parallell of altitude with the parallell of declination, sheweth the Azimuth, and a right line drawne from *P* through this intersection, giveth the houre of the day.

4 The Sphære may be proiected in *plano* by circular lines, after the maner of the old concave hemisphære, by the help of the tangent on the side of the *Sector.*

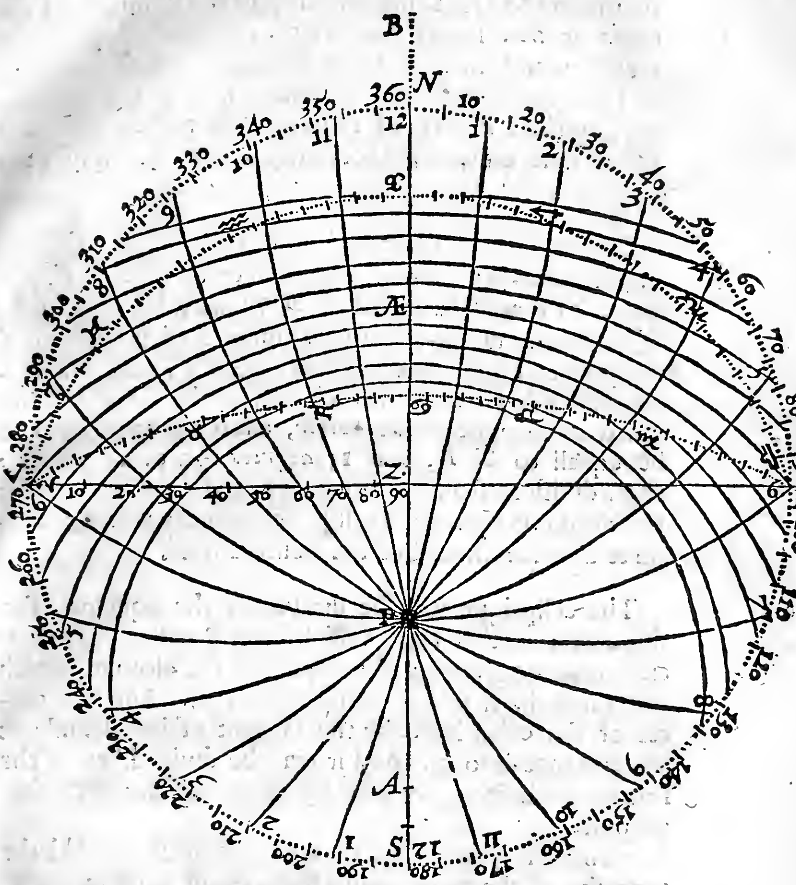
For let the circle given represent the plane of the horizon, let it be divided into foure parts, and crossed at right angles with S *N* the meridian, and *E V* the verticall; so as S may stand for the South, *N* for the North, *E* for the East, *V* the West part of the horizon, and the center Z represent also the zenith. Let each quarter of the horizon be divided into 90 *degrees*, and so the whole into 360 *degre.* beginning from N, and setting to the numbers of 10.20.30. &c. 90 at *E*, 180 at *S*, 270 at *V*, 360 at *N*.

The semidiamiter Z *N*, Z *S*, may be divided according to the tangent of halfe their arkes: so as the arke from the zenith to the horizon being 90 *gr.* and the halfe arke 45 *gr.* the semidiamiters are to be divided in such sort as the tangent of 45 *gr.* as was shewed before in the second proiection. And if from Z we draw circles through each

of

of theſe diviſions, they ſhall be parallels of altitude.

Then having reſpect unto the altitude, we may (the meridian being before divided) number it from Z to Æ, and there place the interſection of the meridian and æquator. The complement of the latitude from Z unto *P*,



ſhall there give the pole of the world, and 90 further from *P* ſhall there give the other interſection of the meridian and æquator.

The middle betweene these intersections shall be *A* the center of the æquator, passing through *E* and *V*, unlesse there be some former errour. The intersections of the tropiques depend on the æquator. From Æ 23 *degrees* 30 *m*. farther shall be ♑, the intersection of the meridian and the Southerne tropique. From Æ 23 *degrees* 30 *m*. nearer shall be ♋, the intersection of the meridian and the Northerne tropique. The intersections of the other intermediate parallels, shall be given in like sort, by their *degrees* of distance from the æquator, and the middle betweene those intersections is alwayes the center.

The houre circles may be here drawne as the Azimuths in the third proiection. For the center of *E* P *V*, the houre of 6 will be found at *B* (somewhat neare unto *N*) by the tangent of the latitude. And if through B we draw an occult line parallell unto *E V*, and divide it on each side from B, in such sort as the tangent is divided on the side of the *Sector*, allowing 45 *degrees* to be equall to *B P*, and 15 *degrees* for every houre: those divisions shall be the centers, and the distanee from the divisions unto *P*, shall be the semidiameters, wheron to describe the rest of the houre circles.

The ecliptique may be drawne as the æquator. For the center of that halfe which hath Southerne declination, shall be given by the tangent of the altitude, which the Sunne hath in his entrance into ♑. And the center of the other halfe, by the tangent of his altitude, at his entrance into ♋. And it may be divided, as in the former proiection, or else by tables calculated to that purpose.

To these circles thus drawne, if we shall addd the moneths of the yeare, and the dayes of each moneth, as we may well doe, at the horizon, on either side betweene

betweene the tropiques; this proiection shall be fitted for the most vsefull conclusions of the Globe.

For the day of the moneth being given, the parallell that shooteth on it, doth shew what declination the Sunne hath at that time of the yeare. And where this parallell crosseth the ecliptique, there is the place of the Sunne. Or the place of the Sunne being first given, the parallel which crosseth it, shall at the horizon shew the day of the moneth. Either of these then being given, or onely the parallell of declination, we may follow it first unto the horizon, there the distance of the end of the parallell from E or *V*, sheweth the amplitude; the same among the houre circles sheweth the time, when the Sunne riseth or setteth. Then having the altitude of the Sunne at any time of the day, the intersection of the parallell of declination with the parallell of altitude, sheweth the houre of the day; and a right line drawne from *Z*, through this intersection to the horizon, giveth the Azimuth.

Thus in either of these proiections, that which is otherwise most troublesome, is easily done by the help of the tangent line: and what I have said of this line, the same may be wrought by scale and numbers out of the table of tangents.

CHAP. IV.

Of the resolution of right-line Triangles.

In

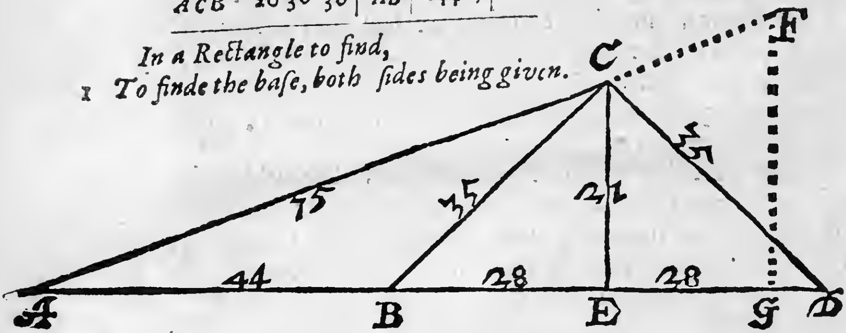
IN all Triangles there being sixe parts, viz. three angles and three sides, any three of them being given, the rest may be found by the *Sector*.

As may appeare by the *Prop.* following, wherein for our practise we may vse these triangles CEA, CEB, CED, are rectangle in B, and AGF rectangle in G, the rest consist of oblique angles.

Ang.	*Gr.*	*M.*	*S.*	*Lin.*	*Parts.*	*Ang.*	*Gr.*	*M.*	*S.*	*Lin.*	*Parts.*
E	90	0	0	*AC*	75	*BCE*	53	7	48	*BD*	28
G	90	0	0	*AF*	100	*ECD*	53	7	48	*AD*	28
A	16	15	36	*FG*	28	*BCD*	106	15	36	*BE*	56
D	36	52	12	*CE*	21	*ACD*	126	52	12	*ED*	100
B	36	52	12	*CD*	35						
B	143	7	48	*CB*	35						
AFG	73	44	12	*AG*	96						
ACE	73	44	12	*AE*	72						
ACB	20	36	36	*AB*	44						

In a Rectangle to find,

1 *To finde the base, both sides being given.*



Let the *Sector* be opend in the line of *lines* to a right angle, (as before was shewed *Cap.* 2. *Prop.* 7.) then take out the sides of the triangle, and lay them, one on one *line*, the other on the other *line*, so as they meete in the center, & marke how farre they extend. For the *line* taken from the termes of their extension, shall be the base required, viz. the side opposite to the right angle.

Or adde the squares of the two sides (as in *Prop.* 4. Superfic.) and the side of the compound square shall be the base.

As if the lines *AE*, *CE*, should be the sides about the right angle, and it were required to find the base subtending the right angle.

To

betweene the tropiques; this proiection shall be fitted for the most vsefull conclusions of the Globe.

For the day of the moneth being given, the parallell that shooteth on it, doth shew what declination the Sunne hath at that time of the yeare. And where this parallell crosseth the ecliptique, there is the place of the Sunne. Or the place of the Snnue being first given, the parallell which crosseth it, shall at the horizon shew the day of the moneth. Either of these then being given, or onely the parallell of declination, we may follow it first unto the horizon, there the distance of the end of the parallell from E or *V*, sheweth the amplitude; the same among the houre circles sheweth the time, when the Sunne riseth or setteth. Then having the altitude of the Sunne at any time of the day, the intersection of the parallell of declination with the parallell of altitude, sheweth the houre of the day; and a right line drawne from Z, through this intersection to the horizon, giveth the Azimuth.

Thus in either of these proiections, that which is otherwise most troublesome, is easily done by the helpe of the tangent line: and what I have said of this line, the same may be wrought by scale & numbers out of the table of tangents.

CHAP. IV.

Of the resolution of right-line Triangles.

IN all Triangles there being sixe parts, viz. three angles and three sides, any three of them being given, the rest may be found by the *Sector*.

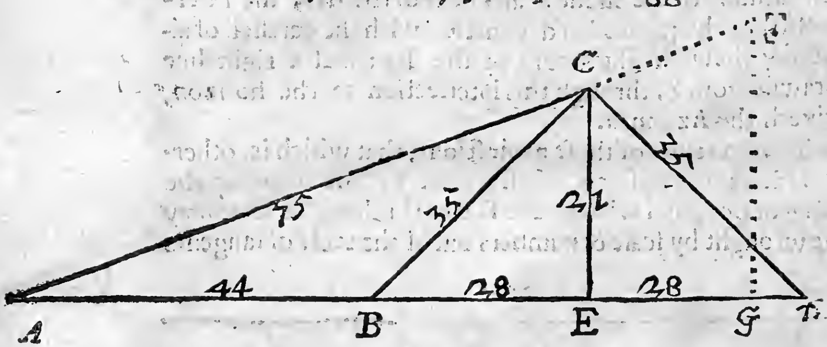
As may appeare by the *Prop.* following, wherein for ou practise we may vse these triangles CEA, CEB, CED are rectangle in E, and AGF rectangle in G the rest consist of oblique angles.

Let

Ang.	Gr. M. S.	Lin.	Parts.	Ang.	Gr. M. S.	Lin.	Parts
E	90 0 0	AC	75	BCE	53 7 48	BD	28
G	90 0 0	AF	100	ECD	53 7 48	AD	28
A	16 15 36	FG	28	BCD	106 15 36	BE	56
D	36 52 12	CE	21	ACD	126 52 12	ED	100
B	36 52 12	CD	35				
B	143 7 48	CB	35				
AFG	73 44 12	AO	96				
ACE	73 44 12	AE	72				
ACB	20 36 36	AB	44				

In a Rectangle right line Triangle,

1 *To finde the base, both sides being given.*



Let the *Sector* be opened in the line of *lines* to a right angle, (as before was shewed *Cap. 2. Prop. 7.*) then take out the sides of the triangle, and lay them, one on one *line*, the other on the other *line*, so as they meete in the center, & marke how farre they extend. For the *line* taken from the termes of their extension, shall be the base required, viz. the side opposite to the right angle.

Or adde the squares of the two sides (as in *Prop.* 4. Superfic.) and the side of the compound square shall be the base.

As if the lines *AE*, *CE*, should be the sides about the right angle, and it were required to find the base subtending the right angle. First

First, I set the line of *Lines* to a right angle by applying the whole line of 10 from 6 in the one line to 8 in the other. Then if the greater of the two lines given be lesse then the line of *Lines*, I take the greater of them *A E*, and transferr it with the compasses into one of the lines of *lines*, and find, that, in my *Sector* (which is 14 inches long, and so, the line of *Lines* almost 7 inches) it reacheth from the center to 518.

Againe, I take the lesser line *C E*, and transferr it into the other line of *Lines*, and find, that it reacheth from the center unto 151. wherefore I take the distance from 151 unto 518, and such is the length of the Base *A C* required.

If either of the lines given be too large for the *Sector*, then I may measure them by feet or inches, and suppose I find the length of *A E* to be about 720, and of *C E*, 210 Then, in the line of *Lines* (being set, one perpendicular to the other, as before) I extend the Compasses from 210 unto 720; and measuring this extent in the line of *lines*, find it to be 750 parts. wherefore, I prick downe 750 parts, in the line *A C*, from the same scale by which I measured *A E*, and *C E*. So, this line *A C* shall be the Base required.

In working by the line of *Superficies*. I need no opening of the *Sector*. For, taking the line *C E* with my compasses, and measuring it in the line of *Superficies* upon my *Sector*, I find it neere 13. parts.

Then taking the line *A E*, I find it to be about 269; These two being added together make 292: and this extent is the length of the base *A C*. required.

2 *To find the base by having the angles and one of the sides given.*

Take the side given, and turne it into the parallell sine of his opposite angle; so the parallell Radius shall be the base.

As if the line *A E* were the side of a rectangle triangle opposite to an angle of 73 *gr.* 45′, and it were required to find the Base.

First, I take the side *A E* with my compasses, and set it it over in the sines of 73 *Gr.* 45′. So, the parallell radius taken from between 90 and 90, will give the Base A C required.

If the side given be such as cannot well be fitted over in the sines of his opposite angle, I may measure it by feet or inches, and suppose I find the length of A E to be 720. then would I take 720 parts, out of the line of lines, and make it a parallell Sine of 73 *gr.* 45′. So, the parallell Radius taken from between 90 and 90, and measured in the line of *lines* will be found to be about 750 parts: wherefore, I pricke downe 750 in the line A C, by the same scale, whereby I measured A E: and this line A C shall be the Base required.

3 *To find a side by having the base, and the other side given.*

Let the Sector be opened in the lines of *lines* to a right angle, and the side given laid on one of those lines from the center: then take the base with a paire of compasses, and setting one foote in the terme of the given side, turne the other to the other line of the Sector, and it shall there shew the side required.

Or take the square of the side out of the square of the base (as in *Prop.* 4. *Superf.*) and the side of the remaining square shall be the side required.

Thus having A C for the Base, and C E, for the side of a rectangle triangle, the other side will be found to be A E.

Or, if A C, being measured, be 750, and C E, 210, the other side A E will be found to be 720.

4 *To find a side having the base. and the angles given.*

Take the base given, and make it a parallell Radius; so the

the parallell *sines* of the angles, shall bee the the opposite sides required.

Thus in the Rectangle A E C, if A C be made a parallell Radius, the parallell sine of *73 gr.* 45′ will give the side *A E*; and the parallell sine of *16 Gr.* 15′ will give the other side *C E*.

5 *To find a side by having the other side and the angles given.*

Take the side given, and turne it into his parallell *sine* of his opposite angle: so the parallell *sine* of the complement shall be the side required.

Thus in the Rectangle D E C, if C E be made a parallell sine of 53 *Gr.* 8′ the parallell sine of 36 *Gr.* 52′. will give the side E D: and the parallell sine of 90 *gr*, will give the Base C D.

6 *To find the angles by having the base and one of the sides given.*

First, take out the base given, and laying it on both sides of the Sector, so as they may meete in the center, and marke how farre it extendeth. Then take out the laterall Radius, and to it open the Sector in the termes of the base. This done, take out the side given, and place it also on the same lines of the Sector from the center. For the parallell taken in the termes of this side, shall be the sine of his opposite angle,

Or take the base given, and make it a parallell Radius; then take the side given, and carrie it parallell to the base, till it stay in like *sines*: so they shall give the quantitie of the opposite angle.

Thus in the Rectangle *A E C* having the Base A C, and the side A E, you may finde the angle C A E, to be 16 *gr.* 15′.

7 *To find the angles by having both the sides given*

Take out the greater side, and lay it on both sides of the Sector, so as they meete in the center, and marke how farre it extendeth. Then take the other side, and to it open the Sector in the termes of the greater side; so the parallell Radius shall be the tangent of the lesser angle. The third angle is alwayes knowne by the complement.

Thus in the Rectangle *D E C*, having the sides *C E*, and *E D*, you may find the lesser angle *E C D* to be 36 g. 52′, and therefore the other angle *E D C* to be 53. 8′.

8. *The Radius being given, to find the tangent, and secant of any arke.*

9 *The tangent of any arke being given, to find the Secant thereof, and the Radius.*

10 *The secant of any arke being given, to find th tangent thereof, and the Radius.*

The tangent, and the secant, together with the Radius of every arke, do make a right angle triangle; whose sides are the *Radius* and *tangent*, and the base alwayes the *secant*; and the angles alwayes knowne by reason of the given arkes. As in the Rectangle *A E C*, if on the center *A*, and semidiameter *A E*, you describe a circle, then make *A E*, to be the Radius, and *E C*, a tangent of 16. 15 and *A C* a secant of 16 *gr.* 15′.

If you describe a circle on the center *C*, and semidiameter *C E*, then is *C E* the Radius and *E* A, a tanget of 73. 45′ and C A a secant of 73. 45.

Wherefore the solution is the same with those before.

In any right-lined triangle whatsoever,

11 *To find a side by knowing the other two sides, and the angle contained by them.*

Let the *Sector* be opened in the lines of *lines* to the angle given

given as I ſhewed before *cap 2 Prop. 7.* Then take out the ſides of the triangle, & laying them the one on the one line, the other on the other; ſo as they meete in the center, marke how far they extend. For the line taken betweene the termes of their extenſion, ſhall be the third ſide required.

As if A C and A *D* were two ſides of a right lined triangle conteining an angle of 16 *gr.* 16′ and it were required, to find the third ſide ſubtending this angle.

Firſt I ſet the lines to an angle of 16. 16′. by applying the ſine of 8 *gr.* 8′ over in the points of 50 and 50, in the line of lines. That done, I take the longer line A D, and transfer it with my compaſſes, into one of the lines of *lines*, and find it to reach from the center to 720.

Againe, I take the leſſer line A C, and transfer it into the other line of *lines*, where it reacheth from the center to 540. wherefore, I take the diſtance from 540 to 720, and ſuch is the length of the 3 ſide *C D* required.

Or (if the lines be given in meaſure) A D 100, and A C 75: I extend the compaſſes from 100 to 75, and meaſuring this extent in the line of *lines*, find to be 35. Whereupon I take 35 parts out of the ſcale, by which A C, and A D were meaſured and prick them downe in the line C D. So, this line C D, ſhall be the third ſide required.

12 *To find a ſide by having the other two ſides, and one of the adiacent angles, ſo it be knowne which of the other angles is acute or oblique*

Let the *Sector* be opened in the line of *lines* to the angle given, and the adiacent ſide laid on one of thoſe lines from the center; then take the other ſide with a paire of compaſſes, and ſetting one foote in the terme of the former given ſide, turne the other to the other line of the *Sector* which here repreſenteth the ſide required, and it ſhall croſſe it in two

 places

places; but with which of them is the terme of the side required, must be judged by the angle.

As if in the triangle following, the side *A C* being given, and the side *C D* and the angle *C A D* 16 *gr.* 16 *m.* it were required to find the side *A D.*

First I open the Sector in the line of *lines* to an angle of 16 *gr.* 16 *m.* and laying the adiacent side from the center *A*, find where it extendeth in *C*. Then I take the other side *C D* with the compasses, and setting one foote in C, & turning the other to the other line of the *Sector* I find that it doth crosse it both in B and D.

Or, (if the lines be given in measure) *A* C 75, and C *D* 35; I may take 35 out of the line of *lines* and setting one foote in 75, I shall find the other foote to crosse the other line of the *Sector*, both at 44 (answerable to *A B*) and at 100 (answerable to *A D.*)

So that it is uncertaine whither the side required be *A B* or *A D*, onely it may be judged by the angle. For if the inward angle where they crosse be obtuse, the side required is the lesser; if it be acute, it is the greater.

13 *To find a side by having the angles, and one of the other sides given.*

Take the side given, and turne it into the parallell sine of his opposite angle; so the parallell sines of the other angle shall be the opposite sides required.

As if in the triangle *A B C*, having the side *A D*, and knowing the angle *C A B* to be 16. *gr.* 16′, and the angle *A B C* to be 143. 8′, it were required, to find the two other sides, *A* C, and *B* C.

The three angles of a right-lined Triangle, are alwayes equall to 180 *Gr.* wherefore, I adde 16 *Gr.* 16′ unto 143. *gr.* 8′. and by the remainder to 180 *Gr.* find the third angle A C B opposite to the knowne side *A B*, to be 20 *gr.* 36′. Then, I take the side *A B*, and make it a parallell sine of 20 *gr.* 36′.

So,

So, his parallell sine of 16. 16′ will be the side *B* C; and the Parallell sine of 143.8′ will be the side *A* C.

Or, if measuring the side *A B* I find it to be 44; I may take 44 parts, either out of the line of *lines*, or out of any other scale of equall parts, and make it a Parallell sine, of 20 *gr* 36′. So his Parallell sine of 16 *gr.* 16′ measured in the same scale, will give 35 for the length of the side *B C* : and the parallell sine of 36 *gr.* 52′ will give 75, for the length of the other side *A C*.

When the angle comes to be above 90 *gr*; the sine of 80 *gr*; doth stand for a sine of 100 *gr*: and the sine of 70 *gr.* for a sine of 110 *Gr.* and so the rest; for those, which are their complements to 180. degrees.

14 *To find the proportion of the side by having the three angles*

Take the laterall sines of the angles, and measure them in the line of lines. For the numbers belonging to those lines do give the proportion of the sides.

Thus, in the two equi-angle triangles *A E C*, A G F, if you take the laterall sine of 90 *gr.* for the right angle at E and G, and measure it in the line of *lines*, you shalll find it to be 100. Then take the laterall sine of 16 *Gr.* 16′ for the common angle at A, you shall find it to be 28. Take the laterall sine of 73 *gr*, 44′ for the third angle at C and F, you shall find it to be 96. Such therefore is the proportion of the sides.

As 100. 96. 28. So are 75. 72. 21.

15 *To find an angle by knowing the Three sides.*

Let the two containing sides be layd on the lines of the *Sector*, from the center, one on one line, and the other on the other; and let the third side, which is opposite to the angle required,

required, be fitted over in their termes: so shall the *Sector* be opened in those lines to the quantitie of the angle required.

The quantitie of this angle is found as in *Cap*: 2 *Prop*. 8.

Thus having the 3 sides of the triangle A C D, to find the angle at *A*. I take the 2 conteining sides A D, A C and transfer them with my compasses into the lines of *Lines*: where I find the one to reach from the center, to 72; the other, to 54.

Then I take C D, (the side opposite to the angle at A) and fit that over betweene 72 and 54.

Or if the 3 sides be given in measure A D, 100; A C 75: C D 35: I might take 35 for the side C D out of the line of *Lines*, and set that over from 100 to 75. This don I take the distance betweene 50 and 50 and measuring it in the line of *Sines* I find it to be about about 8 *gr*. 8′. you double whereof is 16 *gr*. 16′ the angle required.

16 *To finde an angle by having two sides. and one adiacent angle.*

First take out the side opposite to the angle given, and laying it on both sides of the Sector, so as they meete in the center, marke how far it extendeth; then take out the laterall sine of the angle, and to it open the Sector in the termes of the first side: this done, take out the other side given, and place it also on the same lines of the Sector from the center, for the parallells taken in the termes of this side, shall be the sine of the angle opposite to the second side.

Or take out the side opposite to the angle given, and make it a parallell sine of that angle: then take the other side given and carrie it parallell to the former: till it stay in like sines: so they shall give the quantitie of the angle opposite to the second side.

Thus in the triangle A C *D*, knowing two sides A C, C*D*, with the angle C A *D* opposite to the side C D, you may find the angle A D C opposite to the other knowne side A C, to be about 36 *gr*: 52′.

17 To

17 To find an angle by having two sides, and the angle contained by them.

First find the third side by the 11. *Prop.* and then the angles may be found by the 15. or 16. *Prop.*

For observation of angles, the *Sector* may have sights set on the moveable foote; so that by looking through them, the edges of the Sector may be applied to the sides of the angle.

For measuring of the sides of lesser triangles, any scale may suffice, either of feete, or inches, or lesser parts. But for greater triangles, especially for plotting of grounds, I hold it fit to use a chaine of foure perches in length, each perch divided into 25, and the whole chaine an hundred links, wherein, if the whole chaine be (according to $16\frac{1}{2}$ foot in a perch) 66 foote (that is, 792 inches) each severall link will be 7 inches and $\frac{92}{100}$

If (according to 18. in the perch) the whole chaine be 72 feet in length (that is, 864 inches) then, each severall link will be 8 inches and $\frac{64}{100}$

For so the length being multiplied into the bredth, the five last figures give the content in roods and perches by this Table; the other figures toward the left hand, doe shew the number of acres directly.

As in a long square, where the length is 24 chaines $\frac{1}{4}$ the bredth 13. chaines $\frac{1}{2}$, the usuall way is, to resolve the chaines into perches: So the length is 97 perches and the bredth 54 perches. These multiplied one into the other make 5238 square perches and those (divided by 160) give 32. Acres, 2 roods, and 38 perches for the content required.

Links	R	P
100000	4	0
90000	3	24
80000	3	8
70000	2	32
60000	2	16
50000	2	0
40000	1	24
30000	1	8
20000		32
10000		16
9375		15
8750		14
8125		13
7500		12
6875		11
6250		10
5625		9
5000		8
4375		7
3750		6
3125		5
2500		4
1875		3
1250		2
625		1

But, reckoning by chaines and linkes, the length is 24 *ch.* 25 *lin.* the bredth 13 *ch.* 50 *links.* These multiplied one into the other make 32,73750 square linkes. Then, cutting of the 5 last figures, I find 32. *Acres* 73750 *lin.* such as an 100000 do make an acre. Of which 70000 are equall to two roods 32 perches: and the rest 3750 equall to 6 perches more (as appeareth by this table.) So, the whole content is 32 acres, 2 roods, 38 perches, as before.

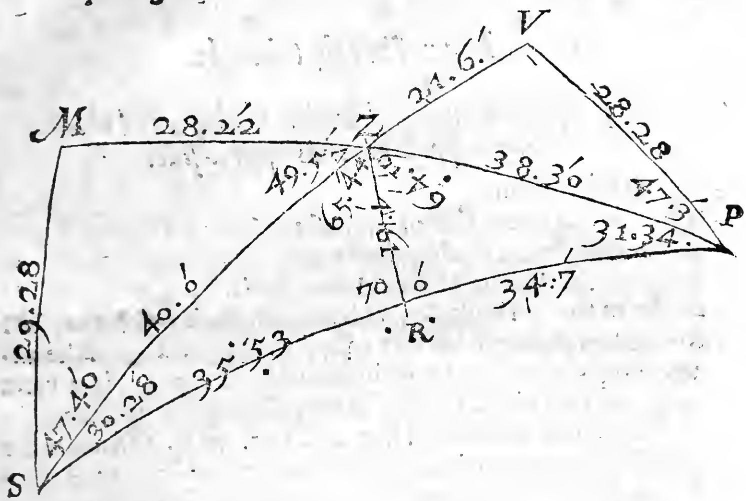
CHAP. V.

Of the resolution of sphæricall Triangles.

For our practise in sphæricall triangle, let *A* be the equinoctiall point, *A B* an arke of the ecliptique representing the longitude of the Sunne in the beginning of ♉, *B C* an arke of the declination from the Sunne to the equator, and *A C* an arke of the equator representing the right ascension.

Let B D be an arke of the horizon representing the amplitude

plitude of the Sunnes rising from the East, and *B E* an arke of the horizon for his setting from the West: so DC shall be the difference of ascension, and *C E* the difference of descension; A D the oblique ascension, and *A E* the oblique descension of the same place of the Sunne in our latitude at *Oxford* of 51 *gr.* 45 *m.* whose complement 38 *gr.* 15 *m.* is the angle at *E* and D. The triangles *A C B*, D *C B*, *E C B*, are rectangle in *C*: the other *A D B*, *A E B*, consist every way of oblique angles.



Or to fit an example nearer to the latitude of *London*. Let Z *P S* represent the zenith pole and Sun, Z *P* being 38 *Gr.* 30 *m.* the complement of the latitude, *P S* 70 *Gr.* the complement of the declination, and Z *S* 40 *Gr.* the complement of the Suns altitude. The angle at Z shall shew the azimuth, and the angle at *P*, the houre of the day from the meridian. Then if from Z to *P* S we let downe a perpendicular Z *R*, we shall reduce the oblique triangle into two rectangle triangles Z *R P*, Z *R* S. Or if from *S* to Z *P* we let downe a perpendicular *S M*, we shall reduce the same *Z P* S into two other triangles, *S M* Z, *S M P*, rectangle at *M*: whatsoever is said

N 2 of

of any of these triangles, the same holdeth for all other triangles in the like cases.

For the resolution of each of these, there be severall wayes. I onely chuse those which are fittest for the *Sector*, wherein if that be remembred which before is shewed in the generall use of the *Sector* concerning laterall and parallell entrance, it may suffice onely to set downe the proposition of the three parts given to the fourth required, and so I shew first by the *sines* alone.

In a rectangle triangle.

1 *To finde a side by knowing the base, and the angle opposite to the required side.*

As the Radius
 is to the sine of the base:
So the sine of the opposite angle
 to the sine of the side required.

As in the rectangle *ACB*, having the base *AB*, the place of the Sunne 30 *gr.* from the Equinoctiall point, and the angle *BAC* of 23 *gr.* 30 *m.* the greatest declination, if it were required to find the side *BC* the declination of the Sunne.

Take either the laterall sine of 23 *gr.* 30 *m.* and make it a parallell Radius; so the parallell sine of 30 *gr.* taken and measured in the side of the *Sector*, shall give the side required 11 *gr.* 30 *m.* Or take the sine of 30 *gr.* and make it a parallell Radius; so the parallell sine of 23 *gr.* 30 *m.* taken and measured in the laterall sines, shall be 11 *gr.* 30 *m.* as before.

So in the triangle *ZPS* having *ZP* 38 *gr.* 30 *m.* and the angle *P* 31 *gr* 34 *m.* given, we shall find the perpendicular *ZR* to be 19 *gr.* 1 *m*, or having *PS* 70 *gr.* and the said angle P 31 *gr.* 34 *m.* given, we may finde the perpendicular *SM* to be 29 *gr.* 28 *m.*

2 *To finde a side by knowing the base and the other side.*

As the sine of the complement of the side given

is to the Radius:
So the sine of the complement of the base
to the sine of the complement of the side required.

So in the rectangle A C B, having AB 30 *gr.* and B C 11 *gr.* 30 *m.* given, the side A C will be found 27 *gr.* 54 *m.*
Or in the rectangle Z *R* P having Z P 38 *gr.* 30 *m.* and Z *R* 69 *gr.* 1 *m.* given, the side *R P* will be found 34 *gr.* 7 *m.*

3 *To find a side by knowing the two oblique angles.*

As the sine of either angle
to the sine of the complement of the other angle:
So is the Radius
to the sine of the complement of the side opposite to the second angle.

So in the rectangle *A C* B, having *C A* B for the first angle 23 *gr,* 30 *m.* and *ABC* for the second 69 *gr.* 22 *m.* the side *A C* will be found 27 *gr.* 54 *m.* Or making *A B C* the first angle, and *C* A *B* the second, the side B *C* will be found 11 *gr.* 30 *m.*

4 *To finde the base by knowing both the sides.*

As the Radius
to the sine of the complement of the one side:
So the sine of the complement of the other side,
to the sine of the complement of the base required.

So in the rectangle *A C B* having A *C* 27 *gr.* 54 *m.* & B C 11 *gr.* 30 *m.* the base *A B* will be found 30 *gr.*

5 *To finde the base by knowing the one side, and the angle opposite to that side.*

As the sine of the angle given,
to the sine of the side given:
So is the Radius

to the sine of the base required.

So in the rectangle B C D, knowing the latitude and the declination, we may find the amplitude; as having B C the side of the declination 11 *gr.* 30 *m.* and B D C the angle of the complement of the latitude 38 *gr.* 15 *m.* the base B D which is the amplitude, will be found to be 18 *gr.* 47 *m.*

6 To find an angle by the other oblique angle, and the side opposite to the inquired angle.

As the Radius
to the sine of the complement of the side:
So the sine of the angle given,
to the sine of the complement of the angle required.

So in the rectangle A C B, having the angle B A C 23 *gr.* 30 *m.* and the side A C 27 *gr.* 54 *m.* the angle A B C will be found 69 *gr.* 21 *m.*

7 To finde an angle by the other oblique angle, and the side opposite to the angle given.

As the sine of the complement of the side
to the sine of the complement of the angle given:
So is the Radius
to the sine of the angle required.

So in the rectangle, A C B, having B A C 23 *gr.* 30 *m.* and B C 11 *gr.* 30 *m.* the angle A B C will be found 69 *gr.* 21 *m.*

8 To finde an angle by the base, and the side opposite to the inquired angle.

As the sine of the base
is to the Radius:
So the sine of the side
to the sine of the angle required.

So in the rectangle B C D, having B D 18 *gr.* 47 *m.* and B C 11 *gr.* 30 *m.* the angle B D C will be found 38 *gr.* 15 *m.*

These

These eight Propositions have been wrought by the *sines* alone; those which follow require joynt helpe of the *tangent*.

And forasmuch as the *tangent* could not well be extended beyond 63 *gr.* 30 *m.* I shall set downe two wayes for the resolution of each Proposition; if the one will not hold, the other may.

9 *To find a side by having the other side, and the angle opposite to the inquired side.*

1 As the Radius
to the sine of the side given:
So the tangent of the angle,
to tangent of the side required.

2 As the sine of the side given,
is to the Radius:
So the tangent of the complement of the angle,
to the tangent of the complement of the side required.

So in the rectangle A C B, having the side A C 27 *Gr.* 54 *m*, and the angle B A C 23 *Gr.* 30 *m.* the side B C will be found to be 11 *gr,* 30 *m.*

10 *To find, a side by having the other side, and the angle next to the inquired side.*

1 As the tangent of the angle,
to the tangent of the side given:
So is the Radius
to the sine of the side required.

2 As the tangent of the complement of the side,
to the tangent of the complement of the angle;
So is the Radius
to the sine of the side required.

This

This and the like, where the tangent standeth in the first place, are best wrought by parallell entrance. And so in the rectangle B C D, having B C the side of declination 11 *gr.* 30 *m.* and B D C the angle of the complement of the latitude 38 *Gr.* 15 *m.* the side D C, which is the ascensionall difference, will be found 14 *Gr.* 57 *m.*

By the ascensionall difference is given the time of the Sunnes rising and setting, and length of the day; allowing an houre for each 15 *gr.* and 4 minutes of times for each severall degree. As in the example the difference betweene the Sunnes ascension in a right sphere, which is alwayes at 6 of the clocke, and his ascension in our latitude being 14 *gr*, 57 *m.* it sheweth that the Sunne riseth very neare an houre before 6, because of the Northerne declination; or after 6, if the Sunne be declining to the Southward.

11 *To find a side by knowing the base, and the angle adiacent next to the inquired side.*

1 As the Radius
to the sine of the complement of the angle:
So is the tangent of the base,
to the tangent of the side required.

2 As the sine of the complement of the angle
is to the Radius:
So the tangent of the complement of the base,
to the tangent of the complement of the side required.

So in the rectangle A C B, knowing the place of the Sun from the next equinoctiall point, and the angle of his greatest declination, we may find his right ascension: viz. the base A B 30 *gr.* and the angle B A C 23 *gr.* 30 *m.* being given, the right ascension A C will be found 27 *gr.* 54 *m.*

12 *To find the base by knowing the oblique angles.*

As the tangent of the one angle,

to the tangent of the complement of the other angle:
So is the Radius
to the sine of the complement of the base.

So in the rectangle A C B, having B A C 23 *gr.* 30 *m.* and A B C 69 *gr:* 22 *m.* the base A B will be found 30 *gr.*

13 *To finde the base, by knowing one of the sides, and the angle adiacent next that side.*

1 As the Radius
is to the sine of the complement of the angle:
So the tangent of the complement of the side,
to the tangent of the complement of the base.

2 As the sine of the complement of the angle
is to the Radius
So the tangent of the side given,
to the tangent of the base required.

So in the rectangle *A C B*, having *A C* 27 *gr.* 54 *m.* and *B A C* 23 *gr;* 30 *m.* the base A B will be found 30 *gr;* 0 *m.*

14 *To find an angle, by knowing both the sides.*

1 As the Radius
is to the sine of the side next the inquired angle:
So the tangent of the complement of the opposite side,
to the tangent of the complement of the angle required.

2 As the sine of the side next the inquired angle,
is to the Radius:
So the tangent of the opposite side,
to the tangent of the angle required.

So in the rectangle A C B, having A C 27 *gr*, 54 *m.* and B C 11 *gr.* 30 *m.* the angle at *A* will be found 23 *gr.* 30 *m.* and the angle at B 69 *gr.* 21 *m.*

15 *To finde an angle, by knowing the base, and the side next adiacent to the inquired angle.*

1 As the tangent of the complement of the side,
to the tangent of the complement of the base:
So is the Radius
to the sine of the complement of the angle required.

2 As the tangent of the base,
to the tangent of the side:
So is the Radius,
to the sine of the complement of the angle required.

So in the rectangle BCD, having the base BD 18 *gr.* 47 *m.* and the side BC 11 *gr.* 30 *m.* the angle D B C between them will be found 53 *gr.* 15 *m.*

16 *To find an angle, by knowing the other oblique angle, and the base.*

1 As the Radius,
to the sine of the complement of the base:
So the tangent of the angle given,
to the tangent of the complement of the angle required.

2 As the sine of the complement of the base,
is to the Radius:
So the tangent of the complement of the angle given,
to the tangent of the angle required.

So in the rectangle A C B, having the angle at *A* 23 *gr.* 30 *m.* and the base A B 30 *gr.* the angle A B C will be found 69 *gr.* 22 *m.*

These sixteen cases are all that can fall out in a rectangle triangle: those which follow do hold.

In

In any sphericall triangle whatsoever

17 *To find a side opposite to an angle given, by knowing one side, and two angles, wherof one is opposite to the given side, the other to the side required.*

As the sine of the angle opposite to the side given,
is to the sine of that side given:
So the sine of the angle opposite to the side required,
to the sine of the side required.

So in the triangle A B E, having the place of the Sunne, the latitude, and the greatest declination, we may finde the amplitude. As having A B 30 *gr.* B A E 23 *gr.* 30 *m.* and AEB 38 *gr.* 15 *m.* the side B E which is the amplitude, will be found 18 *gr.* 47 *m.*

18 *To finde an angle opposite to a side given, by having one angle and two sides, the one opposite to the given angle, the other to the angle required.*

As the sine of the side opposite to the angle given,
is to the sine of that angle given:
So the sine of the side opposite to the angle required,
to the sine of the angle required.

So in the triangle *Z P S*, having the azimuth, and altitude, and declination, we may find the houre of the day. As having *P Z S* 130 *gr.* 3 *m.* *P S* 70 *gr.* and Z *S* 40 *gr.* the angle *Z P S*, which sheweth the houre from the meridian shall be found 31 *gr.* 34 *m.*

19 *To find an angle by knowing the three sides.*

This proposition is most usefull, but most difficult of all others: as in Arithmetique, so by the *Sector*, yet may it be performed severall wayes.

1 According to *Regiomontanus* and others.

As the sine of the lesser side next the angle required,
to the difference of the versed sines of the base and difference of the sides:
So is the Radius
to a fourth proportionall.
Then as the sine of the greater side next the angle required
is to that fourth proportionall:
So is the Radius
to the versed sine of the angle required.

So in the triangle Z *P S*, having the side *P S*, the cõplement of the declination 70 *gr.* 0 *m.* the side Z *P* the complement of the latitude 38 *gr;* 30 *m,* and the base Z *S* the complement of the altitude 40 *gr.* the angle of the houre of the day Z *P S* will be found 31 *gr.* 34 *m.* which is 2 *h.* 6 *m.* from the meridian.

For the base being 40 *gr.* 0 *m.* and the difference of the sides 38 *gr.* 30 *m.* and 70 *gr.* 0 *m.* being 31 *gr.* 30 *m.* the difference of their versed sines will be the same with the distance between the right sine of 50 *gr.* and 58 *gr.* 30 *m.* This difference I take out, and make it a parallell sine of the lesser side 38 *gr.* 30 *m.* so the parallell Radius will be the fourth proportionall. Then coming to the second operation, I make this fourth proportionall a parallell sine of the greater side of 70 *gr.* 0 *m.* and take out his parallell Radius. For this measured from 90 *gr.* toward the center, will be the versed sine of 31 *gr.* 34 *m.*

In the like sort in the same triangle Z *P* S, having the same complements giuen, the angle *P* Z S which is the azimuth from the North part of the meridian, will be found 130 *gr.* 3 *m.* For here the base opposite to the angle required being 70 *gr.* and the difference of the sides 38 *gr.* 30 *m.* and 40 *gr.* being 1 *gr.* 30 *m.* the difference of their versed sines will be the same with the distance betweene the right sines of 20 *gr.* and 88 *gr.* 30 *m.* This difference I take, and make it a parallell sine of the lesser side 38 *gr* 30 *m.* so the parallell Radius will be the fourth proportionall. Then coming to the second operation, I make this fourth proportionall a parallell sine of the

the greater side 40 *gr.* and take out his parallell Radius. For this measured from 90 *gr.* beyond the center in the lines of *sines* stretched forth at their full length, will be the versed sine of 130 *gr.* 3 *m.*

2. I may finde an angle by knowing three sides, by that which I have elsewhere demonstrated upon *Barth. Pitiscus*, and that at one operation in this manner.

As the sine of the greater side
is to the secant of the complement of the other side:
So the difference of sines of the complement of the base, and the arke compounded of the lesser side with complement of the greater,
to the versed sine of the angle required.

So in the same triangle Z *P* *S*, having the same complements given, the angle at *P*, which sheweth the houre from the meridian, will be found as before 31 *gr.* 34 *m.*

For the sides being 38 *gr.* 30 *m.* and 70 *gr.* 0 *m.* I take the secant of the complement of 38 *gr.* 30 *m.* and make it a parallell sine of 70 *gr*; then keeping the Sector at this angle, I consider that the complement of 70 *gr.* being 20 *gr.* added unto 38 *gr* 30 *m.* the compounded side (which is here the meridian altitude) will be 58 *gr* 30 *m*; and that the base being 40 *gr*, the difference of sines of the compounded side and the complement of the base will be (as before) the distance betweene the sines of 50 *gr.* and 58 *gr.* 30 *m.* Wherefore I take out this difference, and lay it on both the lines of *sines* from the center: so the parallell taken in the termes of this difference, and measured from 90 *gr.* toward the center, doth give the versed sine of 31 *gr.* 34 *m.*

This example, of finding the houre of the day might otherwise have been proposed in these termes.

As the sine of the complement of the declination,
is to the secant of the Latitude
So the diference between the sine of the altitude proposed, and the sine of the meridian Altitude.

to the versed sine of the houre from the Meridian.

Then the Latitude being 51 *g.* 30′, the declination 20 gr. northward, and the Altitude 50 gr. the worke would be the same as before.

The other angles *P* Z *S*, *P* S *Z*, may be found in the same sort; but having the sides and one angle, it will be sooner done by that which we shewed before in the 18 *Prop.*

20 *To find a side by knowing the three angles.*

If for the greater angle we take his complement to 180 gr, the angles shall be turned into sides, and the sides into angles, & the operation shall be the same, as in the former *Prop*-

As in the triangle Z P S, having the angle Z P S 51 *Gr.* 34′. Z S P 30 *gr.* 28′ and P Z *S* 130 *gr.* 3′, I would take the greater angle, of 130 *gr.* 3′. out of 180 *gr*, and there remaine 49 *gr.* 57′. Then as if I had a Triangle of 3 knowne sides, one of 51 *gr.* 34′, another of 30 *gr.* 20′ and a third of 49 *gr.* 57′, I would seeke the angle opposite to one of these sides, by the last Prop. So the angle which is thus found, would be the side which is here required.

21 *To find a side, by having the other two sides, and the angle comprehended.*

This proposition being the converse of the nineteenth, may be wrought accordingly; but the best way both for it and those which follow, is to resolve them into two rectangles, by letting downe a perpendicular, as was shewed in the first *Prop.*

So in the triangle Z P S, having Z P the complement of the latitude, and P S the complement of the declination, with *Z* P S the angle of the houre from the meridian, we may find Z S the complement of the altitude of the *Sunne.*

For having let downe the perpendicular Z *R* by the first

Prop

Prop. we have two triangles, *Z R* P, *Z R S*, both rectangle at *R*. Then may we finde the side P *R*, either by the second, or tenth, or eleventh *Prop.* which taken out of P *S*, leaveth the side *R S*: with this *R S* and *Z R* we may find the base *Z* S by the fourth *Prop.*

Or having let downe the perpendicular *S M*, we have two rectangle triangles S *M Z*, *S M P*. Then may we find *M P*, from which if we take *Z P*, there remaineth *M Z*: but with *M Z* and *S M*, we may find the base *Z S*.

22 *To find a side, by having the other two sides, and one of the angles next the inquired side.*

So in the triangle *Z P S* having *Z P* the complement of the latitude, and *P* S the complement of the declination, with *P Z* S the angle of the azimuth, we may finde *Z S* the complement of the altitude of the *S*unne.

For having *Z P*, and the angle at *Z*, we may to *S Z* produced, let downe a perpendicular *P V*. Then we have two rectangle triangles, P V *Z*, P V S, wherein if we find the sides V *Z*, V *S*, and take the one out of the other, there will remain the side required *Z S*.

23 *To finde the side, by having one side, and the two angles next the inquired side.*

So in the triangle A B D, having A B the place of the sun, and *B A D* the angle of the greatest declination, and *A D B* the angle of the equator with the horizon, we may find *A D* the oblique ascention.

For having let downe *B C* the perpendicnlar of declination, we have two rectangles triangles, *A C B*, *D C B*. Then may we find *A C* the right ascention, and D C the ascentionall difference; and comparing the one with the other, there remaineth *A D*.

24 *To find a side, by having two angles, and the side inclosed by them.*

So in the triangle Z *P S*, having the angles at Z and P, with the side intercepted Z P, we may find the side P S. For having let downe the perpendicular P V, we have two rectangles P V Z, P V S. Then may we find the angle V P Z, either by the seventh, or fifteenth or sixteenth *prop.* which added to Z P S, maketh the angle V P S, with this V P S. and P V, we may find the base P S, according to the 13 *Prop.*

25 *To find an angle by having the other two angles and the side inclosed by them.*

So in the triangle Z P *S*, having the angles at Z and P, with the side intercepted Z P, we may finde the other angle *Z S* P. For having let downe the perpendicular Z R, we have two rectangles Z R P, Z R *S*. Then may we finde the angle P Z R by the sixteenth *Prop.* and that compared with P Z *S*, leaveth the angle R Z *S*: with this R Z S and Z R we may find the angle required Z S R, according to the sixth *Proposition.*

26 *To finde an angle, by having the other two angles, and one of the sides next the inquired angle.*

So in the triangle A B D, having the angles at *A* and *D*, with the side A B, we may find the angle A B D. For having let downe the perpendicular B C, we have two rectangles, A C B, D C B. Then may we find the angles A B C, D B C, and take D B C out of A B C; for so there remaineth the angle required A B D.

27 *To find an angle, by knowing two sides, and the angle contained by them.*

So

So in the triangle *Z* P S, having the sides *Z* P, P S, with the angle comprehended *Z* P S, we may find the angle P *Z* S. For having let downe the perpendicular S M, we have two rectangles S *M Z*, S *M* P. Then may we find the side M P, and taking *Z* P out of *M* P, there remaineth *M Z*: with this *M Z* and the perpendicular *M S*, we may find the angle *M Z* S, by the fourteenth Prop. This angle M *Z* S, taken out 180 *gr.* there remaineth P *Z* S.

28 *To finde an angle by knowing the two sides next it and one of the other angles.*

So in the triangle *Z* P S, having the sides *Z* P and P S, with the angle P *Z* S, we may find the angle *Z* P S, For having let downe the perpendicular P *V*, we have two rectangles P V *Z*, P V S. Then may we find the angles *V* P Z, *V* P S; and taking *V* P Z out of *V* P S, there remaineth Z P S which was required.

These 28 cases are all that can fall out in any sphæricall triangle: if any do not presently understand them, let them once more reade over the use of the globes, and they shall soone become easie unto them.

CHAP. VI.

Of the vse of the Meridian line in Navigation.

THe *Meridian* line is here set on the side of the Sector streched forth at full length, on the same plane with the line of *lines* and *Solids*, and is diuided unequally toward 87 *gr.*

(whereof 70 *gr.* are about one halfe) in such sort as the Meridian in the Chart of *Mercators* proiection. The vse of it may be:

1 *To divide a sea Chart according to proiection.*

If a degree of the æquator on the sea-chart be equall to the hundred part of the line of *lines* in the *Sector*, the degrees of the *Meridian* vpon the *Sector*, shall give the like degrees vpon the sea-chart: if otherwise they be unequall, then may the meridians of the sea-chart be divided in such sort as the line of *Meridians* is divided on the *Sector*, by that which we shewed before in the 8 *prop.* of the line of lines.

But to avoid error, I have here set downe a Table, whereby the Meridian line may be divided out of the degrees of the æquator, supposing each degree in the Æquator, to be subdivided into a thousand parts. By which Table, and the vsuall Table of *Sines*, *Tangents* and *Secants*, the proportions following may be also resolved arithmetically. For the manner of division, let the æquator be drawne, and divided, and crossed with parallell meridians, as in the common sea-chart: then looke into the Table, and let the distance betweene the Æquator and 40 *gr.* in the meridian, from the æquator, be equall to 43 *gr.* 711 *parts* of the Æquator; let 50 *gr.* in the meridian from the æquator, be equall to 57 *gr.* 909 *parts* of the equator; and so in the rest.

The making of this Table is, by addition of *Secants*. For the *Parallells* of *latitudes* being lesse then Æquator or Meridian, in such proportion, as the Radius is to the *Secant* of the *Parallell*. For example, the Parallell of 60 *degrees* of *Latitude* is lesse then the Æquator (and consequently, each degree of this *Parallell* of 60 degrees lesse then a degree of the æquator, or *Meridian*) in such proportion as 100000 the *Radius* hath unto 200000 the *Secant* of 60 *degrees*.

M	*Gr*	Par	*M*	*Gr*	par	*M*	*Gr*	Par	*M*	*Gr*	Par	*M*	*Gr*	Par
0	0	0	3	3	001	6	6	011	9	9	037	12	12	088
		100		3	101		6	111		9	138		12	190
		200		3	201		6	212		9	239		12	293
		300		3	301		6	312		9	341		12	395
		400		3	402		6	413		9	442		12	497
		500		3	502		6	514		9	543		12	600
		500		3	602		6	614		9	645		12	702
		700		3	702		6	715		9	746		12	805
		800		3	803		6	816		9	848		12	907
		900		3	903		6	916		9	949		13	010
1	1	000	4	4	003	7	7	017	10	10	051	13	13	112
	1	100		4	103		7	118		10	152		13	215
	1	200		4	204		7	219		10	254		13	318
	1	300		4	304		7	319		10	355		13	421
	1	400		4	404		7	420		10	457		13	523
	1	500		4	504		7	521		10	559		13	626
	1	600		4	605		7	622		10	661		13	729
	1	700		4	705		7	723		10	762		13	832
	1	800		4	805		7	824		10	864		13	935
	1	900		4	906		7	925		10	966		14	038
2	2	000	5	5	006	8	8	026	11	11	068	14	14	141
	2	100		5	106		8	127		11	170		14	244
	2	200		5	207		8	228		11	272		14	347
	2	300		5	307		8	329		11	374		14	450
	2	400		5	408		8	430		11	476		14	553
	2	500		5	508		8	531		11	578		14	656
	2	601		5	609		8	632		11	680		14	760
	2	701		5	709		8	733		11	782		14	863
	2	801		5	810		8	834		11	884		14	967
	2	901		5	910		8	936		11	986		15	070
3	3	001	6	6	011	9	9	037	12	12	088	15	15	174

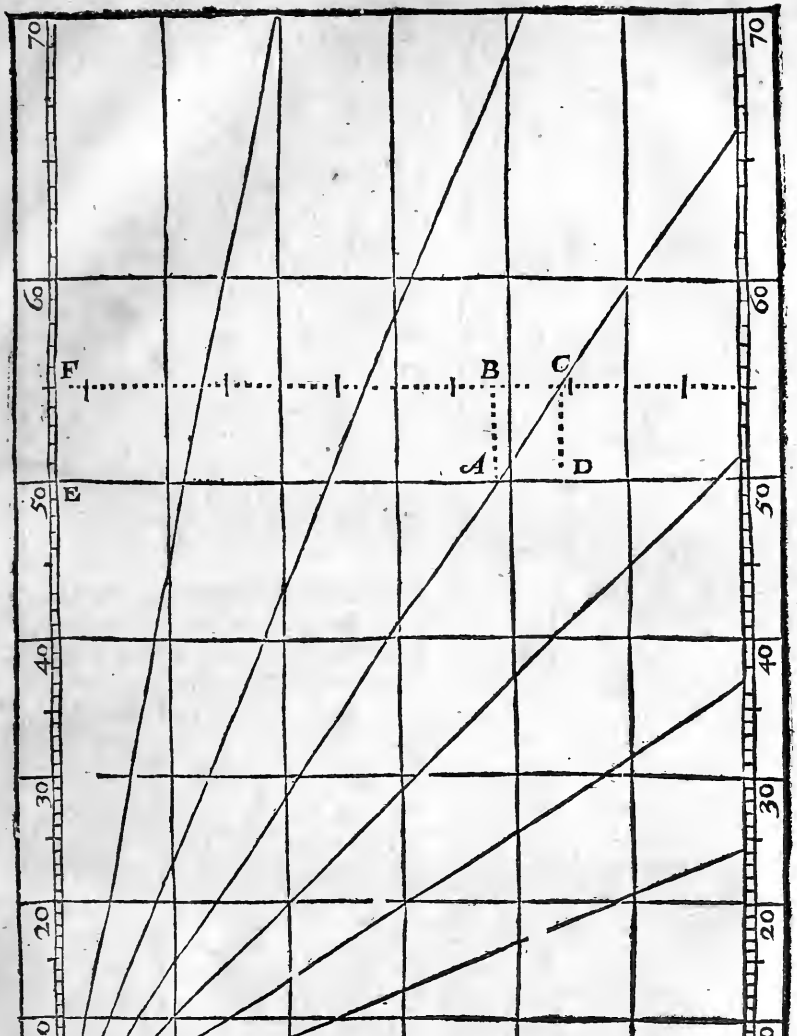
M	Gr	Par	M	Gr	Par	M	Gr	Par	M	Gr	Par	M	Gr	part
15	15	174	18	18	303	21	21	486	24	24	734	27	28	058
	15	277		18	408		21	593		24	844		28	171
	15	381		18	513		21	701		24	953		28	283
	15	485		18	619		21	808		25	063		28	396
	15	588		18	724		21	915		25	173		28	508
	15	692		18	830		21	023		25	282		28	621
	15	796		18	935		22	130		25	392		28	734
	15	900		19	041		22	238		25	502		28	847
	16	004		19	146		22	345		25	613		28	959
	16	107		19	251		22	453		25	723		29	072
16	16	211	19	19	356	22	22	561	25	25	833	28	29	186
	16	316		19	463		22	669		25	943		29	299
	16	420		19	569		22	777		26	054		29	413
	16	524		19	675		22	885		26	164		29	526
	16	628		19	781		22	993		26	275		29	640
	16	732		19	887		23	101		26	386		29	753
	16	836		19	993		23	210		26	497		29	867
	16	941		20	100		23	318		26	608		29	981
	17	045		20	206		23	427		26	719		30	095
	17	150		20	312		23	535		26	830		30	300
17	17	255	20	20	419	23	23	643	26	26	941	29	30	324
	17	359		20	525		23	752		27	052		30	438
	17	464		20	632		23	861		27	164		30	553
	17	568		20	738		23	970		27	275		30	667
	17	673		20	845		24	079		27	387		30	782
	17	778		20	952		24	188		27	499		30	897
	17	883		21	059		24	297		27	610		31	012
	17	988		21	165		24	406		27	722		31	127
	18	093		21	272		24	515		27	834		31	242
	18	198		21	379		24	624		27	946		31	357
18	18	303	21	21	486	24	24	734	27	28	058	30	31	473

M	Gr	Par	M	Gr	Par	M	Gr	Par	M	Gr	Par	M	Gr	Par
30	31	473	33	34	992	36	38	633	39	42	415	42	46	362
	31	588		35	111		38	757		42	544		46	496
	31	704		35	231		38	880		42	673		46	631
	31	820		35	350		39	004		42	802		46	766
	31	936		35	470		39	129		42	931		46	902
	32	052		35	590		39	253		43	061		47	037
	32	168		35	710		39	377		43	191		47	173
	32	284		35	830		39	502		43	320		47	309
	32	409		35	950		39	627		43	451		47	445
	32	517		36	071		39	752		43	581		47	581
31	32	633	34	36	191	37	39	877	40	43	711	43	47	718
	32	750		36	312		40	002		43	842		47	855
	32	867		36	433		40	128		43	973		47	992
	32	984		36	554		40	253		44	104		48	129
	33	101		36	675		40	379		44	235		48	267
	33	218		36	796		40	505		44	366		48	404
	33	336		36	917		40	631		44	498		48	542
	33	453		37	039		40	757		44	630		48	681
	33	571		37	161		40	884		44	762		48	819
	33	688		37	283		41	011		44	894		48	958
32	33	806	35	37	405	38	41	137	41	45	026	44	49	097
	33	924		37	527		41	264		45	159		49	236
	34	042		37	649		41	392		45	292		49	375
	34	161		37	771		41	519		45	425		49	515
	34	279		37	894		41	646		45	558		49	655
	34	397		38	017		41	774		45	691		49	795
	34	516		38	140		41	902		45	825		49	935
	34	635		38	263		42	030		45	959		50	076
	34	754		38	386		42	158		46	093		50	217
	34	873		38	509		42	287		46	227		50	358
33	34	992	36	38	633	39	42	415	42	46	362	45	50	499

M	Gr	Par	M	Gr	Par	M	Gr	Par	M	Gr	Par	M	Gr	Par
45	50	499	48	54	860	51	59	481	54	64	412	57	69	711
	50	641		55	010		59	640		64	582		69	895
	50	783		55	160		59	800		64	753		70	080
	50	925		55	310		59	960		64	924		70	263
	51	068		55	460		60	120		65	096		70	449
—	51	210	—	55	611	—	60	280	—	65	268	—	70	635
	51	353		55	762		60	441		65	440		70	821
	51	496		55	913		60	601		65	613		71	008
	51	639		56	065		60	763		65	786		71	195
	51	783		56	217		60	925		65	960		71	383
46	51	927	49	56	369	52	61	088	55	66	134	58	71	572
	52	071		56	522		61	250		66	308		71	761
	52	215		56	675		61	413		66	483		71	950
	52	360		56	828		61	577		66	659		72	140
	52	505		56	981		61	740		66	835		72	331
—	52	650	—	57	135	—	61	904	—	67	011	—	72	522
	52	795		57	289		62	069		67	188		72	714
	52	941		57	444		62	234		67	365		72	906
	53	087		57	598		62	399		67	543		73	099
	53	233		57	754		62	564		67	721		73	292
47	53	380	50	57	909	53	62	730	56	67	900	59	73	486
	53	526		58	065		62	897		68	079		73	680
	53	673		58	221		63	063		68	258		73	875
	53	821		58	377		63	231		68	438		74	071
	53	968		58	534		63	398		68	618		74	267
—	54	116	—	58	691	—	63	566	—	68	799		74	464
	54	264		58	848		63	734		68	981		74	661
	54	413		59	006		63	903		69	163		74	859
	54	562		59	164		64	072		69	345		75	057
	54	711		59	322		64	242		69	528		75	256
48	54	860	51	59	481	54	64	412	57	69	711	60	75	456

M	Gr	Par	M	Gr	Par	M	Gr	Par	M	Gr	Par	M	Gr	Par
60	75	456	63	81	749	66	88	725	69	96	575	72	105	579
	75	650		81	970		88	971		96	854		105	904
	75	857		82	191		89	219		97	135		106	230
	76	059		82	413		89	467		97	418		105	558
	76	261		82	635		89	716		97	701		106	888
	76	464		82	860		89	967		97	986		107	220
	76	667		83	084		90	218		98	272		107	553
	76	871		83	310		90	470		98	560		107	888
	77	076		83	536		90	723		98	849		108	226
	77	281		83	763		90	978		99	139		108	565
61	77	487	64	83	990	67	91	232	70	99	431	73	108	906
	77	594		84	219		91	489		99	724		109	249
	77	901		84	448		91	746		100	018		109	594
	78	109		84	678		92	005		100	314		109	941
	78	317		84	909		92	264		100	612		110	290
	78	526		85	141		92	525		100	910		110	641
	78	736		85	374		92	787		101	211		110	994
	78	947		85	607		93	050		101	513		111	349
	79	158		85	842		93	314		101	816		111	707
	79	370		86	077		93	579		102	121		112	066
62	79	583	65	86	313	68	93	845	71	102	427	74	112	428
	79	796		86	550		94	113		102	735		112	792
	80	010		86	788		94	382		103	044		113	158
	80	225		87	027		94	652		103	356		113	526
	80	441		87	267		94	923		103	668		113	897
	80	657		87	508		95	195		103	983		114	270
	80	874		87	749		95	468		104	299		114	645
	81	091		87	992		95	743		104	616		115	023
	81	310		88	235		96	019		104	936		115	403
	81	529		88	480		96	296		105	257		115	786
63	81	749	66	88	725	69	96	575	72	105	579	75	116	171

M	Gr.	Par	M	Gr.	Par	M	Gr.	Par	M	Gr.	Par	M	Gr.	part
75	116	171	78	129	075	81	145	650	84	168	947	87	208	705
	116	559		129	558		145	292		169	912		210	649
	116	949		130	045		146	942		170	893		212	668
	117	342		130	536		147	600		171	891		214	745
	117	737		131	031		148	265		172	907		216	909
	118	135		131	530		148	937		173	941		219	158
76	118	536		132	034		149	618		174	994		221	498
	118	939		132	542		150	307		176	067		223	938
	119	345		133	055		151	003		177	160		226	486
	119	755		133	572		151	709		178	275		229	153
	120	160	79	134	094	82	152	423	85	179	411	88	231	950
	120	581		134	620		153	147		180	569		234	891
	121	000		135	151		153	878		181	752		237	991
	121	420		135	687		154	620		182	960		241	268
	121	843		136	228		155	372		184	194		244	744
77	122	270		136	775		156	132		185	454		248	445
	122	700		137	326		156	903		186	743		252	402
	123	133		137	833		157	685		188	062		256	652
	123	570		138	445		158	478		189	411		261	243
	124	009		139	012		159	281		190	793		266	235
	124	452	80	139	585	83	160	096	86	192	210	89	271	705
	124	898		140	164		160	922		193	661		277	753
	125	348		140	748		161	761		195	151		284	517
	125	801		141	339		162	612		196	680		292	191
	126	258		141	936		163	475		198	251		301	058
	126	718		142	138		164	352		199	867		311	563
	127	182		143	147		165	242		201	529		324	455
	127	649		143	763		166	146		203	240		341	166
	128	121		144	385		167	065		205	005		365	039
	128	596		145	014		167	999		206	825		408	011
8	129	075	81	145	650	94	168	947	87	208	705	90	Infinite	



Q

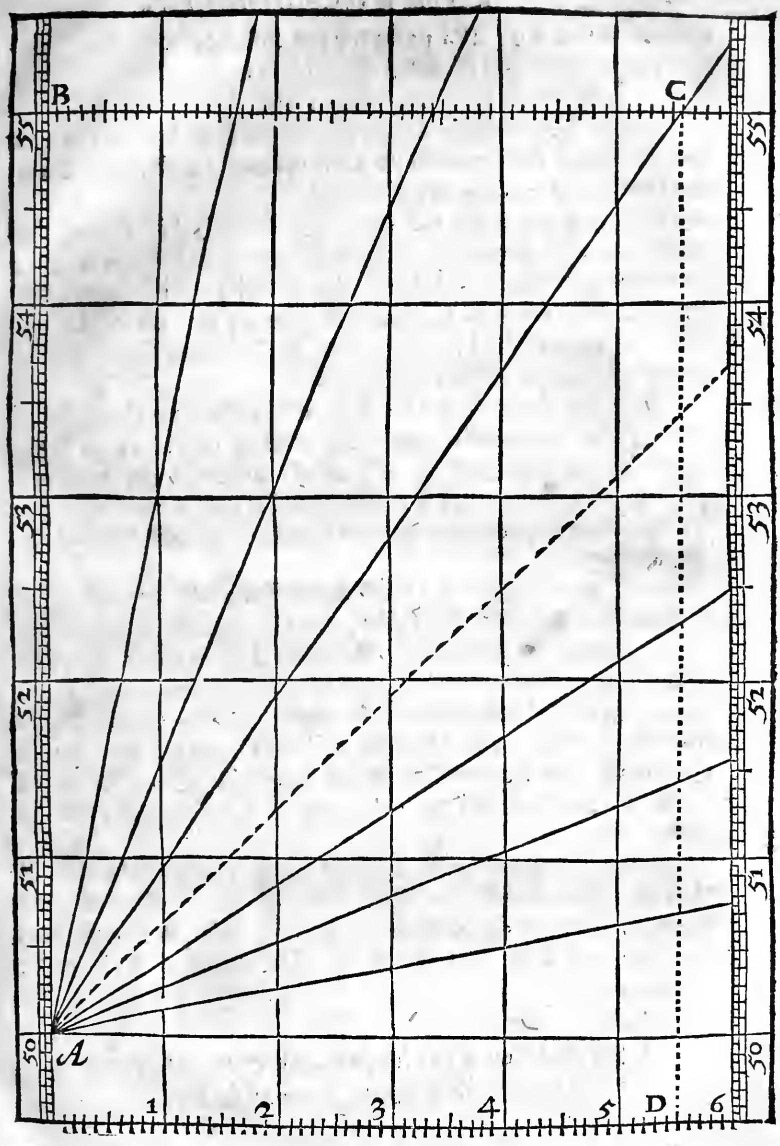
If it be a particular Chart, I would first draw the line *AB* serving for the first Meridian and crosse it with 2 perpendiculars *BC* and *AD*, the one at the upper end, the other at the lower end of the Chart, which may serve for the extreme Parallels of Latitude.

Then considering at what Latitude the Chart is to begin and end, and that this Chart entended for the latitude of these parts, is to begin at 50 *gr.* and so end at 55 *gr.* I looke into the Table, and find that 50 *gr.* of latitude must be drawne at 57 *gr.* 909 parts; and 55 *gr.* of Latitude at 66 *gr.* 134 parts from the Æquator; and that the Meridian distance betweene the Parallell of 50 *gr* and 55 *gr.* of Latitude must be equall to 8 *gr.* 225 parts of the Æquator. Whereupon I take the line *AB* out of the Meridian line and diminish it in such proportion as 8. 225 hath unto 1000 per 3 *Prop. Lin.* and with that extent of the Compasses I divide the two extreme Parallells of Latitude into equall degrees, and through each degree draw meridian lines parallell to the first meridian, noting them with 1. 2. 3. 4. &c. and then, I subdivide either one or all of these degrees into 10 parts, and (if I may) each tenth part into 10 parts more, but howsoever, I suppose each degree to be subdivided into 1000 parts.

The meridians being drawne, I come to the parallells of latitude, beginning at 50 *gr.*

And finding in the Table, that the distance between the Æquator and 50 *gr.* in the meridian should be equall to 57 *gr.* 909 parts in the Æquator and his parallells I may suppose the lowest Parallell to be 57 *gr.* from the Æquator: So the distance between this lowest Parallell and the Parallell of 50 *gr.* will be onely 909 parts. Wherefore I take these 909 odd parts, out of the degree that I divided before, and prick them downe in the two uttermost meridians from the lowest Parallell upwards, and there draw the Parallell of 50 *gr.* of latitude.

In like manner, because I find by the table that the distance betweene the Æquator and 51 *gr.* in the meridian is 59 *gr.* 481 parts of the Æquator, I abate the former 57 gr. and there


B
C
55
54
53
52
51
50
A
1
2
3
4
5
D
6

there remaine 2 gr. 481 parts for the distance betweene the lowest Parallell, and this Parallell of 51: wherefore I take these 2 degrees 481 parts out of the line before divided and pricke them downe in the two uttermost Meridians (as before) from the lowest Parallell upward, and there draw the Parallell of 51 degrees of latitude.

If any desire to have his chart agree with his *Sector*, he may make each degree of longitude æquall to the tenth part of the line of *lines*, and divide the meridian of his chart out out of the Sector: so shall each degree of the chart, be ten times as large as the like degree on the *Sector*, and the worke be easie from the one to the other.

Or he may divide the Meridian of his chart by the side of a Protractor, such as is commonly used by surveiors of land, and is here represented by *A C D E*: wherein the outward part of the semicircle *A B C* is divided æqually into 180 *gr.* The inward part æqually into 16 Rumbs, and each Rumb subdivided into 4.

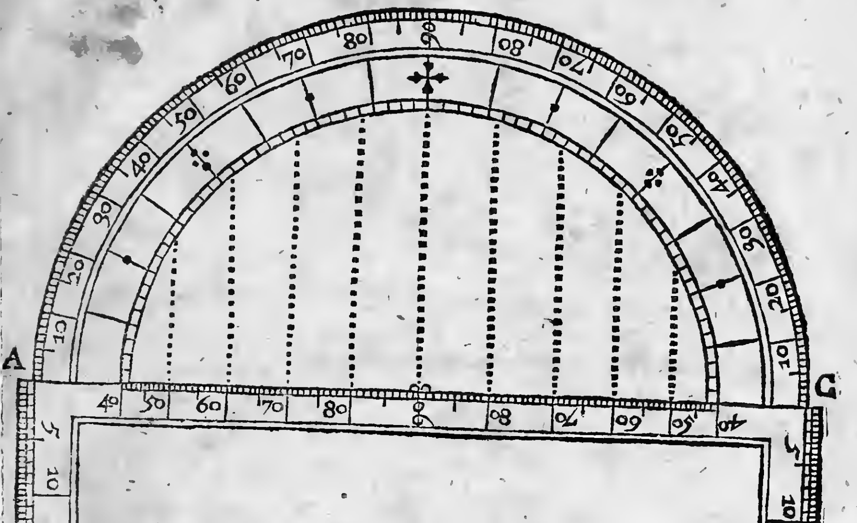
The lines *C D*, *D E*, *E A* divided æqually according to the line of lines upon the *Sector*, or the Parallells upon the Chart. Onely the Diameter *A C* would be divided unequally by letting downe occult perpendicular lines upon it, from each degree in the semicircle which being done the intermediate part betweene the Rumbs and the Diameter may be all cut forth: and the backside of the long square may be filled with 6 lines of chords, or scales of severall parts in the inch,

So may the meridian be divided by the parts of the side *E D*, the angles of each Rumb may readily be pricked downe by the degrees in the Semicircle, and the line of chords and the other scales may serve to doe the like with more variety.

2 *To find how many leagues answer to one degree of longitude in every severall latitude.*

In

The Protractor.



Place the figure page 116 of the *Sector.*

In sailing by the compasse, the course holds sometime upon a great circle, sometime upon a parallel to the æquator; but most commonly upon crooked lines winding towards one of the poles, which lines are well knowne by the name of *Rumbs*.

If the course hold upon a great circle, it is either North or South, under some meridian, or East or West, under the æquator. And in these cases, every degree requires an allowance of twentie leagues, every twentie leagues will make a degree difference in the sailing: so that here needs no further precept then the rule of proportion in the Chapter of *lines*.

But if the course hold East or West, or any of the parallels to the æquator;

As the Radius
is to twenty leagues, the measure of one degree at the æquator:
So the sine of the complement of the latitude
to the measure of leagues answering to one degree in that latitude;

Wherefore I take 20 leagues out of the line of *lines*, and make it a parallell Radius, by fitting it over in the sines of 90 and 90: so his parallell sine taken out of the complement of the latitude, and measured in the line of *lines*, shall shew the number of leagues required,

Thus in the latitude of 18 *gr.* 12 *m.* we shall find 19 leagues answering to one degree of longitude, and 18 leagues in the latitude of 25 *gr.* 15 *m*, as in this Table.

This may be done more readily without opening the Sector, by doubling the sine of the complement of the latitude, as may appeare in the same example.

It may also be done by the line of meridianes, either upon the Sector, or upon the chart. For if we open a paire of compasses to the quantitie of one degree of longitude in the æquator, or one of his Paralells and measure it in the meridian line setting one foote as much above the latitude given, as the other falleth beneath it, so that the latitude may be in the middle betweene the feete of the compasses, the number of leagues intercepted shall be that which was required.

Gr.	1	Lg
0	0	20
18	12	19
25	15	18
31	48	17
36	52	16
41	25	15
45	34	14
49	28	13
53	8	12
56	38	11
60	0	10
63	15	9
66	25	8
69	30	7
72	32	6
75	31	5
78	28	4
81	23	3
84	15	2
87	8	1

But if the course hold upon any of the *rumbs*, betweene a parallell of the æquator and the meridian we are to consides (besides the quarter of the world to which we tend, which must be alwayes knowne.)

1 The difference of longitude at least in generall,
2 The difference of latitude, and that in paticular;
3 The *rumb* whereon the course holds.
4 The distance upon the *rumb*, which is the distance, which we are here to consider, and is alwayes somewhat greater then the like distance upon a greater circle. And for these first I shew in generall this third *Prop.*

3 *To find how many leagues do answer to one degree of latitude in every severall Rumb.*

The Seamans compasse is commonly divided into 32 points, the halfe into 16, the quarter into 8, which have their names of *N*, *N b E*, *N N E*, &c. according to those parts of the world to which they point. Answerable to these points are the *Rumbes* upon their chart; each quarter divided into 8; each *Rumb* 11 *gr.* 15′ distant one from the other. The first *Rumbe* being that which is 11 *gr.* 15′. distant from the Meridian; The second 22 *gr.* 30′ the third 33 *gr* 45′ and so the rest. And (if they have need of smaller parts) they subdivide each *Rumb* into quarters allowing 2 *gr.* 48′. to the first

quarter

quarter 5 *gr.* 37′ to the half. *Rmmb* &c. as in the [illegible] a [illegible] fo owing.

As the si e of the co nplement of the *rumb* frō th meridian.

is to 2 leagues the measure of one degree a the meridian:

So the Radius

to the lea ues answering to one degree upon the Rumb.

As if in failing n E b n, from 50 *gr.* of North latitude, it were required how many leagues the ship should u , before it could come to 51 *gr.* of latitude, Because thi is the third *Rum'* and the inclination thereof 33 *gr.* 45′ I would take 20 leagues &c.

Wherefore I take 20 leagues out of the line of *lines* and make it a parallell sine of 56 *gr.* 15′ the complement of he Rumb from the meridia ; so his parallel Radius taken and measured in the line of *lines*, shall shew me 24, for the number of leagues required.

and thus in the first Rumb from the meridian, we shall fi d 20 *lgs* 39 *parts* answering to one degree of latitude and 21 *lgs* 65 *parts* in the second Rumb, &c. an in this Table, where we subdivide each league into a hundred parts, and shew besides what inclination the rumb hath to the meridian.

This may be done more readily with out opening the *Sector*, by doubling the secant of the Rumbe, as may appeare in the same example.

It may also be done upon the chart, if first we draw the Rumb, then we take the

Rumbe.	Inclinatiō to the Meridiā		Number of leags.	
	Gr.	M.	Lgs	Par
	2	49	20	02
	5	37	20	10
	8	26	20	2
1	11	15	20	39
	14	4	20	62
	16	52	20	90
	19	41	21	24
2	22	30	21	65
	25	19	22	12
	28	7	22	68
	30	56	23	32
3	33	45	24	05
	36	34	24	90
	39	22	25	87
	42	11	26	99
4	45	0	28	8
	47	49	29	78
	50	37	31	52
	53	26	33	57
5	56	15	36	00
	52	4	38	90
	61	52	42	13
	64	41	36	78
6	67	30	52	26
	70	19	59	37
	73	7	68	90
	75	56	82	31
7	78	45	102	52
	81	34	136	30
	84	22	205	24
	87	11	407	60
8	90	0	Infinita.	

the distance upon the Rumb betweene two parallells, & measure it in the meridian line, as farre above the greater latitude as beneath the lesser. For so the number of leagues intercepted, shall be that which was required.

For example: in the second chart Pag 97 I first draw the 8 Rumbs, from the intersection of the meridian with the Parallell of 50 *gr.* of latitude, either by the which I have shewed before in the generall use of sines *Cap.* 11 *Prop.* 10 or by help of the protraction last mentioned. For, laying the center of the Protractor to the point of intersection, (which is to be the center of the Rumbs) and turning the diameter of the protractor, untill it be parallell to the Meridians of the chart (which is then done, when the Meridians and Parallells in the chart fall under like divisions in the Protractor) I may make one pricke at 11 *gr*, 15′ another, at 22 *gr.* 30′ in outward part of the semicircle, and so the rest.

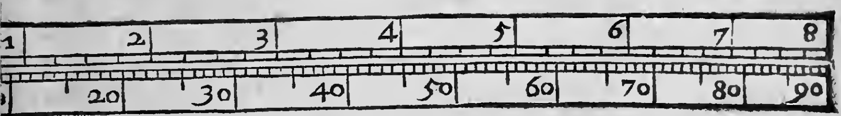
Or, having neither *Sector* nor Protractor I would have a line of chords set on the side of the Ruler which I am to use from which I may take 60 *gr* and with that extent setting one foote of the Compasses in the former point of intersection, draw an occult arke of a circle, and therein pricke downe the former arkes from the Meridian as in *cap.* 11 *Prop.* 10. So, these arkes being pricked downe, by either of these wayes, the right lines drawne through the center and those prickes, shall be the Rumbs required.

The Rumbes being drawne, I take the distance betweene the Parallells of 50 and 51 *gr* upon *A C*, the third Rumb; and mesuring it in the Meridian line I find the compasses to reach from about $\frac{1}{10}$ of a degree below the parallell of 50, but $\frac{1}{10}$ above the parallell of 51 *gr.* intercepting 1 *gr.* $\frac{2}{10}$ or 24 leagues such as 20 make a degree.

Againe, I take the distance upon the same Rumbe betweeen the Parallell of 54 and 55 *gr.* which I find to be somewhat longer then the former distance betweene the Parallells of 50 and 51; but measuring it in the Meridian line according to the latitude of the Parallella I find but 1 *gr.* $\frac{2}{10}$ (or 24 leagues) as before for ahe number of leagues answering to one degree of

Letietud

The Scale of Chords and Rumbs.



pag. 120 *Sector.*

Latitude upon this third Rumb.

And by the same reason, I may finde the number of leagues answering to a degree of Latitude upon the rest of the Rumbs agreeable to the Table.

This considered in generall, I shew more particularly in twelue *Prop.* following, how of these foure any two being given, the other two may be found, both by *Mercators* chart, and by this *Sector*.

1 *By one latitude Rumb and distance, to find the difference of latitudes.*

As the Radius
to the sine of the complement of the Rumb from the meridian;
So the distance upon the Rumb,
to the difference of latitudes.

Let the place given be *A* in the latitude of 50 *gr.* *C* in a greater latitude, but unknowne the distance upon the Rumb being 6 *gr.* betweene them, and the Rumb the third from the meridian.

First I take 6 *gr.* from the distance upon the Rumb, out of the line of *lines* and make it a parallell Radius, by putting it over in the sines of 90 and 90. Then keeping the *Sector* at this angle, I take out the parallell sine of 56 *gr.* 15 *m.* which is the sine of the complement of the third Rumb from the meridian, and measuring it in the line of *lines*, I find it to be 5 *gr.* and such is the difference of latitude required.

Or I may take out the sine of 56 *gr.* 15 *m.* for the complement of the third Rumb from the meridian, and make it a parallell Radius; then keeping the *Sector* at this angle, I take 6 *gr.* for the distance, either out of the line of *lines*, or any other scale of equall parts, or else out of the meridian line, and lay it on both sides of the *Sector* from the center; either on the line of *lines* or *sines*: so the parallell taken from the termes of this distance, and measured in the same scale wherein the distance was measured, shall shew the difference of latitude to be 5 *gr.* as before.

But in shorter distances, such as fall within the compasse of a daies sailing, this worke will hold much better. As may appeare by comparing the worke with the Table following: where the numbers in the front do signifie the leagues; those in the side, the Rumb; and the rest in the middle, the difference of latitude.

In the Chart let a meridian *AB* be drawne through *A*, and in *A* with *AB* make an angle of the Rumb *BAC*. Then open the compasses, according to the latitude of the places, to *EF* the quantitie of 6 *gr.* in the meridian, transferring them into the Rumb from *A* to *C*, and through *C* draw the parallell *BC*, crossing the meridian *AB* in *B*: so the degrees in the meridian from *A* to *B*, shall shew the difference of latitude to be 5 *gr.*

2 *By the Rumb and both latitudes to find the distance upon the Rumb.*

As the sine of the complement of the Rumb from the meridian,
is to the Radius:
So the difference of latitudes,
to the distance upon the Rumb.

As if the places given were *A* in the latitude of 50 *gr.* *C* in the latitude of 55 *gr.* and the Rumb the third from the meridian.

Here I may take 5 *gr.* for the difference of latitude out of the line of *lines*, and put it over in the sine of 56 *gr.* 15 *m.* for the complement of the third Rumb from the meridian. Then keeping the *Sector*, at this angle, I take out the parallell Radius, and measuring it in the line of *lines*, I find it to be 6 *gr.* and such is the distance upon the Rumb, which was required.

Or I may take the laterall Radius, and make it a parallell sine of 56 *gr.* 15 *m.* the complement of the Rumb from the meridian: then keeping the Sector at this angle, I take 5 *gr.* for the difference of latitude, either out of the line of *lines*,

or

Lg. / Rum.	100 G M	80 G.M	60 G.M	40 G.M	20 M	19 M	18 M	17 M	16 M	15 M
	5 0	4 0	3 0	2 0	60	57	54	51	48	45
	4 59	3 59	2 59	1 59	60	57	54	51	48	45
	4 58	3 58	2 59	1 59	60	57	54	51	48	45
	4 56	3 57	2 58	1 58	59	56	53	50	47	44
1	4 54	3 55	2 56	1 57	59	56	53	50	47	44
	4 51	3 53	2 55	1 56	58	56	52	50	47	43
	4 47	3 50	2 52	1 55	57	55	52	49	46	43
	4 42	3 46	2 49	1 53	56	54	51	48	45	42
2	4 37	3 42	2 46	1 51	55	53	50	47	44	41
	4 31	3 37	2 43	1 48	54	52	49	46	43	40
	4 25	3 32	2 39	1 46	53	50	48	45	42	39
	4 17	3 26	2 34	1 43	51	49	46	44	41	38
3	4 10	3 20	2 30	1 40	50	47	45	42	40	37
	4 1	3 13	2 25	1 36	48	46	43	41	39	36
	3 52	3 5	2 19	1 32	46	44	42	39	37	35
	3 42	2 58	2 13	1 28	44	42	40	38	36	33
4	3 32	2 50	2 7	1 25	42	40	38	36	34	32
	3 22	2 41	2 1	1 21	40	38	36	34	32	30
	3 10	2 32	1 54	1 16	38	36	34	32	30	28
	2 59	2 23	1 47	1 12	36	34	32	30	29	27
5	2 47	2 14	1 40	1 7	33	32	30	28	27	25
	2 34	2 3	1 32	1 2	31	29	28	26	25	23
	2 22	1 53	1 25	0 57	28	27	25	24	23	22
	2 8	1 43	1 17	0 52	26	24	23	22	21	19
6	1 55	1 32	1 8	0 46	23	22	21	20	18	17
	1 41	1 20	1 0	0 40	20	19	18	17	16	15
	1 27	1 9	0 52	0 35	17	16	16	15	14	13
	1 13	0 58	0 44	0 30	15	14	13	12	12	11
7	0 59	0 47	0 35	0 24	12	11	11	10	9	9
	0 44	0 36	0 26	0 18	9	8	8	7	7	7
	0 30	0 24	0 18	0 12	6	6	5	5	5	4
	0 15	0 12	0 9	0 9	3	3	3	3	2	2
8	0 0	0 0	0 0	0 0	0	0	0	0	0	0

14 M	13 M	12 M	11 M	10 M	9 M	8 M	7 M	6 M	5 M	4 M	3 M	2 M	1 M	Lg Rum.
40	39	36	33	30	27	24	21	18	15	12	9	6	3	
42	39	36	33	30	27	24	21	18	15	12	9	6	3	
42	39	36	33	30	27	24	21	18	15	12	9	6	3	
42	39	36	33	30	27	24	21	18	15	12	9	6	3	
41	38	35	32	29	26	24	21	18	15	12	9	6	3	1
41	38	35	32	29	26	23	20	17	15	12	9	6	3	
40	37	34	32	29	26	23	20	17	14	11	9	6	3	
40	37	34	31	28	25	23	20	17	14	11	8	6	3	
39	36	33	31	28	25	22	19	17	14	11	8	6	3	2
38	35	33	30	27	24	22	19	16	14	11	8	5	3	
37	34	32	29	26	24	21	19	16	13	11	8	5	3	
36	33	31	28	26	23	21	18	15	13	10	8	5	3	
35	32	30	27	25	22	20	17	15	12	10	7	5	2	3
34	30	29	26	24	22	19	17	14	12	10	7	5	2	
33	30	28	25	23	21	19	16	14	12	9	7	5	2	
31	29	27	24	22	20	18	16	13	11	9	7	4	2	
30	28	25	23	21	19	17	15	13	11	8	6	4	2	4
28	26	24	22	20	18	16	14	12	10	8	6	4	2	
27	25	23	21	19	17	15	13	11	10	8	6	4	2	
25	23	21	20	18	16	14	13	11	9	7	5	4	2	
23	22	20	18	17	15	13	12	10	8	7	5	3	2	5
22	20	18	17	15	14	12	11	9	8	6	5	3	2	
20	18	17	16	14	13	11	10	8	7	6	4	3	1	
18	17	15	14	13	12	10	9	8	6	5	4	3	1	
16	15	14	13	11	10	9	8	7	6	5	3	2	1	6
14	13	12	11	10	9	8	7	6	5	4	3	2	1	
12	11	10	10	9	8	7	6	5	4	3	3	2	1	
10	9	9	8	7	7	6	5	4	4	3	2	1	1	
8	8	7	6	6	6	5	4	3	3	2	2	1	1	7
6	6	5	5	4	5	4	3	3	2	2	1	1	1	
4	4	4	3	3	3	2	2	2	1	1	1	1	0	
2	2	2	2	1	1	1	1	1	1	1	1	0	0	
0	0	0	0	0	0	0	0	0	0	0	0	0	0	8

or out of some other scale of equall parts, and lay it on both sides of the *Sector* from the center, either on the line of *lines* or of *sines*: so the parallell taken from the termes of this difference, and measured in the same scale with the difference, shall shew the distance upon the Rumb to be 6 *gr.* or 120 leagues.

Or keeping the *Sector* at this angle, I may take the difference betweene 50 *gr.* and 55 *gr.* out of the *Meridian* line, and measuring it in the æquator, I shall find it to be equall to 8 *gr.* 22 *p.* of the æquator. Wherefore I take the parallell between 822 and 822 out of the line of *lines*, and measuring it in the line of *lines* I shall find it to be 989; which shewes that according to this projection, the distance upon this third Rumb, answerable to the former difference of latitudes, will be equal to 9 *gr.* 89 *p.* of the equator.

Or the Sector remaining at this angle, I may take the difference betweene 50 *gr.* and 55 *gr.* out of the *Meridian* line, and lay it from the center on both sides of the *Sector*, either on the line of *lines* or of *sines*: so the parallell taken from the termes of this difference, shall be the very line of distance required, the same with *A C* or *E F* upon the chart; which may serve for the better pricking downe of the distance upon the Rumb, without taking it forth of the *Meridian* line as in the former *Prop.*

Or if the Rumb fall nearer to the æquator, that the laterall Radius cannot be fitted over in it, this proposition may be wrought by parallell entrance.

For if I first take out the sine of 56 *gr.* 15 *m.* and make it a parallell Radius, by fitting it over in the sines of 90 and 90, or in the ends of the line of *lines*, and then take 5 *gr.* for the difference of latitudes out of the line of *lines*, and carrie it parallell to the former, I shall find it to crosse both lines of *lines* in the points of 6: and so it gives the same distance as before.

Or if the distance be small, it may be found by the former Table. For the Rumb being found in the side of the Table, and the difference of latitude in the same line; the top of the columne wherein the difference of latitude was found, shall giue the number of leagues in the distance required.

Or we may find this distance in the Table of Rumbs in the fift *Prop*. following. For according to the example looke into the Table of the third Rumb for 5 *gr*. of latitude, and there we shall finde 6 *gr*. 10 parts under the title of distance.

So if the difference of latitude vpon the same Rumb were 50 *gr*. the distance would be 60 *gr*. 13 parts. If the difference of latitude vpon the same Rumb were onely $\frac{1}{2}$ of a degree the distance would be onely 60 parts, such as 100 doe make a degree.

In the chart let a Meridian *A B* be drawne through *A*, and parallels of latitude through *A* and *C*; and then in *A* with *A B* make an angle of the Rumb *B A C*: so the distance taken from *A* to *C*, and measured in the Meridian line, according to the latitude of the places, shall be found to be 6 *gr*. or 120 leagues. And such is the distance required.

3 *By the distance and both latitudes to find the Rumb.*

As the distance vpon the Rumb,
to the difference of latitudes:
So is the Radius
to the sine of the complement of the Rumb from the Meridian.

As if the places given were A in the latitude of 50 *gr*. C in the latitude of 55 *gr*. the distance betweene them being 6 *gr*. vpon the Rumb. First I take 6 *gr*. for the distance vpon the Rumb, & lay it on both sides of the *Sector* from the center; then out of the same scale I take 5 *gr*. for the difference of latitude, and to it open the *Sector* in the termes of the former distance: so the parallell Radius taken and measured in the sines, doth give 56 *gr*. 15 *m*. the complement whereof 33 *gr*. 45 *m*. is the angle of the Rumbs inclination to the Meridian, which was required.

In the chart let a Meridian A B be drawne through A, and parallels of latitude both through A and C; then open the compasses according to the latitude of the places to *E F* the quantitie of *6 gr.* in the meridian, and setting one foote in A turne the other till it crosse the parallell *B C* in *C*, and draw the right line *A C*: so the angle *B A* C shall shew the inclination of the Rumb to the meridian to be 33 *gr*.45 *m*.as before.

These three last *Prop.* depend one on the other, and may be wrought as truely by the common sea-chart as by this of *Mercators* proiection: and therefore in working them by the *Sector*, the distance and the difference of latitudes may as well or better be taken out of the line of *lines* (which here representeth the Æquator) or any other line of equall parts, as out of the inlarged degrees in the *meridian* line. But in the propositions following, the difference of longitude must be taken out of the Æquator; the difference of latitudes and distance vpon the Rumb, must alwayes be taken out of the *meridian* line; which I therefore call the proper difference, and proper distance.

4 *By the longitude and latitude of two places to find the Rumb.*

As if the places given were A in the latitude of 53 *gr* C in the latitude of 55 gr. and the difference of longitude betweene them were 5 gr. 30 *m*.

In the chart let meridians and parallels be drawne through A and C, and a straight line for the Rumb from A to C; then by that we shewed Cap. 2. *Prop.* 9 inquire the quantitie of the angle B A C, and it shall be found to be 33 gr. 45 *m*. which is the third Rumb from the Meridian. Wherefore the proportion holds for the *Sector*,

As *A B* the proper difference of latitude,
is to *B C* the difference of longitude:
So *A B* as Radius,
to B *C* the tangent of the Rumb from the Meridian.

According to this I take the proper difference of latitude from

from 50 *gr.* to 55 *gr.* out of the line of *meridians*, and lay it on both sides of the *Sector* from the center; then I take the difference of longitude 5 *gr.* ½ out of the line of *lines*, and to it open the *Sector* in the termes of the former difference of latitudes: so the parallell *Radius* taken from betweene 90 and 90, and measured in the greater *tangent* on the side of the *Sector*, doth give 33 *gr.* 45 *m.* for the Rumb required.

But if the Rumb fall nearer to the Æquator;

> As *A D* the difference of longitudes,
> is to *D C* the proper difference of latitudes:
> So *A D* as Radius,
> to *D C* the tangent of the rumb from the æquator.

According to this I take the former difference of latitudes from 50 *gr.* to 55 *gr.* out of the line of *Meridians*, and to it open the *Sector* in the termes of the difference of longitude reckoned in the line of *lines* from the center: so the parallell Radius taken and measured in the *tangent*, doth give 56 *gr.* 15 *m.* for the Rumb from the Æquator; which is the complement to the former 33 *gr.* 45. *m.* and so both wayes it is found to be the third rumb from the Meridian.

But if this Rumb were to be found in the common sea-chart, it should seeme to be aboue 47 *gr.* which is more then the fourth Rumb from the Meridian.

5. *By the Rumb and both latitudes, to find the difference of longitude.*

As if the places given were *A* in the latitude of 50 *gr.* and C in the latitude of 55 *gr.* and the Rumb the third from the meridian.

In the chart, let a meridian be drawne through A, and a parallell of latitude through C, then in *A* with *A B* make the angle of the rumb from the meridian *B A C*, (as was shewed *Cap.* 2. *Prop.* 10.) So the degrees in the parallell betweene *B* and *C*, shall be found to 5 *gr.* ½, the difference of longitude

longitude which was required. Wherefore the proportion holds for the *Sector*.

As *A B* the Radius,
to *B C* the tangent of the Rumb from the meridian:
So *A B* as proper difference of the latitudes,
to B C the difference of longitude.

According to this we may take the tangent of the Rumb which is here 33 *gr.* 45 *m.* from the meridian, out of the greater *tangent* on the side of the *Sector*, and putting it over betweene 90 and 90, make it a Radius: then keeping the *Sector* at this angle, take the proper difference of latitudes from 50 gr. to 55 gr. out of the line of *Meridians*, and lay it on both sides of the Sector from the center: so the parallel taken from the termes of this difference, and measured in the line of *lines* shall shew the difference of longitude to be 5 gr. $\frac{1}{2}$.

Or if the Rumb fall nearer the æquator.

As *D C* the tangent of the Rumb from the equator,
to *A D* the Radius:
So *D C* as proper difference of the latitudes,
to *A D* the difference of longitude.

According to this we may best work by parallel entrance, first taking 56 gr. 15 *m.* for the angle of the Rumb from the equator, out of the greater tangent, and make it a parallell Radius: then take the proper difference of latitudes out of the line of meridians, and carrie it parallell to the former: so we shall find it to crosse the line of lines in 5 gr. $\frac{1}{2}$. And this is the difference of longitude required, the same as before.

But if this difference were to be found by the common sea-chart, it should seeme to be onely 3 gr. 20 *m.* which is more then 2 degrees lesse then the truth. And yet this error would be greater, if either the latitude be greater, or the Rumb fall nearer the Æquator: as may appeare by comparing the common sea-chart with the Tables following.

The first Rumbe from the Meridian.	North and by East, South and by East,			North and by West, South and by West.				
La	Long.	Dist.	La	Long.	Dist.	La	Long.	Dist.
Gr	Gr. P.	Gr. P.	Gr	Gr. P.	Gr. P.	Gr	Gr. P.	Gr. P.
0	0	0	30	6 26	30 59	60	15 01	61 18
1	20	1 02	31	6 49	31 61	61	15 4[illegible]	62 20
2	40	2 04	32	6 72	32 63	62	15 83	63 21
3	60	3 06	33	6 96	33 65	63	16 26	64 23
4	80	4 08	34	7 20	34 67	64	16 71	65 25
5	1 00	5 10	35	7 44	35 69	65	17 17	66 27
6	1 20	6 12	36	7 68	36 71	66	17 65	67 29
7	1 40	7 14	37	7 92	37 73	67	18 15	68 31
8	1 60	8 16	38	8 17	38 75	68	18 67	69 33
9	1 80	9 18	39	8 43	39 77	69	19 21	70 25
10	2 00	10 20	40	8 70	40 78	70	19 78	71 37
11	2 20	11 22	41	8 96	41 80	71	20 37	72 39
12	2 40	12 24	42	9 22	42 82	72	21 00	73 41
13	2 61	13 25	43	9 50	43 84	73	21 66	74 43
14	2 81	14 27	44	9 76	44 86	74	22 36	75 45
15	3 02	15 29	45	10 04	45 88	75	23 10	76 47
16	3 22	16 31	46	10 33	46 90	76	23 90	77 49
17	3 43	17 33	47	10 62	47 92	77	24 75	78 51
18	3 64	18 35	48	10 91	48 94	78	25 67	79 53
19	3 85	19 37	49	11 21	49 96	79	26 67	80 55
20	4 06	20 39	50	11 52	50 98	80	27 76	81 57
21	4 27	21 41	51	11 83	52 0	81	28 97	82 59
22	4 49	22 43	52	12 15	53 2	82	30 32	83 61
23	4 70	23 45	53	12 47	54 4	83	31 84	84 63
24	4 92	24 47	54	12 81	55 6	84	33 61	85 65
25	5 14	25 49	55	13 16	56 8	85	35 69	86 67
26	5 36	26 51	56	13 50	57 10	86	38 24	87 69
27	5 58	27 53	57	13 86	58 12	87	41 52	88 71
28	5 80	28 55	58	14 23	59 14	88	46 15	89 73
29	6 03	29 57	59	14 62	60 16	89	54 06	90 75
30	6 26	30 50	60	15 01	61 18	90		

The second Rumb from the Meridian.			North North-east, South South-east,			North North-west, South South-west.		
La	Long.	Dist.	La	Long.	Dist.	La	Long.	Dist.
Gr	Gr. P.	Gr. P.	Gr	Gr. P.	Gr. P.	Gr	Gr, P,	Gr. P.
0	0	0	30	13 03	32 47	60	31 25	64 94
1	0 42	1 08	31	13 51	33 50	61	32 09	66 03
2	0 83	2 16	32	14 00	34 64	62	32 96	67 11
3	1 24	3 25	33	14 49	35 72	63	33 86	68 19
4	1 65	4 33	34	15 00	36 80	64	34 79	79 27
5	2 07	5 41	35	15 50	37 88	65	35 75	70 35
6	2 49	6 49	36	16 09	38 97	66	36 75	71 44
7	2 91	7 57	37	16 51	40 05	67	37 80	72 52
8	3 32	8 66	38	17 03	41 13	68	38 88	73 60
9	3 74	9 74	39	17 56	42 21	69	40 00	74 68
10	4 16	10 82	40	18 10	43 30	70	41 19	75 77
11	4 59	11 90	41	18 65	44 38	71	42 43	76 85
12	5 01	12 99	42	19 20	45 46	72	43 74	77 93
13	5 43	14 07	43	19 76	46 54	73	45 11	79 01
14	5 85	15 15	44	20 33	47 62	74	46 57	80 10
15	6 28	16 23	45	20 92	48 71	75	48 12	81 18
16	9 71	17 32	46	21 50	49 79	76	49 78	82 26
17	7 14	18 40	47	22 11	50 85	77	51 55	83 34
18	7 58	19 48	48	22 72	52 95	78	53 46	84 42
19	8 01	20 56	49	23 35	53 03	79	55 54	85 51
20	8 45	21 65	50	23 98	54 12	80	57 82	86 59
21	8 90	22 73	51	24 63	55 20	81	60 33	87 67
22	9 34	23 81	52	25 30	56 28	82	63 13	88 76
23	9 79	24 89	53	25 98	57 37	83	66 32	89 84
24	10 24	25 98	54	26 69	58 45	84	69 99	90 92
25	10 70	27 06	55	27 39	59 53	85	74 32	92 00
26	11 16	28 14	56	28 12	60 61	86	79 63	93 09
27	11 62	29 22	57	28 87	61 7	87	86 46	94 17
28	12 08	30 31	58	29 64	62 78	88	96 10	95 25
29	12 55	31 39	59	30 44	63 86	89	112 57	96 33
30	13 03	32 47	60	31 25	64 94	90		

The third Rumbe from the Meridian	North-east by North, South-east by South,			North-west by North, South-west by South.				
La	Long.	Dist.	La	Long.	Dist.	La	Long.	Dist.
Gr	Gr. P.	Gr. P.	Gr	Gr. P.	Gr. P.	Gr	Gr. P.	Gr. P.
0	0	0	30	21 03	36 08	60	50 42	72 16
1	0 66	1 20	31	21 80	37 28	61	51 78	73 36
2	1 33	2 40	32	22 58	38 49	62	53 18	74 56
3	2 00	3 61	33	23 38	39 69	63	54 63	75 77
4	2 67	4 81	34	24 18	40 89	64	56 12	76 97
5	3 34	6 01	35	25 00	42 09	65	57 66	78 17
6	4 01	7 22	36	25 82	43 30	66	59 29	79 37
7	4 68	8 42	37	26 64	44 50	67	60 69	80 58
8	5 36	9 62	38	27 48	45 70	68	62 71	81 78
9	6 03	10 82	39	28 34	46 90	69	64 53	82 98
10	6 71	12 03	40	29 21	48 11	70	66 44	84 19
11	7 39	13 23	41	30 09	49 31	71	68 45	85 39
12	8 07	14 43	42	30 98	50 51	72	70 75	86 59
13	8 76	15 64	43	31 88	51 71	73	72 77	87 79
14	9 44	16 84	44	32 80	52 92	74	75 12	89 00
15	10 13	18 04	45	33 74	54 12	75	77 62	90 20
16	10 83	19 24	46	34 69	55 32	76	80 30	91 40
17	11 53	20 45	47	35 67	56 52	77	83 15	92 61
18	12 23	21 65	48	36 66	57 73	78	86 25	93 81
19	12 93	22 85	49	37 67	58 93	79	89 60	95 01
20	13 64	24 05	50	38 69	60 13	80	93 27	96 22
21	14 35	25 26	51	39 74	61 33	81	97 32	97 42
22	15 07	26 46	52	40 82	62 54	82	101 85	98 62
23	15 80	27 66	53	41 91	63 74	83	106 97	96 82
24	16 53	28 86	54	43 03	64 94	84	112 90	101 03
25	17 26	30 07	55	44 19	66 15	85	119 90	102 23
26	18 00	31 27	56	45 37	67 35	86	128 45	103 43
27	18 75	32 47	57	46 58	68 55	87	139 47	104 64
28	19 50	33 67	58	47 82	69 75	88	155 00	105 84
29	20 26	34 88	59	49 11	70 96	89	181 58	107 04
30	21 03	36 08	60	50 42	72 16	90		

The fourth Rumbe from the Meridian.					North-east, South-east,					North-west, South-west.				
La	Long.		Dist.		La	Long.		Dist.		La	Long.		Dist.	
Gr	Gr.	P.	Gr.	P.	Gr	Gr.	P.	Gr.	P.	Gr	Gr.	P.	Gr.	P.
0		0		0	30	31	47	42	43	60	75	46	84	85
1	1	00	1	41	31	32	63	43	84	61	77	49	85	27
2	2	00	2	83	22	33	81	45	25	62	79	58	87	68
3	3	00	4	24	33	34	99	46	67	63	81	75	89	09
4	4	00	5	66	34	36	19	48	07	64	83	99	90	51
5	5	01	7	07	35	37	41	49	50	65	86	31	91	92
6	6	01	8	49	36	38	63	50	91	66	88	73	93	34
7	7	02	9	90	37	39	88	52	33	67	91	23	94	75
8	8	03	11	31	38	41	14	53	74	68	93	85	96	17
9	9	04	12	73	39	42	42	55	15	69	96	58	97	58
10	10	05	14	14	40	43	71	56	57	70	99	43	98	99
11	11	07	15	56	41	45	03	57	98	71	102	43	100	41
12	12	09	16	97	42	46	36	59	40	72	105	58	101	82
13	13	11	18	38	43	47	72	60	81	73	108	91	103	24
14	14	14	19	80	44	49	10	62	22	74	112	43	104	65
15	15	17	21	21	45	50	50	63	64	75	116	17	106	06
16	16	21	22	63	46	51	93	65	05	76	120	17	107	48
17	17	25	24	04	47	53	38	66	46	77	124	45	108	89
18	18	30	25	45	48	54	86	67	88	78	129	08	110	31
19	19	36	26	87	49	56	37	69	29	79	134	10	111	72
20	20	42	28	28	50	57	91	70	71	80	139	59	113	14
21	21	49	29	70	51	59	48	72	12	81	145	65	114	55
22	22	56	31	11	52	61	09	73	54	82	152	42	115	96
23	23	64	32	53	53	62	73	74	95	83	160	10	117	38
24	24	73	33	94	54	64	41	76	37	84	168	95	118	79
25	25	28	35	35	55	66	13	77	73	85	179	41	120	21
26	26	94	36	77	56	67	90	79	20	36	192	21	121	62
27	28	06	38	18	57	69	71	80	61	87	208	71	123	04
28	29	18	39	60	58	71	57	82	02	83	231	95	124	45
29	30	32	41	01	59	73	49	83	44	89	271	71	125	86
30	31	47	42	43	60	75	45	84	85	90				

The fift Rumbe from the Meridian.	} North-east and by East, North-west and by West, South-east and by East, South-west and by West							
La	Long.	Dist.	La	Long.	Dist.	La	Long.	Dist.
G	Gr. P	Gr. P	Gr	Gr. P	Gr. P	Gr	Gr, P,	Gr. P
0	0	0	30	47 10	54 00	60	112 93	108 00
1	1 49	1 80	31	48 84	55 80	61	115 97	109 80
2	2 99	3 60	32	50 60	57 60	62	119 10	111 60
3	4 49	5 40	33	52 37	59 40	63	122 34	113 40
4	6 00	7 20	34	54 16	61 20	64	125 70	115 20
5	7 50	9 00	35	55 98	63 00	65	129 18	117 00
6	9 00	10 80	36	57 82	64 80	66	132 78	118 80
7	10 50	12 60	37	59 68	66 60	67	139 54	120 60
8	12 01	14 40	38	61 57	68 40	68	140 45	122 40
9	13 52	16 20	39	63 48	70 20	69	144 53	124 20
10	15 04	18 00	40	65 42	72 00	70	148 81	126 00
11	16 56	19 80	41	67 39	73 80	71	153 30	127 80
12	18 09	21 60	42	69 39	75 60	72	158 00	129 60
13	19 62	23 40	43	71 42	77 40	73	163 00	131 40
14	21 16	25 20	44	73 48	79 20	74	168 26	133 20
15	22 70	27 00	45	75 58	81 00	75	173 86	135 00
16	24 62	28 80	46	77 72	82 80	76	179 84	136 80
17	25 82	30 60	47	79 89	84 60	77	186 26	138 60
18	27 39	32 40	48	82 10	86 40	78	193 17	140 40
19	28 97	34 20	49	84 36	88 20	79	200 69	142 20
20	30 55	36 00	50	86 67	90 00	80	208 91	144 00
21	32 15	37 80	51	89 03	91 80	81	217 98	145 80
22	33 76	39 60	52	91 43	93 60	82	228 13	147 60
23	35 38	41 40	53	93 88	95 40	83	239 61	149 40
24	37 01	43 20	54	96 40	97 20	84	252 85	151 20
25	38 66	45 00	55	98 98	99 00	85	268 51	153 00
26	40 32	46 80	56	101 62	100 80	86	287 67	154 80
27	42 00	48 60	57	104 33	102 60	87	312 36	156 60
28	43 67	50 40	58	107 12	104 40	88	345 1[illegible]	158 40
29	45 38	52 20	59	109 98	106 20	89	406 72	160 20
30	47 10	54 00	60	112 93	108 00	90		

The sixe Rumbe from the Meridian			East North-east, West North-west.			East South-east, West South-west.		
La.	Long.	Dist.	La.	Long.	Dist.	La.	Long.	Dist.
Gr.	Gr. P.	Gr. P.	Gr.	Gr. P.	Gr. P.	Gr.	Gr. P.	Gr. P.
0	0	0	30	75 98	87 39	60	182 18	156 78
1	2 41	2 61	31	78 78	81 00	61	187 07	155 40
2	4 83	5 23	32	81 61	83 62	62	192 13	162 01
3	7 25	7 84	33	84 48	86 23	63	197 36	164 62
4	9 66	10 45	34	87 37	88 84	64	202 77	167 24
5	12 08	13 06	35	90 30	91 46	65	208 38	169 85
6	14 51	15 68	36	93 27	94 07	66	214 20	172 46
7	16 94	18 29	37	96 27	96 68	67	220 25	175 08
8	19 37	20 90	38	99 31	99 30	68	226 57	177 69
9	21 81	23 52	39	102 40	101 91	69	233 15	180 30
10	24 26	26 13	40	105 53	104 52	70	240 06	182 92
11	26 71	28 74	41	108 71	107 14	71	247 27	185 53
12	29 17	31 36	42	111 93	109 75	72	254 90	188 14
13	31 65	33 97	43	115 20	112 36	73	262 92	190 75
14	34 14	36 58	44	118 53	114 97	74	271 43	193 37
15	36 63	39 20	45	121 92	117 59	75	280 46	195 98
16	39 13	41 81	46	125 36	120 20	76	290 11	198 59
17	41 65	44 42	47	128 87	122 81	77	300 46	201 21
18	44 18	47 03	48	132 44	125 43	78	311 62	203 82
19	46 73	49 65	49	136 09	128 04	79	323 73	206 43
20	49 29	52 26	50	139 81	130 65	80	337 00	209 05
21	51 87	54 87	51	143 60	133 27	81	351 64	211 66
22	54 47	57 49	52	147 47	135 88	82	368 00	214 27
23	57 08	60 10	53	151 44	138 49	83	386 51	216 89
24	59 71	62 71	54	155 50	141 10	84	407 89	219 50
25	62 36	65 33	55	159 66	143 72	85	433 13	222 11
26	65 04	67 94	56	163 93	146 33	86	464 05	224 73
27	67 74	70 55	57	168 31	148 95	87	503 88	227 34
28	70 46	83 17	58	172 80	151 56	88	560 00	229 95
29	73 20	75 78	59	177 42	154 17	89	656 08	232 56
30	75 98	78 39	60	182 18	156 78	90		

The seventh Rumbe from the Meridian.			East and by North, West and by North,			East and by South, West and by South		
La	Long.	Dist.	La	Long.	Dist.	La	Long.	Dist.
G	Gr. P.	Gr. P	Gr	Gr. P	Gr. P	Gr	Gr, P,	Gr. P:
0	0	0	30	15823	15377	60	37935	30755
1	502	512	31	16406	15890	61	38956	31267
2	1005	1025	32	16996	16402	62	40010	31780
3	1508	1538	33	17592	16915	63	41098	32293
4	2012	2050	34	18195	17428	64	42226	32805
5	2516	2563	35	18804	17940	65	43394	33318
6	3021	3075	36	19422	18453	66	44603	33830
7	3527	3588	37	20048	18965	67	45866	34343
8	4034	4100	38	20682	19478	68	47180	34855
9	4542	4613	39	21324	19990	69	48552	35368
10	5052	5126	40	21976	20503	70	49989	35881
11	5563	5638	41	22637	21016	71	51494	36393
12	6077	6151	42	23308	21528	72	53079	36906
13	6592	6663	43	23990	22041	73	54752	37418
14	7109	7176	44	24684	22553	74	56522	37931
15	7628	7688	45	25389	23066	75	58403	38443
16	8150	8201	46	26105	23579	76	60413	38956
17	8675	8714	47	26836	24091	77	62567	39469
18	9201	9226	48	27580	24604	78	64891	39981
19	9731	9739	49	28340	25116	79	67415	40494
20	10264	10251	50	29113	25629	80	70175	41006
21	10801	10764	51	29903	26141	81	73225	41519
22	11342	11277	52	30711	26654	82	76630	42032
23	11887	11789	53	30537	27169	83	80486	42544
24	12435	12302	54	32382	27679	84	84938	43057
25	12987	12814	55	33248	28192	85	90198	43569
26	13544	13327	56	34136	28704	86	96631	44082
27	14105	13840	57	35047	29217	87	104926	44594
28	14671	14352	58	35981	29730	88	116611	45107
29	15244	14865	59	36945	30242	89	136623	45620
30	15823	15377	60	37935	30755	99		

The eight Rumbe of East and West, with the Longitude answering to one degr. of distance, and the distance belonging to one degree of Longitude.

La	Long.	Dist.	La	Long.	Dist.	La	Long.	Dist.
Gr	Gr, P.	Parts.	Gr	Gr. P.	Parts.	Gr	Gr. P.	Parts.
0	0	10000	30	1 25	86 60	60	2 00	50 00
1	1 00	99 98	31	1 17	85 71	61	2 06	48 48
2	1 00	99 94	32	1 18	84 80	62	2 13	46 94
3	1 00	99 86	33	1 19	83 86	63	2 20	45 40
4	1 00	99 75	34	1 21	82 90	64	2 28	43 83
5	1 00	99 62	35	1 22	81 91	65	2 37	42 26
6	1 01	99 45	36	1 24	80 90	66	2 46	40 67
7	1 01	99 25	37	1 25	79 86	67	2 56	39 07
8	1 01	99 02	38	1 27	78 80	68	2 67	37 46
9	1 01	98 76	39	1 29	77 71	69	2 79	35 83
10	1 02	98 48	40	1 31	76 60	70	2 92	34 20
11	1 02	98 16	41	1 33	75 47	71	3 07	32 55
12	1 02	97 81	42	1 35	74 31	72	3 24	30 90
13	1 03	97 43	43	1 37	73 13	73	3 42	29 23
14	1 03	97 03	44	1 39	71 93	74	3 63	27 56
15	1 03	96 59	45	1 41	70 71	75	3 86	25 88
16	1 04	96 12	46	1 44	69 46	76	4 13	24 19
17	1 04	95 63	47	1 47	68 20	77	4 44	22 49
18	1 05	95 10	48	1 49	66 91	78	4 81	20 79
19	1 06	94 55	49	1 52	65 60	79	5 24	19 08
20	1 06	93 97	50	1 55	64 28	80	5 76	17 36
21	1 07	93 35	51	1 59	62 93	81	6 39	15 64
22	1 08	92 72	52	1 62	61 56	82	7 18	13 91
23	1 09	92 05	53	1 66	60 18	83	8 20	12 18
24	1 09	91 35	54	1 70	58 77	84	9 57	10 45
25	1 10	90 63	55	1 74	57 35	85	11 47	8 71
26	1 11	89 88	56	1 79	55 92	86	14 33	6 97
27	1 12	89 10	57	1 84	54 46	87	19 11	5 23
28	1 13	88 29	58	1 89	52 99	88	28 65	3 49
29	1 14	87 46	59	1 94	51 50	89	57 30	1 74
30	1 15	86 60	60	2 00	50 00	90		0

These tables are calculated for each of the Rumbs. The first seven have three columnes, and of them the first containeth the degrees of Latitude from the Æquinoctiall to the Pole: the second doth give the difference of Longitude; and the third the distance, both of them belonging to that Rumb and latitude.

As in the Table of the third Rumb; at the *latitude* of 50 *Gr.* I find under the title of *Longitude* 38 *gr.* 69 *parts*, and under the title of *distance* 60 *gr.* 13 *parts.* This shewes that if the course held constantly on the third Rumb from the Æquinoctiall to the Latitude of 50 *gr.* the difference of Longitude would be 38 *gr.* 69 *parts* of 100 and the distance upon the Rumbe 60 gr. 13 parts. For here I reckon the distance by degrees, rather then by leagues or miles, and subdivide each degree into 100 parts, rather then into 60 minutes, for the more ease in calculation, and withall to make the calculation to agree the better, both with this, and my *Crosse staffe* and other instrumentr.

The use of these Tables, for the finding of the difference of Longitude, is this. Turne to the table of the Rumb, and there see what longitude belongeth to either latitude, then take the one longitude out of the other, the remainder will be the difference of longitude required.

As in the former example, where the places given were *A* in the latitude of 50 Gr. *C* in the latitude of 55 Gr. and the Rumb the third from the meridian: I looke into the table of the third Rumb and there find,

Latitude 50 gr.	Longitude 38 gr. 69 parts.
Latitude 55 *gr.*	Longitude 44 *gr.* 19.
Therefore the diff. of longitude 5 *gr.*	50.

There is another use of these tables, for the describing of the Rumbs both on the *Globe*, and all sorts of *Charts.* For having drawne the circles of longitude and latitude, and finding by the tables, the difference of longitude belonging to each Rumb and latitude: If we make a pricke in the chart, at every

every degree of latitude, according to that difference of longitude, and draw lines through those prickes, so as they make no angles, the lines so drawne shall be the Rumbs required.

The use of the eight Rumb is something different from the rest. For there being here no change of latitude, I have set to each latitude, the difference of longitud, belonging to one degree of distance, and the distance belonging to one degree of longitude.

As if two places shall be 20 leagues, or one degree distant one from the other, in the latitude of 50 *gr.* the difference of longitude betweene them will be 1 *gr.* 55 *parts.* But if they differ one degree in longitude, the distance betweene them will be onely 64 parts, which fall short of 13 leagues, or at the most 64 *gr.* 28 parts, such as 10000 do make a degree.

6 *By the difference of longitude, Rumb, and one latitude, to find the other latitude.*

As if the places given were *A*, in the latitude of 50 *gr.* C in a greater lattiude but unknowne, the difference of longitude 5 *gr.* $\frac{1}{12}$, and the Rumb the third from the Meridian.

In the chart let A *B*, *D C*, meridians, be drawne through A and *C*, according to the difference of longitnde, one 5 *gr.* $\frac{1}{3}$ from the other; and a parallell of latitude through *A*, crossind the meridian C D in D: then in A, with A B, make an angle of the Rumbe B A C: so the degrees in the meridian betweene D and C, shall be found to be 5 *gr.* the proper difference of latitude which was required. Wherefore the proportion holds for the *Sector*,

As A D the Radius
to D C the tangent of the Rumb from the æquator
So A D as difference of longitude,
to D C the proper difference of latitude.

According to this, I take 56 *gr*, 15 *m.* for the angle of the Rumb from the æquator, out of the greater *Tangents*, and make it a parallell Radius. Then I Reckon 5 *gr.* $\frac{1}{2}$ in the line of *lines* from the center, for the difference of longitud. So the parallell taken from the termes of this difference, and measured in the line of *meridians*, shall reach from 50 gr. the latitude given, to 55 gr. which is the latitude required.

Or if the Rumb fall nearer to the meridian.

As *BC* the tangent of the Rumb from the meridian,
is to *AB* the Radius :
So *BC* as difference of longitude,
to *AD* the proper difference of latitude.

According to this we may best work by parallel entrance; first take 35 gr. 45 *m.* for the angle of the Rumb from the meridian, out of the greater *Tangent*, and make it a parallell Radius; then take 5 gr. $\frac{1}{2}$ for the difference of longitude out of the line of *lines*, and carry it parallell to the former, till the feete of the compasses stay in like points: so the line between the center and the place of this stay, being taken and measured in the line of *meridians* from 50 gr. forward, shall shew the latitude required to be 55 gr. as in the former way.

The like may be found by the tables of Rumbs. For in the table of the third Rumb, at the latitude of 50 gr. I finde the longitude of 38 gr. 69 *p*; to this if I adde 5 *gr.* 50 *p.* for the difference of longitude given, the compound longitude will be 44 gr. 19 *p.* and this answers to the latitude of 55 gr.

But if this difference of latitude were to be found by the common sea-chart, it should seeme to be 8 gr. 13 *m.* and so the second latitude should be 58 gr. 13 *m.* which is aboue 3 gr. more then the truth.

7 By one latitude, rumb, and distance, to find the difference of longitude.

As if the places given were *A* in the latitude of 50 gr. *C* in a greater latitude but unknowne, the distance upon the Rumb being 6 gr. betweene them, and the Rumb the third from the meridian.

In

In the chart, let a meridian A B, and a parallell A D be drawne through A, and in A, with A B, make an angle BAC for the Rumb from the meridian; then open the compasses according to the latitude of the places to *E* F, the quantitie of 6 gr. in the meridian, transferring them into the Rumb from A to C, and through C draw another meridian D C, crossing the parallell drawne through A in D: so the degrees intercepted in the parallell from A to D, shall shew the difference of longitude required to be about 5 gr. $\frac{5}{6}$. Wherefore the proportion holds for the Sector.

As A C the Radius, (meridian:
is to A D, equall to B C, the sine of the *Rumb* from the
So A C as proper distance upon the *Rumb*,
to A D the difference of longitude.

According to this I take the sine of 33 *gr.* 45 *m.* for the angle of the Rumb from the meridian, and make it a parallell Radius; then keeping the *Sector* at this angle, I take 6 *gr.* for the distance out of the *meridian* line, according to the estimated latitudes of both places, and lay it on both sides of the *Sector* from the center: so the parallell taken from the termes of this distance, and measured in the lines of *lines*, shall shew the difference of longitude to be about 5 *gr.* $\frac{5}{6}$.

In this and some of the *Prop.* following, where there is but one latitude knowne, there may be sometimes an error of a minute or two, in the estimation of the proper distance, yet it may be rectified at a second operation.

This proposition may also be wrought by the Tables of Rumbs. For according to the example, in the Table of the third Rumb, at the latitude of 50 *gr.* I find the longitude of 38 *gr.* 69 *p.* and the distance of 60 *gr.* 13 *p:* to this I adde 6 *gr.* for the distance given; so the compound distance will be 66 *gr.* 13 *p.* and this answers to the longitude of 44 *gr.* 19 *p*; then if I take the one longitude out of the other, the difference will be 5 gr. 50 *p.* as before.

But if this difference were to be found by the common sea-chart, it should seeme to be onely 3 gr. 20 *m.* which is more then 2 gr. lesse then the truth.

8. *By one latitude, Rumb, and difference of longitudes, to find the distance.*

As if the places were given *A*, in the latitude of 50 gr. *C* in a greater latitude but unknowne, the difference of longitude betweene them being 5 gr. ½, and the Rumb the third from the meridian.

In the chart let A B, D C, meridians be drawne through *A* and *C*, according to the difference of longitude, and a parallell of latitude through *A*, crossing the meridian D C in D; then in *A*, with AB, make an angle of the Rumb B A C: so the distance on the Rumb from *A* to *C* taken and measured in the *meridian*, according to the estimated latitude of the places, shall be found to be 6 gr. Wherefore the proportion holds for the *Sector*.

As A D, equall to BC, the sine of the Rumb from the meridian,
is to *A C* the Radius:
So AD as difference of longitudes,
to *A C* the proper distance upon the Rumb.

According to this, I take the laterall Radius, and make it a parallell sine of 33 gr. 45 *m.* which is here the angle of the Rumb from the meridian; then I reckon 5 gr. ½ in the lines of *lines* from the center, for the difference of longitude: so the parallell taken from the termes of this difference, and measured in the line of *meridians*, according to the latitudes of the places, shall there shew the distance required to be about 6 gr. which are 120 leagues.

Or if the Rumb fall nearer to the meridian, that the lateral Radius cannot be fitted over in his sine, this *Prop.* must be wrought by parallell entrance, and so also it gives the same distance as before.

Or we may find this distance by the Table of Rumbs. For in the tabl of the third Rumb, at the latitude of 50 gr. I find the longitude of 38 gr. 69 *p.* and the distance of 60 gr. 13. *p.*

To

To this longitude here found, I adde 5 gr. 50 *p.* for the difference of longitude given: so the compound longitude will be 44 gr. 19 *p.* and this answers to the distance of 66 gr. 15 *p.* Then if I take the one distance out of the other, the remainder will be 6 gr. 02 *p.* for the distance required.

But if this distance were to be measured on the common sea-chart, it should seeme to be almost 10 gr. or at the least 197 leagues, above 77 leagues more then the truth.

9 *By one latitude, distance, and difference of longitudes, to find the Rumb.*

As if the places given were *A*, in the latitude of 50 gr. *C* in a greater latitude but unknowne, the difference of longitude betweene them being 5 gr. ½, and the distance of 6 gr. upon the Rumb.

In the chart let AB, DC, meridians, be drawne through *A* and *C*, and a parallell of latitude through *A*; then open the compasses according to the latitudes of the places, to *EF* the quantity of 6 gr. in the *meridian*, and setting the one foote in *A*, the other foote shall crosse the other meridian in C; and if we draw the right line AC, the angle BAC shall shew the inclination of the Rumb to the meridian to be about 33 gr. 45 *m.* Wherefore the proportion holds for the *Sector*.

As AC the proper distance upon the Rumb,
is to AD the difference of longitude:
So AC as Radius,
to AD, equall to BC, the sine of the Rumb from the meridian.

According to this, I take the proper distance 6 gr. out of the line of *meridians*, and lay it on both sides of the *Sector* from the center; then I take the difference of longitude 5 gr. ½ out of the line of *lines*, and to it open the *Sector* in the terms of the former distance: so the parallell Radius taken from betweene 90 and 90, and measured in the *sines*, doth give about 33 gr. 45 *m.* for the Rumb required.

But if this Rumb were to be found by the common sea-

chart,

chart, it should seeme to be above 66 *gr.* and so almost the sixt Rumb from the Meridian.

10 *By the longitude and latitude of two places, to find their distance from the Rumb.*

Let the *Sector* be opened in the lines of *lines*, unto a right angle (as was shewed before *Cap.* 2. *Prop.* 7.) then take out the proper difference of latitude, and lay it on the one line, and the difference of longitude, and lay it on the other line, so as they may both meete in the center, marking how far they extend. For the line taken from the termes of their extension, and measured in the *meridian*, according to their latitudes, shall shew the distance required.

So if the places given were A and C, A in the latitude of 50 *gr.* C in the latitude of 55 *gr.* the proper difference of latitude shall be the line A B, and let B C the difference of longitude be 5 *gr.* ½, we shall find that *A C* the distance upon the Rumb is about 6 *gr.* which make 120 leagues.

For in the chart, let an occult meridian be drawne through A, and a parallell of latitude through C, crossing the former meridian in B, and a right line for the Rumb from A to C, so have we a rectangle triangle *A B C*, whose base *A C*, taken and measured in the meridian from E below 50 *gr* to F, as much above 55 *gr*, doth containe the quantitie of 6 *gr.*

In the same manner the *Sector* being opened to a right angle, in the lines of *lines:* if we take the difference of latitude out of the line of *meridians*, in his proper place from 50 *gr.* to 55 *gr.* and place it on one of the sides from the center, to resemble *A B*, then reckon the difference of longitude on the other perpendicular line from the center to 5 *gr.* ½, in stead of B C, we shall have the like rectangle triangle on the *Sector*, to that which we had before on the chart; and if we take out the base of it, and measure it in the line of *meridians* from below 50 *gr.* to as much aboue 55 *gr* we shall finde as before, that it containeth about 6 *gr.* or 120 leagues.

But if this distance were to be measured on the common

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sea-chart, it should seeme to be almost 7 *gr.* $\frac{1}{4}$, or 145 leagues; which is 25 leagues more then the truth.

11 *By the latitude of two places, and the distance upon the Rumb, to find the difference of longitude.*

Let the *Sector* be opened in the lines of *lines* to a right angle, then take out the proper difference of latitudes, and lay it on one of the lines from the center, then take the proper distance with a paire of compasses, and setting one foote in the termes of the difference, turne the other foote to the other line of the *Sector*, and it shall there shew the difference of longitude required.

So if the places given were *A*, in the latitude of 50 *gr.* *C* in the latitude of 55 *gr.* with 6 *gr.* of distance one from another, we shall find their difference of longitude to be about 5 *gr.* $\frac{1}{2}$.

For in the chart let a meridian *A* B be drawne for the one, and B *C*, *A D*, parallells of latitude for them both: Then open the compasses according to the latitude of the places, to *E* F the quantitie of 6 *gr.* in the *meridian*, and setting one foote in *A*, having latitude of 50 *gr.* turne the other to the parallell of 55 *gr.* and it shall there cut off the required difference of longitude B *C* 5 *gr.* $\frac{1}{2}$.

In the same maner, the *Sector* being opened to a right angle, in the lines of *lines*: if we take the difference of latitude out of the line of *meridians* in his proper place from 50 *gr.* unto 55 gr. and place it on one of the lines from the center; then take 6 gr. the distance upon the Rumb out of the same line of *meridians*, according to the latitudes of the places, and set the one foote in the terme of the former difference, turning the other foote to the other perpendicular line, we shall finde that it will crosse it about 5 gr. $\frac{1}{2}$ from the center: which is the difference of longitude required.

But if this difference of longitude were to be found by the common sea chart, it would seeme to be onely 3 gr. 20 *m* which is more then 2 gr. 10 *m.* lesse then the truth.

12. *By one latitude, distance and difference of longitudes, to finde the difference of latitudes.*

Let the *Sector* be opened in the line of *lines* to a right angle, and let the difference of longitude be reckoned in one of those lines from the center; then take the proper distance with a paire of compasses, and setting the one foote in the terme of the former difference, turne the other foote to the other line of the *Sector*, and it shall thence cut off a line, equall to the proper difference of latitude required.

So if the places given were *A* and *C*, *A* in the latitude of 50 *gr.* *C* in a greater latitude but unknowne, the difference of longitude betweene them 5 *gr.* $\frac{1}{2}$, and the distance upon the Rumb 6 *gr.* or 120 leagues, we shall find the difference of latitude to be 5 *gr.*

For in the chart, let occult meridians be drawne through *A* and *C*, and a parallell of latitude through *A*; then open the compasses according to the estimated latitudes of the places to E F the quantity of 6 *gr.* in the meridian, and setting the one foote in *A*, turne the other to the meridian drawne through *C*, and it shall there cut off the line *D C*, which is the difference of latitude required.

In the same maner, the *Sector* being opened to a right angle, in the lines of *lines*, if in the one line we reckon the difference of longitude from the center to 5 *gr.* $\frac{1}{2}$, then taking 6 *gr.* for the distance out of the line of *Meridians*, according to the latitude of the places, we set the one foote in the terme of the given difference, and turne the other foote to the other perpendicular line, we shall find that it cuts a line from it, which taken and measured in the line of *meridians*, from 50 *gr.* on forward, doth shew the difference of latitude to be as before 5 *gr.*

But if this difference of latitude were to be found by the common sea-chart, it would seeme to be onely 2 *gr.* 25 *m.* which is 2 *gr.* 35 *m.* lesse then the truth. Such is the difference betweene both these charts.

THE

THE THIRD BOOKE

Containing the use of the particular *Lines*.

THE lines of *lines*, of *superficies*, of *solids*, of *sines*, with the laterall lines of *tangents* and *meridians*, whereof I haue hitherunto spoken, are those which I principally intended: that little roome on the *Sector* which remaineth, may be filled up with such particular lines as each one shall thinke convenient for his purpose. I have made choise of such as I thought might be best pricke on without hindring the sight of the former, viz. lines of *Quadrature*, of *Segments*, of *Inscribed bodies*, of *Equated bodies*, and of *Mettals*.

CHAP. I.

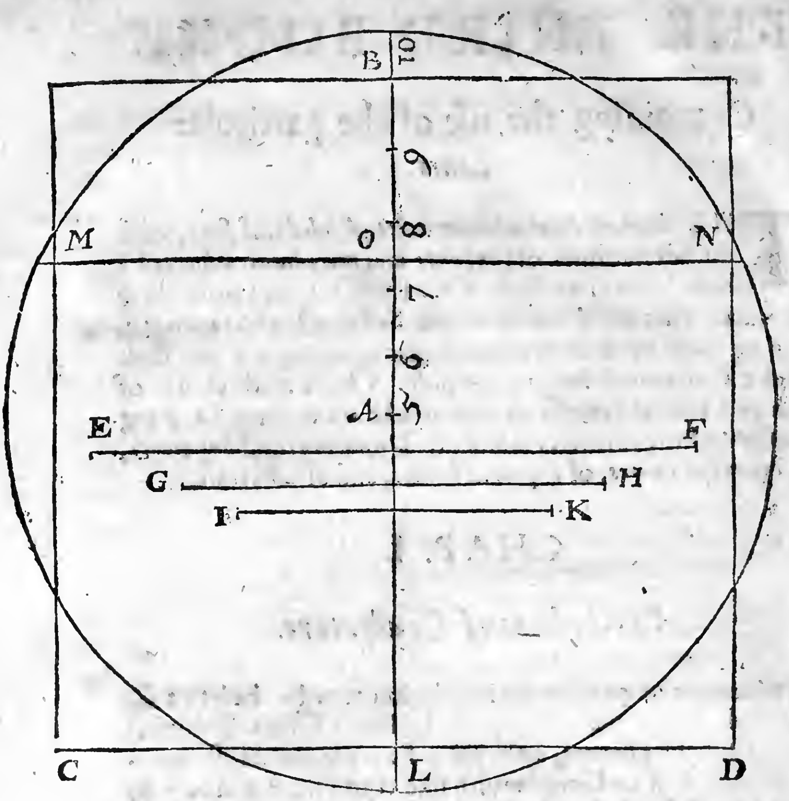
Of the lines of Quadrature.

THe lines of *quadrature* may be knowne by the letter *Q*, and by their place betweene the lines of *sines*. *Q* signifieth the side of a *square*; 5 the side of a *pentagon* with five equall sides. 6 of an *hexagon* with sixe equall sides, and so 7 8, 9, and 10. *S* stands for the Semidiameter of a circle, and 90 for a line equall to 90 *gr.* in the circumference. The use of them may be,

1 *To make a square equall to a circle given:*
2 *To make a circle equall to a square given.*

If the circle be first given, take his semidiameter; and to it open the *Sector* in the points at S: so the parallell taken from betweene the points at *Q*, shall be the side of the square required.

 If



If the square be given take his side, and to it open the *Sector*, in the points at *Q*: so the parallell taken from betweene the points at *S*, shall be the Semidiameter of the circle required.

Let the Semidiameter of the circle given be *A* B, the side of the square equall unto it shall be found to be *CD*.

3 *To reduce a circle given, or a square into an equall pentagon, or other like sided and like angled figure.*

Take the side of the figure given, and fit it over in his due points: so the parallells taken from betweene the points of the

the other figures, shall be the sides of those figures: which being made up with equall angles, shall be all equall one to the other.

Let the Semidiameter of the circle given be *AB*, the side of an *hexagon* equall to this circle, shall by these meanes be found to be *G H*; and the sides of an octagon to be *I K*. Other planes not here set downe, may first be reduced into a square, by the sixt *Prop. Superf.* and then into a circle, or other of these equall figures, as before.

4 *To find a right line, equall to the circumference of a circle, or other part thereof.*

Take the Semidiameter of the circle given, and to it open the *Sector* in the points at *S*; so the parallell taken from betweene the points at 90 in this line, shall be the fourth part of the circumference: which being knowne, the other parts may be found out by the second and third *Prop.* of *lines.*

Thus if the Semidiameter of the circle given be *A B*, the right line *E F* shall be found to be the fourth part of the circumference. Therefore the double of *E F* shall be equall to the circumference of 180 *gr.* and the halfe of *E F* shall be the circumference of 45 *gr.* and so in the rest.

CHAP. II.

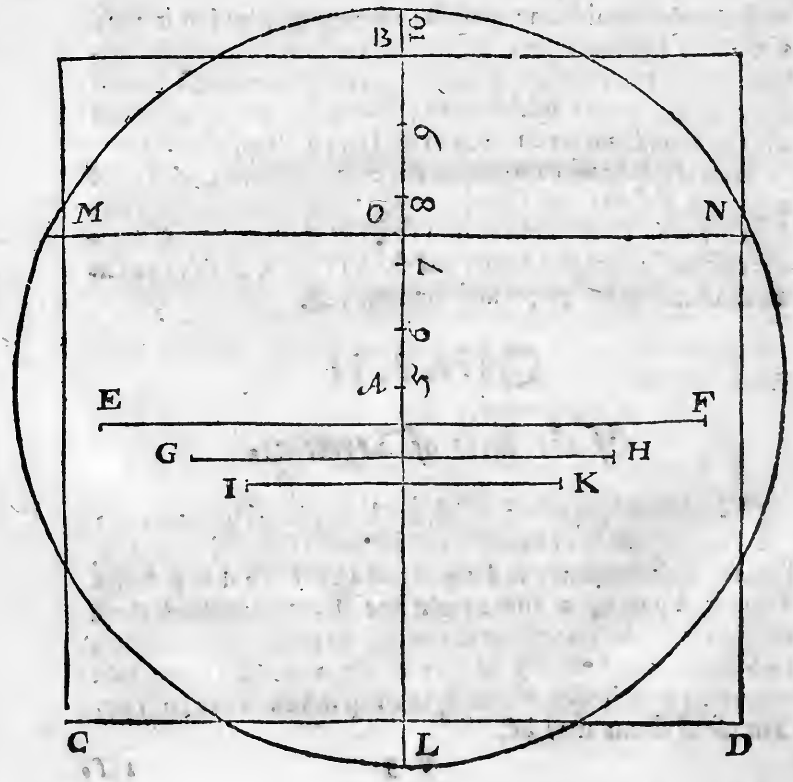
Of the lines of Segments.

THe lines of *segments* which are here placed between the lines of *sines* and *superficies*, and are numbred by 5, 6, 7, 8, 9, 10. do represent the diameter of a circle, so divided into a hundred parts, as that a right line drawne through these parts, perpendicular to the diameter, shall cut the circle into two segments, of which the greater segment shall have that proportion to the whole circle, as the parts cut have to 100. The use of them may be.

1 *To divide a circle given into two segments according to a proportion given.*
2 *To find a proportion betweene a circle and his segments given.*

Let the *Sector*. be opened in the points of an 100, to the diameter of the circle given: so a parallell taken from the points proportionall to the greater segment required, shall giue the depth of that greater segment.

Or if the segments be given, let the *Sector* be opened as before; then take the depth of the greater segment, and carry it



parallell

parallell to the diameter: so the number of points wherein they stay, shall shew the proportion to 100.

As if the diameter of the circle given were *B L*, the depth of the greater segment *L* O being 75, doth shew the proportion of the segment O *M L N* to the circle to be as 75 to 100 viz. three parts of foure.

Hence I might shew, if there were any use of it,

To find the side of a square, equall to any knowne segment of a circle.

The side of a square equall to the whole circle, may be found by the former *Cap.* and then having the proportion of the segment to the circle, we may diminish the square in such proportion, by that which hath beene shewed *Lib.* 1. *Cap.* 3. *Prop*, 3.

CHAP. III.

Of the lines of Inscribed bodies.

THe lines of *inscribed bodies* are here placed betweene the lines of *lines*, and may be knowne by the letters, *D*, *S*, I C, O, *T*, of which D signifieth the side of a *dodecahedron*, I of an *Icosahedron*, C of a *cube*, O of an *octahedron* and *T* of a *tetrahedron*, all inscribed into the same sphære, whose semidiameter is here signified by the letter *S*.

The use of these lines may be,

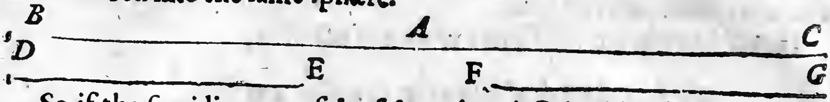
1 *The semidiameter of a sphære being given, to find the sides of the fiue regular bodies, which may be inscribed in the said sphære.*

2 *The side of any of the fiue regular bodies being given, to find the semidiameter of a sphære, that will circumscribe the said bodie.*

If the sphære be first given, take his semidiameter, and to it

open

open the *Sector* in the points at S: if any of the other bodies be firſt given, take the ſide of it, and fit it over in his due points: ſo the parallell taken from betweene the points of the other bodies, ſhall be the ſides of thoſe bodies, and may be inſcribed into the ſame ſphære.



So if the ſemidiameter of the ſphære be *A C*, the ſide of the *dodecahedron* inſcribed ſhall be *D E*.

CHAP. IIII.

Of the lines of Equated bodies.

THe lines of *equated bodies* are here placed betweene the lines of *lines* and *ſolids*, noted with theſe letters, *D*, *I*, *C*, *S*, *O*, *T*, of which *D* ſtands for the ſide of a *dodecahedron*, *I*, for the ſide of an *Icoſahedron*, *C* for the ſide of a *cube*, *S* for the diameter of a *ſphere*, O for the ſide of an *octahedron*, and *T* for the ſide of a *tetrahedron*, all equall one to the other. The uſe of theſe lines may be.

1 *The diameter of a ſphære being given, to find the ſides of the fiue regular bodies, equall to that ſphære.*

2 *The ſide of any of the fiue regular bodies being given, to find the diameter of a ſphære, and the ſides of the other bodies, equall to the firſt body given.*

If the ſphære be firſt given, take his diameter, and to it open the *Sector* in the points at S: if any of the other bodies be firſt given, take the ſide of it, and fit it over in his due points: ſo the parallels taken from betweene the points of the other bedies, ſhall be the ſides of thoſe bodies equall to the firſt body given.

Thus in the laſt diagram, if the diameter of a ſphære given be *B C*, the ſide of the *dodecahedron* equall to this ſphære, would be found to be F G.

CHAP.

CHAP. V.

Of the lines of Mettals.

THe lines of *Mettalls* are here ioyned with those before of *equated bodies*, and are noted with these characters ☉.☿.♄.☽.♀.♂.♃. of which ☉ stands for gold, ☿ for quicksilver, ♄ for leade, ☽ for silver, ♀ for copper, ♂ for iron, and ♃ for tin. The use of them is to give a proportion betweene these severall mettals, in their magnitude and weight, according to the experiments of *Marinus Ghetaldus*, in his booke called *Promotus Archimedes*.

1 *In like bodies of severall mettalls and equall weight, having the magnitude of the one, to finde the magnitude of the rest.*

Take the magnitude given out of the lines of *Solids*, and to it open the *Sector* in the points belonging to the mettall given: so the parallells taken from between the points of the other mettalls, and measured in the lines of *Solids*, shall giue the magnitude of their bodies.

Thus having cubes or sphæres of equall weight, but severall mettalls, we shall finde that if those of tin containe 10000 *D*, the others of iron will containe 9250, those of copper 8222, those of silver 7161, those of lead 6435, those full of quicksilver 5453, and those of gold 3895.

2 *In like bodies of severall mettalls and equall magnitude, having the weight of one to finde the weight of the rest.*

This proposition is the converse of the former, the proportion not direct, but reciprocall, wherefore having two like bodies, take the given weight of the one out of the lines of *Solids*, and to it open the *Sector* in the points belonging to the mettall of the other body: ſo the parallell taken from the points belonging to the body given, and meaſured in the lines of *Solids*, ſhall give the weight of the body required.

As if a cube of gold weighed 38 ℓ. and it were required to know the weight of a cube of lead having equall magnitude. Firſt I take 38 ℓ. for the weight of the golden cube, out of the lines of *Solids*, and put it over in the points of ♄ belonging to lead: ſo the parallell taken from betweene the points of ☉ ſtanding for gold, and meaſured in the lines of *Solids*, doth give the weight of the leaden cube required to be 25 ℓ.

Thus if a ſphære of gold ſhall weigh 10000. we ſhall finde that a ſphære of the ſame diameter full of quickſilver ſhall weigh 7143, a ſphære a lead 6053, a ſphære of ſilver 5438, a ſphære of copper 4737, a ſphære of iron 4210, and a ſphære of tinne 3895.

3 *A body being given of one mettall, to make another like unto it, of another mettall, and equall weight.*

Take out one of the ſides of the body given, and put it over in the points belonging to his mettall: ſo the parallell taken from betweene the points belonging to the other mettall, ſhall give the like ſide, for the body required. If it be an irregular body, let the other like ſides be found out in the ſame manner.

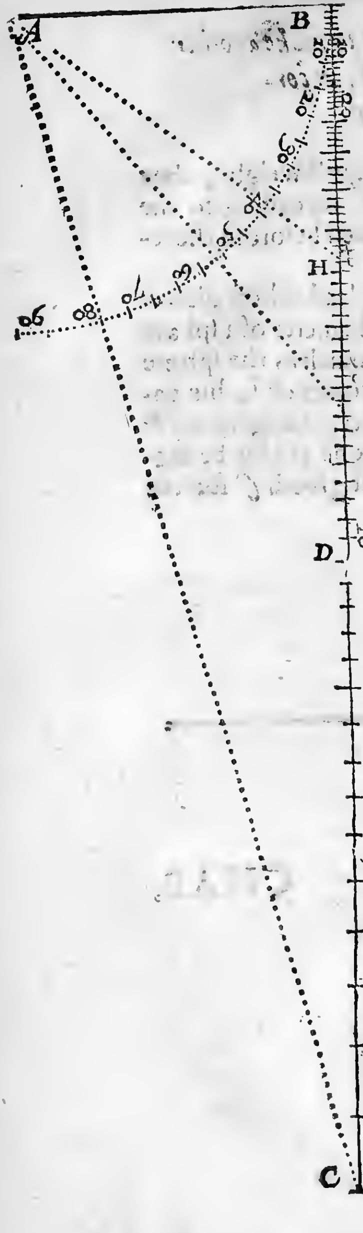
Let the body given be a ſphære of lead containing in magnitude 16 *d*, whoſe diameter is *A*, to which I am to make a ſphære of iron, of equall waight: If I take out the diameter *A*, and put it over in the points of ♄ belonging to lead, the parallell taken from betweene the points of ♂ ſtanding for iron, ſhall be *B*, the diameter of the iron ſphære required. And this compared with the other diameter, in the lines of *Solids*,

Solids will be found to be 23 *d.* in magnitude.

4 *A body being given of one mettall, to make another like unto it of another mettall, according to a weight given.*

First find the sides of a like body of equall weight, then may we either augment or diminish them according to the proportion given by that which we shewed before in the second and third *Prop:* of *Solids.*

As if the body given were a sphære of lead, whose diameter is *A*, and it were required to find the diameter of a sphære of iron which shall weigh three times as much as the sphære of lead: I take *A*, and put it over in the points of ♄, his parallell taken from betweene the points of ♂, shall give me *B* for the diameter of an equall sphære of iron: if this be augmented in such proportion as 1 unto 3, it giveth *C* for the diameter required.



CHAP. VI.

Of the lines on the edges of the Sector.

HAving shewed some use of the lines on the flat sides of the *Sector*, there remaine onely those on the edges. And here one halfe of the outward edge is divided into inches, and numbred according to their distance from the ends of the *Sector.* As in the Sector of fourteene inches long, where we find 1 and 13, it sheweth that division to be 1 inch from the nearer end, and 13 inches from the farther end of the *Sector*.

The other halfe containeth a line of lesser *tangents*, to which the gnomon is Radius. They are here continued to 75 *gr.* And if there be need to produce them farther, take 45 out of the number of degrees required, and double the remainder: so the *tangent* and *secant* of this double remainder being added, shall make up the *tangent* of the degrees required.

As if *A B* being the Radius, and *B C* the tangent line, it were required to find the tangent of 75 *gr.* If we take 45 *gr.* out of 75 *gr.* the remainder is 30 *gr.* and the double 60 *gr.* whose tangent is B D, and the secant is A D: if then we adde A D to B D, it maketh *B C* the tangent of 75 *gr.* which was required. In like sort the secant of 61 *gr* added to the tangent of 61 *gr.* giveth the tangent of 75 *gr.* 30 *m.* and the secant of 62 *gr.* added to the tangent of 62 *gr.* giveth the tangent of 76 *gr.* and

and so in the rest. The use of this line may be

To obserue the altitude of the Sunne.

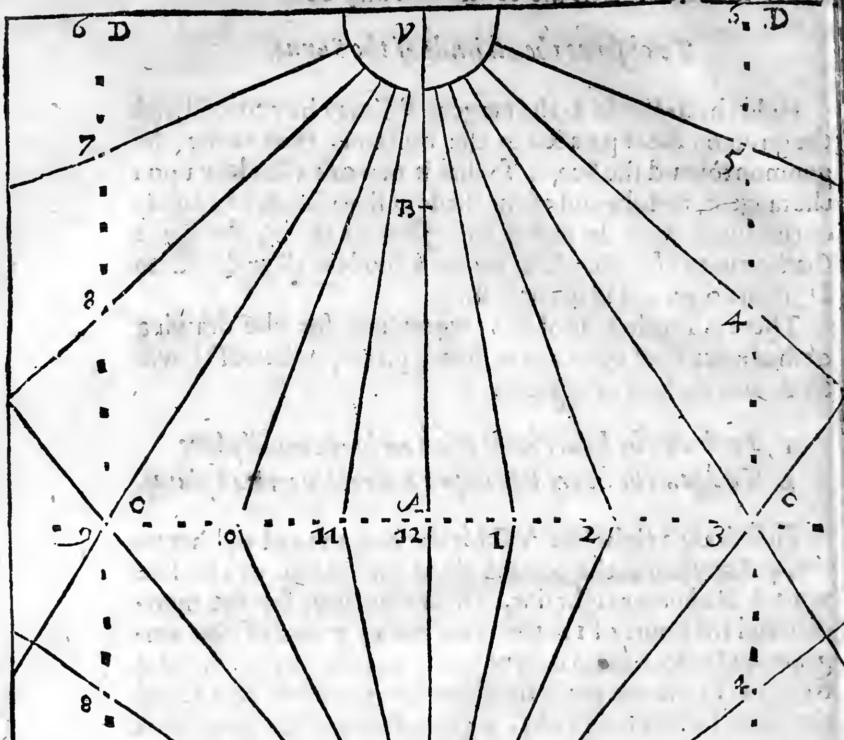
Hold the *Sector* so as the tangent *BC* may be verticall, and the gnomon *B A* parallell to the horizon; then turne the gnomon toward the Sunne, so that it may cast a shadow upon the tangent, and the end of the shadow shall shew the altitude of the *Sunne*. So if the end of the gnomon at *A*, do give a shadow unto *H*, it sheweth that the altitude is 38 *gr.* $\frac{1}{2}$, if unto D, then 60 *gr.* and so in the rest.

There is another use of this *tangent* line, for the drawing of the houre lines upon any ordinary plane, whereof I will set downe these propositions.

1 *To draw the houre lines upon an horizontall plane.*
2 *To draw the houre lines upon a direct verticall plaine.*

First draw a right line A C for the horizon and the æquator, and crosse it at the point A about the middle of the line with A B another right line, which may serve for the meridian and the houre of 12; then take out 15 *gr.* out of the tangents, and pricke them downe in the æquator on both sides from 12: so the one point shall serve for the houre of 11, and the other for the houre of 1. Againe, take out the tangent of 30 *gr.* and pricke it downe in the æquator on both sides from 12: so the one of these points shall serue for the houre of 10, and the other for the houre of 2. In like maner may you pricke downe the tangent of 45 *gr.* for the houres of 9 and 3 and the tangent of 60 *gr.* for the houres of 8 and 4, and the tangent of 75 *gr.* for the houres of 7 and 5.

Or if any please to set downe the parts of an houre, he may allow 7 *gr.* 30 *m.* for every halfe houre, and 3 *gr.* 45 *m.* for every quarter. This done, you are to consider the latitude of the place, and the qualitie of the plane: For the *secant* of the latitude shall be the semidiameter in a vertical plane, & the *secant* of the complement of the latitude in an horizontall plane.


D
V
D
B
A
C
C
H
D
D
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5

For example, about London the latitude is 51 *gr.* 30 *m.* and let the plane be verticall. If you take *A V* the secant of 51 *gr.* 30 *m.* out of the *Sector*, and pricke it downe in the meridian line from *A* unto *V*, the point *V* shall be the center: and if you draw right lines from *V* unto 11, and 10, and the rest of the houre points, they shall be the hourelines required.

But if the plane be horizontall, then you are to take out *A H* the secant of 38 *gr.* 30 *m.* for the semidiameter, and prick it downe in the meridian line from *A* unto *H*: so the right lines drawne from the center *H* unto the houre points, shall be the hourelines required; onely the houre of 6 is wanting, and that must alwaves be drawne parallell to the æquator, through the center V in a verticall, through the center *H* in a horizotall plane.

This being done, if you set the lines *A H*, *H V*, to a right angle (*H A V*) the right line *H V* the base of this triangle shall be the axis of the style for either plaine.

3 *To draw the houre lines on a polar plane.*

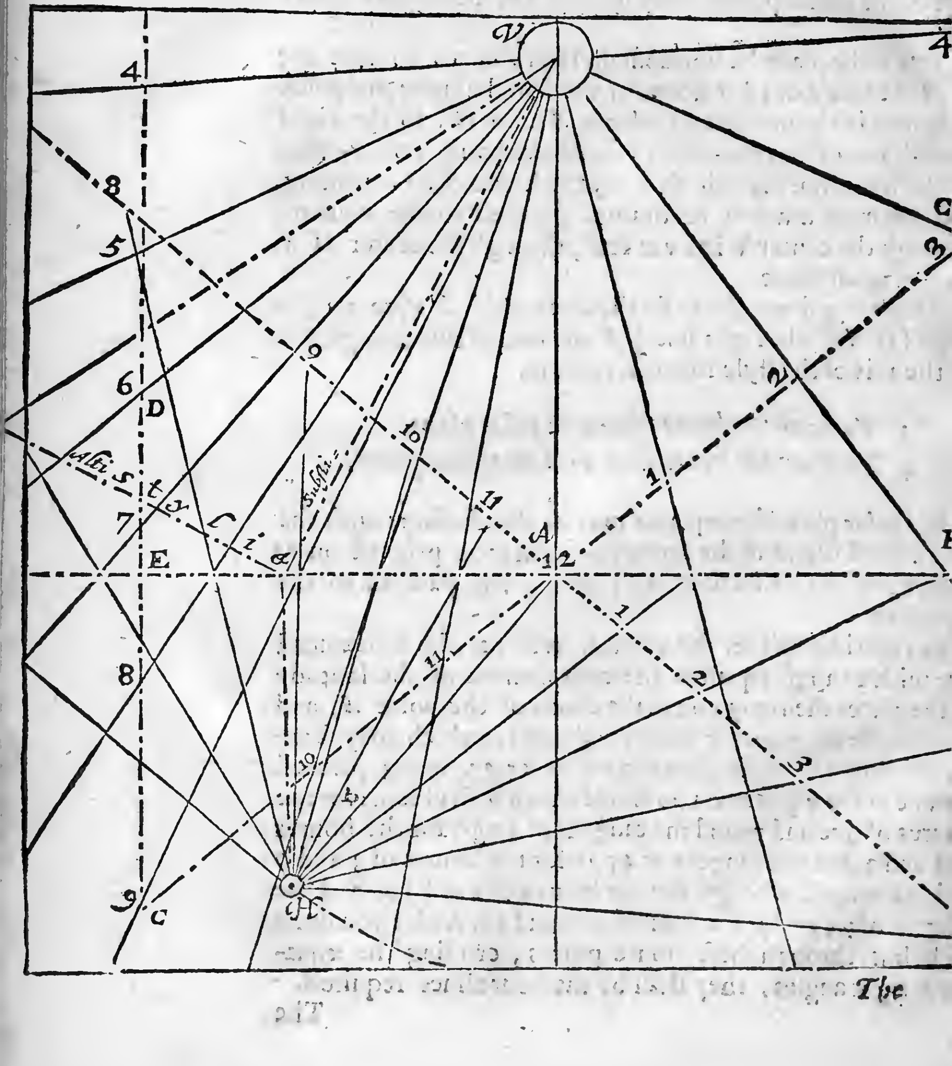
4 *To draw the houre lines on a meridian plane.*

In a *polar* plane the æquator may be also the same with the horizontall line, and the houre points may be pricked on as before, but the houre lines must be drawne parallell to the meridian.

In a meridian plane, the æquator will cut the horizontall line with an angle equall to the complement of the latitude of the place; then may you make choise of the point *A*, and there crosse the æquator with a right line, which may serve for the houre of 6: so the tangent of 15 *gr.* being pricked downe in the æquator on both sides from 6, shall serue for the houres of five and 7; and the tangent of 30 *gr.* for the houres of 8 and 4, and the tangent of 45 *gr.* for the houres of 3 and 9 and the tangent of 60 *gr.* for the houres of 2 and 10; and the tangent of 75 *gr.* for the houres of 1 and 11. And if you draw right lines through these houre points, crossing the æquator at right angles, they shall be the houre lines required.

The

The substilar will be the same with the houre of 12 in th Polar plane, and with the houre of 6 in the Meridian plan the axis of the stile may be parallell to the substilar in eith plane according to the distance of the third houre from t substilar.



The

5 *To draw the houre lines in a verticall declining plane.*

First, draw *A V* the meridian, and *A E* the horizontall line crossing one the other at right angles in the point *A*.

2 Then take out *A V*, the secant of the latitude of the place, which you may suppose to be 51 *gr*, 30 *m*. and pricke it downe in the meridian line from *A* unto V.

3 Because it is a declining plane, and you may suppose it to decline 40 *gr*. Eastward, you are to make an angle of the declination upon the center *A*, below the horizontall line, and to the left hand of the meridian line, because the declination is Eastward, for otherwise it should have bin to the right hand, if the declination had bin Westward.

4 Take A *H*, the secant of the complement of the latitude out of the *Sector*, & pricke it downe in the line of declination from A unto *H*, as you did before for the semidiameter in the horizontall plane.

5 Draw a line at full length through the point A, which must be perpendicular unto A *H*, and cut the horizontall line according to the angles of declination, and it will be as the æquator in the horizontall plane.

6 Take the houre points out of the *Tangent* line in the *Sector* and pricke them downe in this æquator on both sides from the houre of 12 at A.

7 Lay your ruler, & draw right lines through the center H & each of these houre points: so have you all the houre lines of an horizontall plane, onely the houre of 6 is wanting, and that may be drawne through H perpendicular to H A.

Lastly, you are to obserue and marke the intersections, which these houres lines do make with A E the horizontall line of the plane: and then if you draw right lines through the center V, and each of these intersections, they shall be the houre lines required.

The line H F drawne up to the Horizon and parallell to the meridian, will give the substilar V F: The line F G drawne Perpendicular to V F and equall to F H will give V G the axis of the stile.

6 *To pricke downe the houre points another way.*

Having drawne a right line for the æquator as before, and made choise of the point A, for the houre of 12: you may at pleasure cut of two equall lines A 10, and A 2. Then upon the distance betweene 10 and 2, make an equilaterall triangle, and you shall have *B* for the center of your æquator, and the line A B shall give the distance from A to 9, and from A to 3. That done take out the distance betweene 9 and 3, and this shall give the distance from B unto 8, and from 8 unto 7; and from 8 unto 1. and againe from B unto 4, and from 4 unto 5 and from 4 unto 11. So have you the houre points, and if you take out the distance B 1, B 3, B 5, &c. You may finde the points not onely for the halfe houres, but also for the quarters.

But if it so fall out, that some of these houre points fall out of your plane, you may helpe your selfe by the larger *tangent*, both in the verticall, and horizontall planes.

For if at the houre points of 3 and 9, in schem. *p.* 158 you draw occult lines parallell to the meridian; the distances D C betweene the houre line of 6, and the houre points of 3 and 9, will be equall to the semidiameter A V in a verticall, and A H in a horizontall plane, and if they be divided in such sort as the line A C is divided, you shall have the points of 4, and 5, and 7, and 8, with their halfes and quarters.

As in the horizontall plane, take out the semidiameter A H, and make it a parallell Radius by fitting it over in the *sines* of 90 and 90: Then take 15 *gr.* our of the larger *tangent* and lay them on the lines of *sines*, where they will reach from the center unto the sines of 15 *gr.* 32 *m.* therefore take out the parallell sine of 15 *gr.* 32 *m.* and it shall give the distance from 6 unto 5, and from 6 unto 7, in your horizontall plane. That done take out 30 *gr.* out of the larger *tangent*, and lay them on the *sines*, from the center unto the sines of 35 *gr.* 16 *m* and the parallell sine of 35 *gr.* 16 *m.* shall give you the distance from 6 unto 4, and from 6 unto 8, in your horizontall plane.

plane. The like may be done for the halfe houres and quarters.

So also in the verticall declining plane. If you first take out the *secant* of the declination of the plane, and prick it downe in the horizontall line from A unto E, and through E draw right lines parallell to the meridian, which will cut the former houre lines of 3 and 9, or one of them in the point C: then take out the semidiameter A V, and prick it downe in those parallells from C unto D, and draw right lines from A unto C, and from V unto D; the line V D shall be the houre of 6, and if you divide these line A C and D C, in such sort as you divided the like line D C in the horizontal plane, you shall have all the houre points required.

Or you may find the point D, in the houre of 6, without knowledge either of H or *C*. For having prickt downe A V in the meridian line, and A E in the horizontall line, and drawne parallels to the miridian through the points at E, you may take the *tangent* of the latitude out of the *Sector*, and fit it over in the sines of 90 and 90: so the parallell sine of the declination measured in the same *tangent* line, shall there shew the complement of the angle D V A, which the houre line of 6 maketh with the meridian; then having the point D, take out the semidiameter V A, and pricke it downe in those parallels from D unto C: so shall you have the lines D C and A C to be divided as before.

The like might be used for the houre lines upon all other planes. But I must not write all that may be done by the *Sector*. It may suffice that I have wrote something of the use of each line, and thereby given the ingenuous Reader occasion to thinke of more.

The conclusion to the Reader.

IT is well knowne to many of you, that this Sector was thus contrived, the most part of this booke written in latin, many copies transcribed and dispersed more then sixteene yeares since. I am at the last contented to give way that it come forth in English. Not that I thinke it worthy either of my labour or the publique view, but partly to satisfie their importunity, who not understanding the Latine, yet were at the charge to buy the instrument, and partly for my owne ease. For as it is painefull for others to transcribe my copie, so it is troublesome for me to give satisfaction herein to all that desire it. If I finde this to give you content, it shall incourage me to do the like for my Crosse-staffe, *and some other Instruments. In the meane time beare with the Printers faults, and so I rest.*

Gresham Coll. 1. Maij. 1623. *E. G.*

FINIS.

THE FIRST BOOKE OF THE *CROSSE-STAFFE.*

CHAP. I.

Of the description of the Staffe.

THe *Crosse Staffe* is an instrument well knowne to our Sea-men, and much used by the ancient Astronomers & others, serving Astronomically for obseruation of altitude and angles of distance in the heauens, Geometrically for perpendicular heights and distances on land and sea.

The description and seuerall vses of it are extant in print, by *Gemma Frisius* in Latine, in English by *Dr. Hood.* I differ something from them both, in the proiection of this *Staffe*, but so, as their rules may be applied vnto it, and all their propositions be wrought by it: and therefore referring the Reader to their bookes, I shall be briefe in the explanation of that which may be applied from theirs vnto mine, and so come to the vse of those lines which are of my addition, not extant heretofore.

The necessary parts of this Instrument are five: the *Staffe*, the *Crosse*, and the three *sights*. The Staffe which I made for my owne use, is a full yard in length, that so it may serue for measure.

The Crosse belonging to it is 26 inches $\frac{1}{5}$ betweene the two outward sights. If any would have it in a greater forme, the proportion betweene the Staffe and the Crosse, may bee such as 360 vnto 262.

The lines inscribed on the Staffe are of foure sots. One of them serues for measure and protraction: one for obseruation of angles: one for the Sea-cart; and the foure other for working of proportions in seuerall kindes.

The line of measure is an *inch line*, and may be knowne by his equall parts. The whole yard being divided equally into 36 inches, and each inch subdiuided, first into ten parts, and then each tenth part into halfes.

The line for obseruation of angles may bee knowne by the double numbers set on both sides of the line, beginning at the side at 20, and ending at 90: on the other side at 40, and ending at 180: and this being divided according to the degrees of a quadrant, I call it the *tangent line on the Staffe*.

The next line is the meridian of a Sea-chart, according to *Mercators* proiection from the Equinoctiall to 58 *gr.* of latitude, and may be knowne by the letter *M*, and the numbers 1.2.3.4. unto 58.

The lines for working of proportions, may be knowne by their vnequall diuisions, and the numbers at the end of each line.

1 The line of *numbers* noted with the letter N, diuided vnequally into 1000 parts, and numbred with 1.2.3.4. vnto 10.

2 The line of *artificiall tangents* is noted with the letter *T*, divided unequally into 45 degres, and numbred both wayes, for the Tangent and the complement.

3 The line of *artificiall sines*. noted with the letter *S*, divided unequally into 90 degrees, and numbred with 1.2.3.4. unto 90.

4 The line of *versed sines* for more easie finding the houre and azimoth, noted with V, divided vnequally into about 164 *gr.* 50 *m.* numbred backward with 10.20.30. vnto 164.

Thus there are seven lines inscribed on the Staffe: there are five lines more inscribed on the Crosse.

1 A Tangent line of *36 gr. 3 m.* numbred by 5. 10. 15. unto 35: the midst whereof is at 20. *gr*; and therefore I call it the *tangent of* 20; and this hath respect vnto 20 *gr.* in the Tangent on the Staffe.

2 A Tangent line of *49 gr. 6 m.* numbred by 5. 10. 15. unto 45; the midst whereof is at 30 *gr.* and hath respect unto 30 *gr.* in the Tangent on the Staffe, whereupon I call it the *tangent of* 30.

3 A line of *inches* numbred with 1. 2. 3. vnto 26; each inch equally subdiuided into ten parts, answerable to the inch line upon the Staffe.

4 A line of seuerall *chords*, one answerable to a circle of twelue inches semidiameter, numbred with 10. 20. 30. unto 60. another to a semidiameter of a circle of six inches; and the third to a semidiameter of a circle of three inches; both numbred with 10. 20. 30. unto 90.

5 A continuation of the *meridian* line from 57 *gr.* of latitude unto 76 *gr*; and from 76. to 84 *gr.*

For the inscription of these lines. The first for measure is equally diuided into inches and tenth parts of inches.

The tangent on the Staffe for obseruation of angles, with the tangent of 20 and the tangent of 30 on the Crosse, may all three be inscribed out of the ordinary *table of tangents.* The Staffe being 36 inches in length; the Radius for the tangent on the Staffe will be 13 inches and 103 parts of 1000: so the whole line will be a tangent of 70 *gr.* and must be numbred by their complements, and the double of their complements, the tangent of 10 *gr.* being numbred with 80 and 160.

The Radius for the tangent of 20 on the Crosse, will bee 36 inches, and the whole line betweene the sights a tangent of *36 gr. 3 m.* according as it is numbred. The Radius for the tangent of 30 *gr.* on the Crosse, will be 22 inches and 695 parts of 1000: so the whole line betweene the sights will containe a tangent of *49 gr. 6 m.* in such sort as they are numbred.

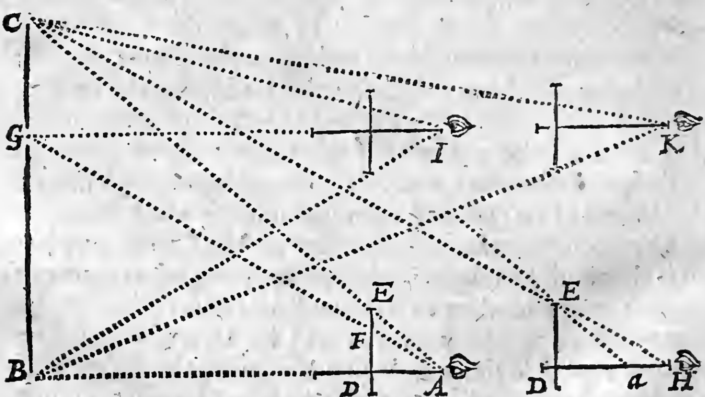
The meridian line may be inscribed out of the Table which I set downe for this purpose in the vse of the Sector.

The line of numbers may be inscribed out of the first Chiliad of Master *Briggs* Logarithmes: and the rest of the lines of proportion out of my *Canon of artificiall sines and tangents*; and in recompence thereof this booke will serue as a comment to explaine the use of my *Canon.*

CHAP. II,

The use of the lines of inches *for perpendicular heights and distances.*

IN taking of heights and distances, the Staffe may be held in such sort, that it may be even with the distance, and the Crosse parallel with the height: and then if the eye at the beginning of the Staffe shall see his markes by the inward sides of the two first sights, there will be such proportion between the distance and the height, as is betweene the parts intercepted on the Staffe and the Crosse. Which may be farther explained in these propositions.



1 *To find an height at one station, by knowing the distance.*

Set the middle sight unto the distance upon the Staffe, the height

height will bee found vpon the Crosse. For

As the segment of the Staffe
vnto the segment on the Crosse:
So is the distance given,
unto the height.

As if the distance A B being knowne to bee 256 feete, it were required to find the height B C: first I place the middle sight at 25 *inches* and 6 *parts* of 10; then holding the Staffe levell with the distance, I raise the Crosse, parallell vnto the height, in such sort, as that my eye may see from *A* the beginning of the inches on the Staffe by the sight *E*, at the beginning of the inches on the Crosse unto the mark *C*: which being done, if I find 19 inches and 2 parts of 10 intercepted on the Crosse betweene the sights at *E* and *D*, I would say the height *B C* were 192 feete.

Or if the obseruation were to be made before the distance were measured, I would set the middle sight either vnto 10 inches, or 12, or 16, or 20, or, 24, or some such other number as might best be divided into severall parts, and then worke by proportion. As if in the former example the middle sight were at 24 on the Staffe, and 18 on the Crosse, it should seem that the height is $\frac{3}{4}$ of the distance; and therefore the distance being 256, the height should be 192.

2 *To finde an height, by knowing some part of the same height.*

As if the height from G to C were knowne to be 48, and it were required to find the whole height B C: either put the third sight or some other running sight vpon the Crosse betweene the eye and the marke G. For then

As the difference betweene the sights,
vnto the whole segment of the Crosse:
So is the part of the height given,
vnto the whole height.

If then the difference betweene the sights E and F, shall

 be

be 45, and the segment of the Crosse ED 180, the whole height *B C* will be found to be 192.

3 *To find an height at two stations, by knowing the difference of the same stations.*

As the difference of segments on the Staffe,
unto the difference of stations:
So is the segment of the Crosse,
unto the height.

Suppose the first station being at *H*, the segment of the Crosse *E D* were 180, and the segment of the Staffe *H D* 300: then comming 64 feete nearer vnto *B*, in a direct line, vnto a second station at *A*, and making another obseruation; suppose the segment of the Crosse *E D* were 180 as before, and the segment of the Staffe *A D* 240; take 240 out of 300, the difference of segments will be 60 parts. And

As 60 parts unto 64 the difference of stations:
So *D E* 180 unto *B C* 192 the height required.

In these three *Prop.* there is a regard to be had of the height of the eye. For the height measured, is no more then from the levell of the eye upward.

4 *To finde a distance, by knowing the height.*

As the segment of the Crosse,
unto the segment of the Staffe:
So is the height giuen,
unto the distance.

So the segment E D being 18, and D A 24, the height *C B* 192, will shew the distance *A B* to be 256.

5 *To finde a distance, by knowing part of the height.*

As the difference betweene the sights,
unto the segment of the Staffe:

So

So is the part of the height given,
unto the distance.

And thus the difference betweene *E* and *F* being 45, and the segment D A 240; the part of the height *G C* 48, will give the distance *A* B to be 256.

6 *To finde a distance at two stations, by knowing the difference of the same stations.*

As the difference of segments on the Staffe,
unto the difference of stations:
So is the whole segment,
unto the distance.

And thus the segment of the Crosse being 180, the segment of the Staffe at the first station 240, at the second 300, the difference of the segments 60, and the difference of stations 64, the distance A B at the first station will be found to be 256, and the distance H B at the second station 320.

7 *To find a breadth by knowing the distance perpendicular to the breadth.*

This is all one with the first *Prop.* For this bredth is but an height turned sidewayes: and therefore
As the segment of the Staffe,
unto the segment of the Crosse;
So is the distance
unto the breadth.

And thus the segment of the Staffe being 24, and the segment of the Crosse 18, the distance A B 256, will give the breadth B C to be 192.

8 *To find a breadth at two stations in a line perpendicular to the bredth, by knowing the difference of the same stations.*

This is also the same with the third *Prop.* and therefore

As the difference of ſegments on the Staffe,
unto the difference of ſtations:
So the ſegment on the Croſſe betweene the two ſights,
unto the bredth required.

And thus the difference betweene the ſtations at *A* and *H* being 64, the difference of ſegments on the Staffe 60, the ſegment of the Croſſe 180, the bredth B C will bee found to be 192.

In like manner may we finde the breadth *G C* for having found the bredth B C the proportion will hold.

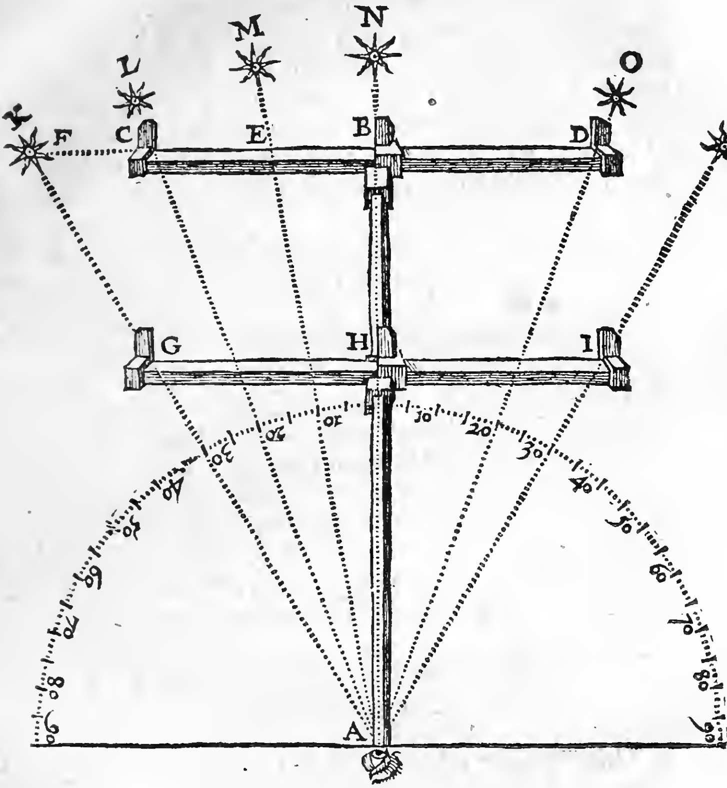
As D E is unto F E, ſo B *C* unto *G C*. Or otherwiſe,
As H *a* unto H A, ſo F E unto *G C*.

Neither is it materiall whether the two ſtations be choſen at one end of the bredth propoſed, or without it, or within it, if the line betweene the ſtations be perpendicular unto the bredth: as may appeare if in ſtead of the ſtations at *A* and H, we make choiſe of the like ſtations at *I* and *K*.

There might be other wayes propoſed to work theſe *Prop.* by holding the Croſſe even with the diſtance, and the Staffe parallell with the height: but theſe would proove more troubleſome, and thoſe which are delivered are ſufficient, and the ſame with thoſe which others have ſet down under the name of the *Iacobs Staffe*.

CHAP.

CHAP. III.
The use the Tangent lines in taking of Angles.



1 To finde an angle by the Tangent on the Staffe.

LEt the midle sight be alwaies set to the middle of the Crosse, noted with 20 and 30, and then the Crosse drawne nearer the eye, untill the markes may be seene close within the sights. For so if the eye at *A* (that end of the Staffe which is noted with 90 and 180) beholding the marks *K* and *N*, betweene the two first sights, *C* and *B*, or the markes *K* and *P* betweene the two outward sights, the Crosse being drawne downe unto H, shall stand at 30 and 60, in the Tangent on the Staffe: it sheweth the angle *K A N* is 30 *gr.* the angle *K A P* 60 *gr.* the one double to the other; which is the reason of the double numbers on this line of the Staffe: and this way will serve for any angle from 20 *gr.* toward 90 *gr.* or from 40 *gr.* toward 180 *gr.* But if the angle bee lesse then 20 *gr.* we must then make use of the Tangent vpon the Crosse

2 *To finde an angle by the Tangent of 20 upon the Crosse.*

Set 20 unto 20, that is, the middle sight to the middest of the Crosse at the end of the Staffe, noted with 20: so the eye at *A*, beholdng the marks *L* and *N*, close betweene the two first sights, *C* and *B*, shall see them in an angle of 20 *gr.*

If the markes shall be nearer together, as are *M* and *N*, then draw in the Crosse from *C* vnto *E*: if they be farther asunder, as are K and N, then draw out the Crosse from C vnto F; so the quantity of the angle shall be still found in the Crosse in the Tangent of 20 *gr.* at the end of the Staffe; and this will serue for any angle from 20 toward 35 *gr.*

3 *To finde an angle by the Tangent of 30 upon the Crosse.*

This Tangent of 30 is here put the rather, that the end of the Staffe resting at the eye, the hand may more easily remooue the Crosse: for it supposeth the Radius to be no longer then *A H*, which is from the eye at the end of the Staffe unto 30 *gr.* about 22 *inches* and 7 *parts.* Wherefore here set the middle sight unto 30 *gr.* on the Staffe, and then either draw the Crosse in or out, untill the markes be seene between the two first sights; so the quantitie of the angle will be found in the Tangent of 30, which is here represented by the line *G H*; and this will serve for any angle from 0 *gr.* toward 48 *gr.*

4 *To obserue the altitude of the Sunne backward.*

Here it is fit to have an horizontall sight set to the beginning of the Staffe, and then may you turne your backe toward the Sun, and your Crosse toward your eye. If the altitude be vnder 45 *gr.* set the middle sight to 30 on the Staffe, and looke by the middle sight through the horizontall vnto the horizon, mouing the Crosse vpward or downeward, untill the upper sight doe shadow the upper halfe of the horizontall sight: so the altitude will be found in the Tangent of 30.

If the altitude shalbe more then 45 *gr.* set the middle sight unto the middest of the Crosse, and look by the inward edge of the lower sight throungh the horizontall to the horizon, moving the middle sight in or out, untill the upper sight doe shadow the upper halfe of the horizontall sight: so the altitude will be found in the degrees on the Staffe betweene 40 and 180.

5 *To sit the Staffe to any angle given.*

This is the conuerse of the former *Prop.* For if the middle-sight be set to his place and degree, the eye looking close by the sights as before, cannot but see his obiect in the angle given.

6 *To obserue the altitude of the Sunne another way.*

Set the middle sight to the middle of the Crosse, and hold the horizontall sight downward, so as the Crosse may be parallell to the horizon, then is the Staffe verticall; and if the outward sight of the Crosse do shadow the horizontall sight, the complement of the altitude will be found in the Tangent on the Staffe.

7 *To obserue an altitude by thread and plummet.*

Let the middle sight be set to the middest of the Crosse, and to that end of the Staffe which is noted with 90 and 180; then having a thread and a plummet at the beginning of the Crosse, and turning the Crosse upward, and the Staffe toward the Sunne, the thread will fall on the complement of the altitude above the horizon. And this may be applied to other purposes.

8 *To apply the lines of inches to the taking of angles.*

If the angles be observed betweene the two first sights, there will be such proportion betweene the parts of the Staffe and the parts of the Crosse, as betweene the Radius and the Tangent of the angle.

As if the parts intercepted on the Staffe were 20 inches, the parts on the Crosse 9 inches. Then by proportion as 20 vnto 9, so 100000 unto 45000 the tangent of 24 *gr.* 14. *m.*

But if the angle shall be observed betweene the two outward sights, the parts being 20 and 9 as before, the angle will be 48 *gr.* 28 *m.* double vnto the former.

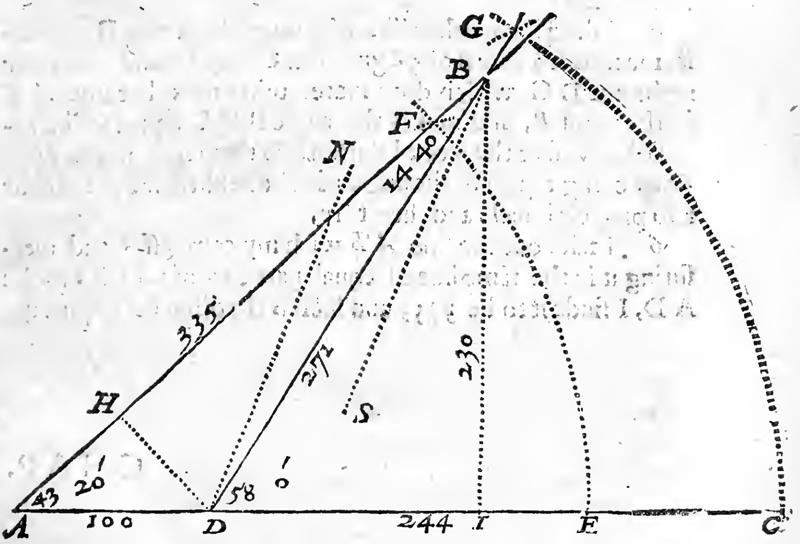
In all these there is a regard to be had to the parrallax of the eye, and his height above the Horizon in observations at Sea; to the semidiameter of the sunne, his parallax and refraction, as in the vse of other staves. And so this will be as much, or more then that whch hath beene heretofore performed by the *Crosse-Staffe.*

CHAP.

CHAP. IIII.

The use of the lines of equall parts ioyned with the lines of Chords.

THe lines of equall parts doe serue also for protraction, as may appeare by the former *Diagrams*; but being ioyned with the lines of Chords, which I place upon one side of the Crosse, they will farther serve for the protraction and resolution of right line triangles; whereof I will give one example in finding of a distance at two stations otherwise then in the second *Chap.*



Let the distance required be *A B*. At *A* the first station I make choise of a station line toward *C*, and obserue the angle *B A C* by the tangent lines, which may be 43 *gr*. 20 *m*; then

 having

having gon an hundred paces toward *C*, I make my second station at D, where suppose I finde the angle B D C to be 58 *gr.* or the angle *B D A* to be 112 *gr*, this being done, I may finde the distance *A B* in this maner.

1 I draw a right line *A C*, representing the station line.

2 I take 100 out of the lines of equall parts, and pricke them downe from *A* the first station unto *D* the second.

3 I open my compasses to one of the chords of 60 *gr.* and setting one foote in the point *A*, with the other I describe an occult arke of a circle intersecting the station line in *E*.

4 I take out of the same line of *chords* a chord of 43 *gr.* 20 *m.* (because such was the angle at the first station) and this I inscribe into that occult arke from *E* unto *F*, which makes the angle F A D equall to the angle obserued at the first station.

5 I describe another like arke upon the center D, and inscribe into it a chord of 58 *gr.* from *C* unto *G*, and draw the right line D G, which doth meete with the other line *A E* in the point *B*, and makes the angle B D C equall to the angle observed at the second station. So the angles in the *Diagram* being equall to the angles in the field, their sides will be also proportionall: and therefore,

6 I take out the line *A B* with my compasses, and measuring it in the same line of equall parts, from which I tooke A D, I finde it to be 335, and such is the distance required.

CHAP.

CHAP. V.

The use of the Meridian line.

1 THe Meridian line, noted with the letter *M*, may serue for the more easie diuision of the plane sea-cart, according to *Mercators* proiection, For if you shall draw parallell meridians, each degree being halfe an inch distant from other, the degree of this meridian line on the Staffe, shall giue the like degrees for the meridians on the chart, from the Equinoctiall toward to Pole: and then if through these degrees you draw straight lines perpendicular to the meridians, they shall be parallels latitude.

If any desire to haue the degrees of his chart larger then those which I haue put on the Staffe, he may take these and increase them in a double, or treble, or a decuple proportion at his pleasure.

2 This *meridian* line being ioyned with the line of *chords*, may serue for the protraction and resolution of such right line triangles as concerne latitude, longitude, rumb and distance in the practice of navigation. As may appeare by this example.

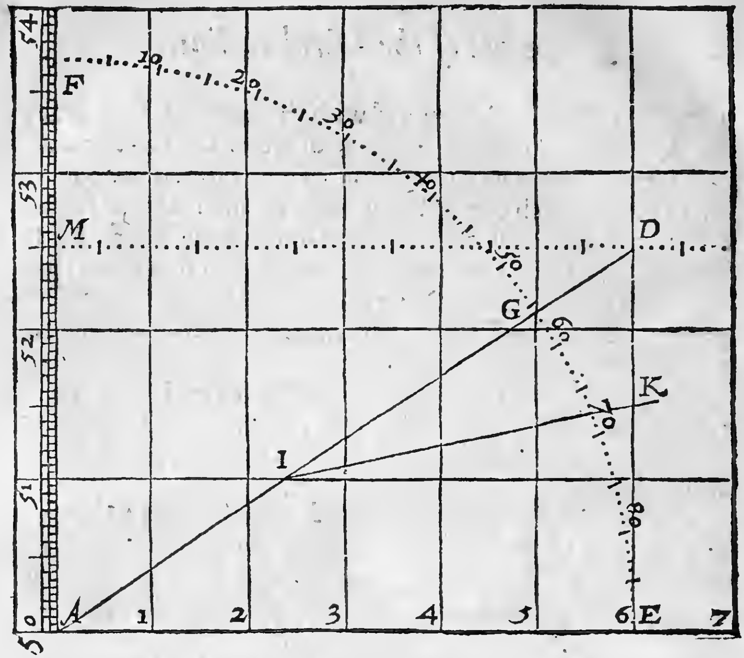
Suppose two places giuen, *A* in the latitude of 50 *gr.* D in the latitude of 52 *gr.* $\frac{1}{2}$, the differēce of longitude between them being 6 *gr.* and let it be required to know, first what Rumbe leadeth from the one place to the other, secondly how many degrees distant they are a sunder.

1 I draw a right line *A E*, representing the parallell of the place from whence I depart.

2 I take 6 *gr.* for the difference of longitude, either out of the line of *inches*, allowing halfe an inch for every degree, or out of the beginning of the *Meridian* line; (for there the meridian degrees differ very little from the equinoctiall degrees) and these 6 *gr.* I pricke downe in the parallell from *A* to *E*.

3 In

3 In *A* and *E*, I erect two perpendiculars, A M and E D, represnting the meridians of both places.



4 I take the difference of the latitude from 50 *gr*. to 52 *gr*. 30 *m*. out of the *meridian* line, and prick it down in the meridians from *A* vnto *M*, and from E to D, and draw the right line M D for the parallell of the second place, and the right line A D for the line of distance betweene both places: so the angle M A D shall give the Rumb that leadeth from the one place to the other.

5 To find the quantitie of this angle M A D, I may either make use of the Protractor, or else of a line of *chords*, and so I open my compasses vnto one of the chords of 60 *gr*. and setting one foote in the point *A*, with the other I describe an

an occult arke of a circle, intersecting the meridian in *F*, and the line of distance in G; then I take the chord F G with my compasses, and measuring it in the same line of *chords* as before, I finde it 56 *gr* $\frac{1}{4}$: and such is the inclination of the Rumb to the Meridian, which is the first thing that was required.

6 To finde the quantitie of the line of distance A D, I take it out with my compasses, and masuring it in the meridian line, setting one foote beneath the lesser latitude, and the other foote as much above the greater latitude, I find about 4 *gr.* $\frac{1}{2}$ intercepted betweene both feet: and such is the distance upon the Rumb, which is the second thing that was required.

But if this example were protracted according to the common Sea-chart, where the degrees of the equinoctiall and meridian are both alike; the Rumb M A D would be found to be aboue 67 *gr.* and A D the distance upon the Rumbe about 6 *gr.* $\frac{1}{2}$.

Suppose farther, that having set forth from *A* toward D, upon the former Rumb of 56 *gr.* 15 *m.* *N E b E*, after the ship had run 36 leagues, the wind changing, it ran 50 leagues more upon the seuenth Rumb of *E b N*, whose inclination to the meridian is 78 *gr.* 45 *m.* And let it be required to know what longitude and latitude the ship is in, by pricking downe the way thereof upon the Chart.

Having drawne a blank chart as before, with meridians and parallels, according to the latitude of the places proposed.

1 I would make an angle *M A D* of 56 *gr.* 15 *m.* for the Rumb of *N E b E*, which is done after this manner: I open my compasses to one of the *chords* of 60 *gr.* and setting one foote in the point *A*, with the other I describe an occult arke of a circle, interfecting the meridian in *F*; then I take 56 *gr.* 15 *m.* out of the same line of *chords*, and pricke them downe from F unto G: so the right line *A G* shall be the Rumb of *N E b E*.

2 I would take 36 leagues out of the *meridian line*, ex-

 tending

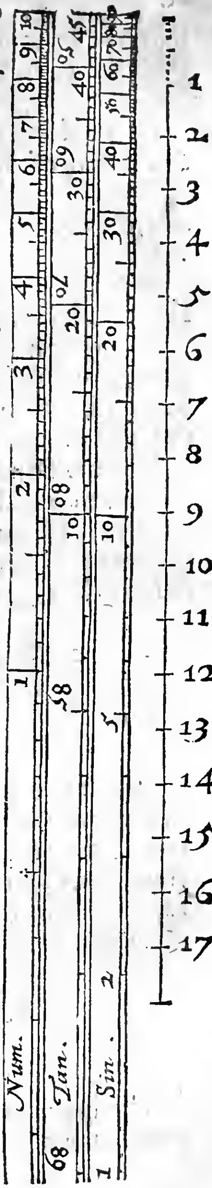
tending my compasses from 50 *gr.* 51 48 *m.* or rather from much below 50 as above 51, and prick them downe upon the Rumb from *A* unto *I*; so the point *I* shall represent the place wherein the ship was when the winde changed. And this is in the latitude of 51 *gr.* 0 *m.* and in the longitude of 2 *gr.* 21 *m.* Eastward from the meridian *A M.*

3 By the same reason, I may draw the right line *I K* for the Rmmb of *E b N*, and pricke downe the distance of 50 leagues from I unto K: so the point K shall represent the place whither the ship came, after the running of these 50 leagues: and this is in the latitude of 51 *gr.* 30 *m.* and in longitude 6 *gr.* 16 *m.* Eastward from the first meridian *A M* and therefore 16 *m.* Eastward from the second meridian, E D.

But if these two courses were to be pricked downe by the common Sea-chart, the point *I* would fall in the latitude of 51 *gr.* 0 *m.* and the point *K* in the latitude of 51 *gr.* 30. *m.* But the longitude of *I* would be onely 1 *gr.* 30 *m.* and the longitude of *K* only 3 *gr.* 57 *m.* more: both these do make but 5 *gr.* 27 *m.* for the difference of longitde betweene the first Meridian A M, and the point K: whereby it should seeme that the point K is yet 33 *m.* Westward from the Meridian of the place to which the ship was bound.

Such is the difference betweene both these charts.

CHAP.



CHAP. VI.

The use of the line of Numbers.

THe line of Numbers here noted with 1. 2. 3. 4 unto 10, is compleat in those divisions which are betweene 1 and 10: the other like divisions at the beginning of the line doe serue rather to answere to the first degrees of the two other lines of Sines and Tangents then for any necessity, which is the cause why some of them are omitted. And here as in the use of other Scales the figures 1. 2. 3. 4. and set downe upon the line doe sometimes signifie themselues alone, sometimes 10. 20. 30. 40. sometimes 100. 200. 300, 400, and so forward as the matter shall require. The first figure of every number is alwayes that which is here set downe, the rest must be supplied according to the nature of the question.

1 *Having two numbers given to finde a third in continuall proportion, a fourth, a fifth, and so forward.*

Extend the compasses from the first number unto the second; then may you turne them, from the second to the third, and from the third to the fourth, and so forward.

Let the two numbers given bee 2 and 4.

Extend the compasses from 2 to 4, then may you turne them from 4 to 8, and from 8 to 16, and from 16 to 32, and from 32 to 64, and from 64 to 128.

Or if one foote of the compasses being set to 64, the other fall out of the line, you may set it to another 64 neerer the beginning of the line, and there the other foot will reach to 128, and from 128 you may turne them to 256, and so forward.

Or if the two first number given were 10 and 9: extend the compasses from 10 at the end of the line, backe unto 9, then may you turne them from 9 unto 8. 1, and from 8. 1 unto 7. 29. And so if the two first numbers given were 1 and 9, the third would be found to be 81, the fourth 729, with the same extent of the compasses

In the same maner, if the two first numbers were 10 and 12, you may finde the third proportionall to be 14. 4, the fourth 17. 28. And with the same extent of the compasses, if the two first numbers were 1 and 12, the third would bee found to be 144, and the fourth to be 1728.

2 *Having two extreme numbers given, to find a meane proportionall betweene them.*

Divide the space betweene the extreame numbers into two equall parts, and the foote of the compasses will stay at the meane proportionall. So the extreme numbers given being 8 & 32, the meane betweene them will be found to be 16, which may be prooved by the former *Prop.* where it was shewed, that as 8 to 16, so are 16 to 32.

3 *To find the square roote of any number given.*

The square roote is alwayes the meane proportionall betweene 1 and the number given [illegible] e found by

dividing

dividing the ſpace betweene them into two equall parts. So the roote of 9 is 3, and the roote of 81 is 9, and the roote of 144, is 12, and the roote of 1440 almoſt 38.

If you ſuppoſe pricks under the number given, (as in Arithmeticall extraction) and the laſt pricke to the left hand ſhall fall under the laſt figure, which will be as oft as there be odde figures the unitie will be beſt placed at 1 in the middle of the line: ſo the roote and the ſquare will both fall forward toward the end of the line. But if the laſt pricke ſhall fall under the laſt figure but one, which will bee as oft as there be euen figures, then the unitie may be placed at 1 in the beginning of the line and the ſquare in the ſecond length or rather the unitie may be placed at 10 in the end of the line of the roote and the ſquare will both fall backward toward the middle of the line, in the ſecond length.

4 *Having two extreme numbers given, to find two meane proportionals betweene them.*

Divide the ſpace betweene the two extreme numbers given, into three equall parts. As if the extreme numbers given were 8 and 27, divide the ſpace betweene them into three equall parts, the feete of the compaſſes will ſtand in 12 and 18.

5 *To find the cubique roote of a number given.*

The cubique roote is alwayes the firſt of two meane proportionals betweene 1 and the number given, and therefore to be found by dividing the ſpace betweene them into three equall parts.

So the roote of of 1728 will be found to be 12. The roote of 17280 is almoſt 26: and the roote of 172800 is almoſt 56.

If you ſuppoſe pricks under the number given after the maner of Arithmeticall extraction, & the laſt prick to the left hand ſhall fall under the laſt figure as it doth in 1728, the unitie will be beſt placed at 1 in the middle of the line, and the roote the ſquare and the cube will all fall forward toward the end of the line.

If the laſt pricke ſhall fall vnder the laſt figure but one as in 17280, the unitie may be placed at 1 in the beginnning of the line, & the cube in the ſecond length. or the unitie may be placed at 10 in the end of the line: and the cube in the firſt length; or if the cube fall out of the line you may helpe your ſelfe as in the firſt *Prop.*

But if the laſt prick ſhall fall under the laſt figure but two, as in 172800, then place the unitie alwaies at 10 in the ende of the line: ſo the roote the ſquare and the cube will all fall backward and be found in the ſecond length between the middle and end of the line.

6 *To multiply one number by another.*

Extend the compaſſes from 1 to the multiplicator; the ſame extent applied the ſame way, ſhall reach from the multiplicand to the product.

As if the numbers to be multiplied were 25 and 30: either extend the compaſſes from 1 to 25, and the ſame extent will give the diſtance from 30 to 750; or extend them from 1 to 30, and the ſame extent ſhall reach from 25 to 750.

7 *To divide one number by another.*

Extend the compaſſes from the diviſor to 1, the ſame extent ſhall reach from the dividend to the quotient.

So if 750 were to be divided by 25, the quotient would be found to be 30.

8 *Three*

8 *Three numbers being given to finde a fourth proportionall.*

This golden rule, the most usefull of all others, is performed with like ease. For extend the compasses from the first number to the second, the same extent shall give the distance from the third to the fourth.

As for example, the proportion betweene the diameter and the circumference, is said to bee such as 7 to 22: if the diameter be 14, how much is the circumference? Extend the compasses from 7 to 22, the same extent shall give the distance from 14 to 44: or extend them from 7 to 14, and the same extent shall reach from 22 to 44.

Either of these wayes may be tried on severall places of this line; but that place is best, where the feete of the compasses may stand nerest together.

9 *Three numbers being given to finde a fourth in a duplicated proportion.*

If any haue daily use of this proposition he may cause another line of Numbers to be made.

This proposition concernes questions of proportion betweene *Lines* and *Superfices*; where if the denomination be of lines, extend the compasses from the first to the second nmmber of the same denomination: so the same extent being doubled, shall give the distance from the third number unto the fourth.

The diameter being 14, the content of the circle is 154: the diameter being 28, what may the content be? Extend the compasses from 14 to 28, the same extent doubled will reach from 154 to *616*. For first it reacheth from 154 unto 308; and turning the compasses once more, it reacheth from 308, unto 616; and this is the content required.

But

But if the first denomination be of the superficiall content, extend the compasses unto the halfe of the distance, betweene the first number and the second of the same denomination: so the same extent shall give the distance from the third to the fourth.

The content of a circle being 154, the diameter is 14: the content being 616, what may the diameter be? Divide the distance betweene 154 and 616 into two equall parts, then set one foote in 14, the other will reach to 28 the diameter required.

10 *Three numbers being given to find a fourth in a triplicated proportion.*

This proposition concerneth questions of proportion betweene *lines* and *sollids*; where if the first denomination bee of lines, extend the compasses from the first number to the second of the same denomination: so the extent being tripled, shall give the distance from the third number unto the fourth.

Suppose the diameter of an iron bullet being 4 inches, the weight of it was 9 ℔: the diameter being 8 inches, what may the waight be? Extend the compasses from 4 to 8, the same extent being tripled, will reach from 9 unto 72. For first it reacheth from 9 unto 18; then from 18 to 36; thirdly from 36 to 72. And this is the weight required.

But if the first denomination shall be of the Solid content, or of the weight, extend the compasses to a third part of the distance betweene the first number and the second of the same denomination: so the same extent shall give the distance from the third number unto the fourth.

The weight of a cube being 72 ℔, the side of it was 8 inches: the weight being 9 ℔, what may the side be? Divide the distance betweene 72 and 9, into three equall parts; then set one foote to 8, the other will reach to 4, the side required.

CHAP.

CHAP. VII.

The use of the line of artificiall Sines.

THis line of *sines* hath such use in finding a fourth proportionall, as the ordinary *Canon* of *Sines*: and the maner of finding it, is alwayes such as in this example.

As the sine of 90 *gr.* unto the sine of 30 *gr.*
So the sine of 20 *gr.* unto a fourth sine.

Extend the compasses from the Sine of 90 *gr.* unto the sine of 30 *gr.* the same extent will reach from the sine of 20 *gr.* unto the sine of 9 *gr.* 50 *m.*

Or you may extend them from the sine of 90 *gr.* unto the sine of 20 *gr.* the same extent will reach from the sine of 30 *gr.* unto the sine of 9 *gr.* 50 *m.* and such is the fourth proportionall sine required,

In like maner if the question proposed were

As the sine of 30 *gr.* unto the sine of 52 *gr.*
So the sine of 38 *gr.* to a fourth sine.

Extend the compasses in the line of *sines* from 30 *gr.* unto 52 *gr*; the same extent shall give the distance from 38 *gr.* unto 76 *gr.* Or extend them from 30 *gr.* unto 38 *gr.* the same extent will reach from 52 *gr.* unto 76 *gr.* which is the fourth proportionall sine required.

And thus may the rest of all sinicall proportions bee wrought two wayes. The minutes which are wanting in the first degree, may be supplied by the line of *Numbers*, as I shew in the next *Chapter*.

CHAP. VIII.

The use of the line of artificiall Tangents.

THis line of *Tangents* hath like use, but commonly ioyned with the line of *sines*: the manner of working by it, may appeare by this example.

As the Tangent of 38 *gr.* 30 *m.*
is the Tangent of 23. *gr.* 30. *m.*
So the Sine of 90 *gr.*
to a fourth Sine.

This *Prop.* and such others upon two lines, may bee wrought two wayes. For extend the compasses from the Tangent of 38 *gr.* 30 *m.* to the Tangent of 23 *gr.* 30 *m*; the same extent shall give the distance from the sine of 90 *gr.* to the sine of 33 *gr.* 8 *m.* Or else extend them from 38 *gr.* 30. *m.* in the Tangents unto 90. *gr.* in the line of *Sines*; the same extent from the Tangent of 23 *gr.* 30 *m.* shall reach to the sine of 33 *gr.* 8 *m.* which is the fourth proportionall sine required.

And this crosseworke in many cases is the better, in regard the tangents which should passe on from 40 *gr.* to 50 *gr.* and so forward, doe turne backe at 45 *gr.* These two lines of *Sines* and *Tangents*, may serue for the resolution of all sphericall triangles, according to those Canons which I have set downe in the use of the Sector. Onely two cases the 19 and 20 will bee more easily resolued by that which followeth in the last Chapter of this booke.

Or if at any time one meete with a *Secant*, Let him account the sine of 80 *gr.* for a *Secant* of 10 *gr.* and the sine of 70 *gr.* for a *Secant* of 20 *gr.* and so take the sine of of the complement in stead of the *Secant.*

As if the proposition were,
As the Radius to the secant of 51 *gr.* 30. *m.*
So the sine of 23 *gr.* 30. *m.*
to a fourth sine.

Extend the compasses from the Radius that is the sine of 90 *gr.* to the sine of 38 *gr.* 30 *m.* the same extent will give the distance from the sine of 23. *gr.* 30 *m.* both to the sine of 14. *gr.* 22 *m* to the sine of 39 *gr.* 50 *m.* But in this case, the sine of 39 *gr* 50 *m.* is the fourth required. For the first number being lesse then the second, that is, the Radius lesse then the secant, the sine of 23 *gr.* 30 *m.* which is the third, must also be lesse then the fourth.

If the fourth proportionall number shall at any time fall out of the line, by reason of the minutes that are wanting in the first degree, it may be supplied by resoluing the third number given into minutes, and then working by the line of numbers.

As if the proposition were,

As the Sine of 90 *gr.*
to the Sine of 10 *gr.*
So the sine of 5 *gr.*
to a fourth sine.
Or the Tangent of 5 *gr.*
to a fourth Tangent.

Extend the compasses from the sine of 90 *gr.* unto the sine of 10 *gr.* the same extent will reach from the Sine or Tangent of 5 *gr.* beyond the end of the staffe. Wherefore I resolve these 5 *gr.* into 300 minutes and find the former extent to reach in the line of numbers from 300 *m.* unto 52 *m.* and such is the fourth proportionall required.

If the the extent from the sine of 90 *gr.* unto the sine of 10 *gr.* be too large for the compasses we may use the Sine of

 5 *gr.*

5 *gr.* 44 *m.* instead of the signe of 90 *gr.*

And so extending the compasses from the sine of 5 *gr.* 44 *m.* unto the sine of 10 *gr.* we shall finde the same extent to reach in the line of Numbers from 300 unto 52 as before.

And by the same reason wee may use the tangent of 5 *gr.* 43 *m.* instead of the tangent of 45 *gr.* as I farther shew in the next *Chapter.*

CHAP. IX.

The use of the line of Sines and Tangents ioyned with the line of Numbers.

THe lines of *Sines* and *Tangents* another like use joyned with the the line of *Numbers*, especially in the resolution of right line triangles, where the angles are measured by degrees and minutes, and the sides measured by absolute numbers, whereof I will set downe these propositions.

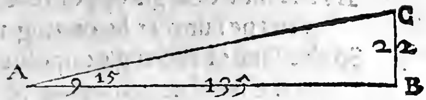
1 Having three angles and one side, to finde the two other sides.

If it be a rectrangle triangle wherein one side about the right angle being knowne it were required to finde the other. This may be found by the line of Tangents and line of Numbers. For

As the Tangent of 45 *gr.*
to the tangent of the angle opposite to the side required,
So the number belonging to the side given

to the number belonging to the side required.

As in the rectangle A B C knowing the angle C A B to



be 9 *gr.* 15 *m.* and the side A B to be 135 parts, if it were required to finde the other side B C about the right angle.

Extend the compasses from the Tangent of 45 *gr.* unto the Tangent of 9 *gr.* 15 *m.* the same extent will reach in the line of Numbers from 135 unto 22, and such is the length of the side B C. Or in the crosse worke extend the compasses from the Tangent of 45 *gr.* unto 135 in the line of numbers the same extent will reach from the Tanget of 9 *gr.* 15 *m.* unto 22 in the line of Numbers.

If this extent from the tangent of 45 *gr.* to 9 *gr.* 15 *m.* or 135 parts bee too large for the compasses, you may use the Tangent of 5 *gr.* 43 *m.* instead of the Tangent of 45 *gr.* because both alike answer to 10. &c, parts in the line of Numbers.

And then either extend the compasses from 5 *gr.* 43 *m.* unto 9 *gr.* 15 *m.* in the line of Tangents the same extent will reach from 135 unto 22 in the line of numbers, or else extend them from the tangent of 5 *gr.* 43 *m.* unto 135 in the line of Numbers the same extent will reach from the Tangent of 9 *gr.* 15 *m.* unto 22 in the line of Numbers as before.

In like manner if the same rectangle A B C knowing the angle A C B to be 80 *gr.* 45 *m.* and the side B C to bee 22 parts, it were required to finde the other side B A. You may use the Tangent of 84 *gr.* 17 *m.* instead of the Tangent of 45 *gr.* and so the side B A will be found to bee 135 parts.

This holdeth for finding of the sides of rectangle triangles but generally in all triangles, whither they be right or obtuse angles having three angles and one side wee may finde the two other sides by the line of *Sines* and line of *Numbers*.

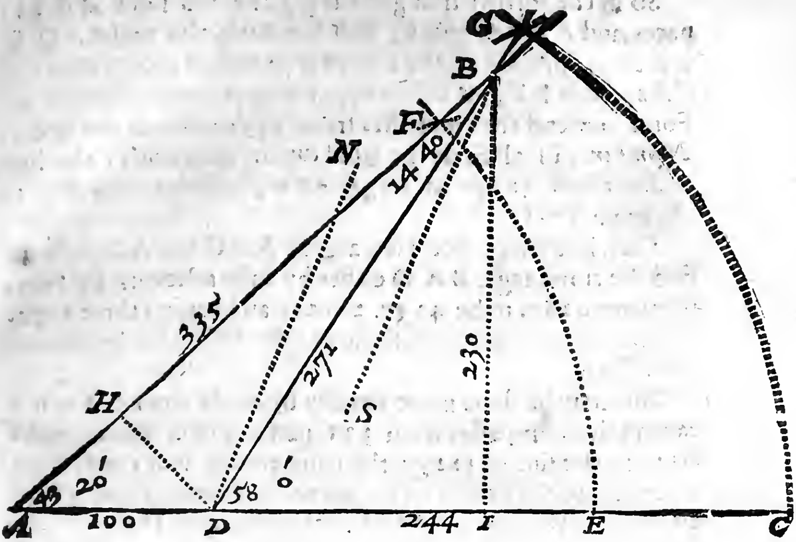
As the Sine of angle opposite to the side given,
is to the number belonging to that side given,
So the Sine of the angle opposite to the side required,
to the number belonging to the side required.

As in the example of the fourtth *Chapter.* of this booke, where knowing the distance betweene two stations at *A* and *D* to be 100 paces, the angle *B A C* to be 43 *gr.* 20 *m* and the angle *B D C* to be 58 *gr.* it was required to find the distance *A B.*

First having these two angles, I may finde the third angle *A B D* to be 14 *gr.* 40 *m.* either by substraction or by complement unto 180. Then in the Triangle *B A D*, I have three angles, and one side, whereby I may finde both *A B* and *D B.*

I know the angle *A B D* opposite to the measured side *A D* to bee 14 *gr.* 40 *m.* and the angle *A D B* opposite to the side required, to bee 122 *gr.*: wherefore I extend the compasses in the line of *Sines* from 14 *gr.* 40 *m.* unto 122 *gr.* or (which is all one) to 58 *gr.* (for after 90 *gr.* the sine of 80 *gr.* is also the sine of 100 *gr.* and the sine of 70 *gr.* the sine of 110 *gr.* and so in the rest) so shall I finde the same extent to reach in the line of *numbers*, from 100 unto 335. And such is the distance required betweene *A* and *B.*

In



In like maner if I extend my compaſſes from the ſine of 14 *gr.* 40 *m.* to the ſine 43 *gr.* 20. *m.* the ſame extent will reach in the line of *Numbers* from 100 to 271. And ſuch is the diſtance betweene D and B.

Or in croſſe worke, I may extend the compaſſes from 14 *gr.* 40 *m.* in the *Sines*, unto 100 parts in the line of *Numbers*: ſo the ſame extent will give the diſtance from 58 *gr.* to 335 parts, and from 43 *gr.* 20 *m.* to 271 parts.

2 *Having two ſides given, and one angle oppoſite to either of theſe ſides, to finde the other two Angles and the third ſide.*

As the ſide oppoſite to the angle given,
is to the ſine of the angle given:
So the other ſide given,
to the ſine of that angle to which it is oppoſite.

So in the former triangle, having the two sides *AB* 335 paces, and A D 100 paces, and knowing the angle A D B, which is opposite to the side *A* B, to be 122 *gr.* I may find the angle A B D, which is opposite to the other side A D. For if I extend the compasses from 335 to 100 in the line of *Numbers*, I shall finde the same extent to reach in the line of *Sines* from 122 *gr.* to 14 *gr.* 40 *m*; and therefore such is the angle A B D.

Then knowing these two angles A B D and A D B, I may find the third angle B A D either by subtraction or by complement to 180, to be 43 *gr.* 20 *m*; and having three angles and two sides, I may well finde the third side D B, by the former. *Prop.*

This may be done more readily by crosse worke. For if I extend the compasses from 335 parts, in the line of *numbers*, to the sine of 122 *gr.* the same extent will reach from 100 parts to the sine of 14 *gr.* 40 *m.* and backe from 43 *gr.* 20 *m.* to 271 parts; and such is the third side D B.

3 *Having two sides and the angle betweene them, to find the two other angles and the third side.*

If the angle contained betweene the two sides bee a right angle, the other two angles will be found readily by this Canon.

As the greater side given,
is to the lesser side:
So the tangent of 45. *gr.*
to the tangent of the lesser angle.

So in the rectanle triangle *A I B*, knowing the side *A I* to be 244, and the side *I B* to be 230: if I extend the compasses from 244 to 230 in the line of *numbers*, the same extent will reach from 45 *gr.* to about 43 *gr.* 20 *m.* in the line of of Tangents; and such is the lesser angle *B A I*, and the complement 46 *gr.* 40 *m* shewes the greater angle *A B I.* The angles being knowne, the third side A B may bee found by the first *Prop.*

So likewise in the example of the third Chapter of this booke, concerning taking of angles by the line of Inches, where the parts intercepted on the Staffe being 20 Inches, and the parts on the Crosse 9 Inches, it was required to finde the angle of the altitude. For,

I may extend the compasses in the line of *Numbers*, from 20 unto 9, the same extent will reach in the line of *Tangents*, from 45 *gr.* to 24 *gr.* 14 *m.*

Or in crosse worke,

I may extend the compasses from 20 parts in the line of *Numbers* to the tangent of 45 *gr*; the same extent shall give the distance from 9 parts unto the Tangent of 24 *gr.* 14 *m.*

And such is the angle of the altitude required.

If the parts intercepted on the staffe being 20 inches and the parts on the Crosse 9 tenth parts of an inch it were required to finde the angle of the altitude. Here the angle would be much lesse, and the 9 would fall out of the line of numbers.

To supplie this defect, I use the Tangent of 5 *gr.* 43. *m.* instead of the tangent of 45 *gr.* And then if I extend the compasses in the line of Numbers from 20 unto 9 the same extent will reach in the line of Tangents from 5 *gr.* 43 *m.* unto 2 *gr.* 35 *m.*

Or in Crosse worke if I extend them from 20 partes in the one line of numbers unto the Tangent of 5 *gr.* 43 *m.* the same extent will give the distance from 9 in the line of Numbers unto the Tangent of 2 *gr.* 35 *m.*

And such is this angle of the altitude required.

But if it be an oblique angle that is contained betweene the the two sides given. the triangle may be reduced into two rectangle triangles and then resolued as before.

As in the triangle ADB, where the side A B is 335, and the side A D 100, and the angle B A D 43 *gr.* 20 *m:* if I let downe the perpendiculor D H upon the side A B, I shall have two rectangle triangles, A H D, D H B; and in the rectangle A H D, the angle at *A* being 43. *gr.* 20 *m.* the other ang'e A D H will be 46. *gr.* 40 *m*; and with these angles and the side *AD*, I may find both *AH* and DH, by the first *Prop.*

Then taking *AH* out of *AB*, there remaines H B for the side of the rectangle D H B; and therefore with this side H B and the other side H D, I may finde both the angle at B, and the third side D B, as in the former part of this *Prop.*

Or I may find the angles required, without letting downe any perpendicular, For,

As the summe of the sides,
is to the difference of the sides:
So the tangent of the halfe summe of the opposite angles,
to the Tangent of halfe the difference betweene those angles.

As in the former triangle *AD* B, the summe of the sides *A* B, *A* D, is 435, and the difference betweene them 235; the angle contained 43 *gr.* 20. *m*; and therefore the summe of the two opposite angles 136 *gr.* 40 *m.* and the halfe summe 68 *Gr.* 20 *m.* Hereupon I extend the compasses in the line of *Numbers* from 455 to 235, and I finde them to reach in the line of *Tangents* from 68 *Gr.* 20 *m.* unto 53 *Gr.* 40 *m*; and such is the halfe difference betweene the opposite angles at B and *D*. This halfe difference being added to the halfe sum, doth give 122 *Gr.* for the greater angle *AD* B: and being subtracted, it leaueth 14 *Gr.* 40 m. for the lesser angle *A* B D. Then the three angles being knowne, the third side B D may be found by the first Prop.

4 *Having*

4 *Having the three sides of a right line triangle, to find the three angles.*

Let one of the three sides given be the base, but rather the greater side, that the perpendicular may fall within the triangle; then gather the summe, and the difference of the two other sides, and the proportion will hold.

As the base of the triangle,
is to the summe of the sides:
So the difference of the sides
to a fourth, which being taken forth of the base, the perpendicular shall fall on middle of the remainder.

As in the former triangle A D B, where the base A B is 335, the summe of the sides A D and D B 371, and the difference of them 171. If I extend the compasses in the line of *Numbers* from 335 unto 371, I shall finde the same extent to reach from 171 unto 189 4. This fourth number I take out of the base 335. 0, and the remainder is 145.6, the halfe whereof is 72. 8, and doth shew the distance from A unto *H*, where the perpendicular shall fall, from the angle D, upon the base *A* B, dividing the former triangle A D B into two right angle triangles, D H A and D H B, in which the angles may be found by the second *Prop.*

And this may suffice for the right line triangles. But for the more easie protraction of these triangles, I will set downe one proposition more concerning *chords.*

5 *Having the semidiameter of a circle, to finde the Chords of every Arke.*

As the sine of the Semiradius of 30 *gr.*
to the sine of halfe the arke proposed :
So is the semidiamiter of the circle given,
to the chord of the same arke.

As if in the protracting the former triangle A D B, it were required to find the length of a chord of 43 *gr.* 20 *m.* agreeing to the semidiameter *A E*, which is known to be 3 inches. The halfe of 43 *gr.* 20 *m.* is 21 *gr.* 40 *m* ; wherefore I extend the compasses from the sine of 30 *gr.* to the sine of 21 *gr.* 40 *m.* and I finde the same extent to reach in the line of *Numbers* from 3. 000 *parts* to 2. 215 ; which shewes, that the semidiameter being 3 inches, the chord of 43 *gr.* 20 *m.* will be 2 *inches* and 215 parts of 100.

In like maner the chord of 58 *gr.* agreeing to the same semidiameter, would be found to be 2 inches and 909 parts. For the halfe of 58 being 29 ; if I extend the compasses in the line of Sines from 30 *gr.* to 29 *gr.* the same extent will reach in the line of *Numbers* from 3. 000. unto 2. 909.

Or in crosse worke, if I extend the compasses from the *Sine* of 30 *gr.* to 3. 000 in the line of *Numbers*, I shall finde the same extent to reach from 21 *gr.* 40 *m.* to 2. 215 *parts*, and from 29 *gr.* to 2 909 *parts*, and from 7 *gr.* 20 *m.* to 765 *parts* ; for the chord of 14 *gr.* 40 *m.* for the third angle A B D.

CHAP.

CHAP. X.

The use of the line of versed Sines.

THis line of *versed Signes* is no necssary line. For all triangles, both right lined and sphericall, may be resolued by the three former lines of Numbers, Sines and Tangents; yet I thought good to put it on the *Staffe* for the more easie finding of an angle having three sides, or a side having three angles of a sphericall triangle given.

Suppose the three sides to be, one of them 100 *gr.* the other 78 *gr.* and the third 38 *gr.* 30 *m.* and let it be required to find the angle, whose base is 110 *gr.*

I first adde them together, and from halfe the summe subtract the base, noting the difference after this maner.

The base	110 *gr.*	0 *m.*
The one side	78	0
The other side	38	30
The summe of all three	226	30
The halfe summe	113	15
The difference	3	15

For so the proportion will holde,

1 As the Radius the Sine of the one side
So the Sine of the other Side to the fourth Sine.

2 As this fourth Sine to the Sine of the halfe Summe
So the Sine of the difference to a seventh Sine.

3 The meane proportionall betweene this seventh sine and the Radivs will shew the sine of the complement of halfe the angle required.

This

This done, I come to the Staffe, and extend the compasses from the sine of 90 *gr.* to the sine of 78 *gr.* which is one of the sides; and applying this extent from the sine of the other side 38 *gr.* 30 *m.* I find it to reach to a fourth sine, about 37 *gr.* 30 *m.* From this fourth sine of 37 *gr.* 30 *m.* I extend the compasses againe, to the sine of the halfe summe 113 *gr.* 15 *m.* (which is all one with the sine of 66 *gr.* 45 *m.*) and this second extent will reach from the sine of the difference 3 *gr.* 15 *m.* to the sine of 4 *gr.* 54 *m.*

Then to finde the meane proportionall sine betweene this seventh sine of 4 *gr.* 54 *m.* and the sine of 90 *gr.* I might divide the space betweene them into two equall parts, and so I should finde the compasses to stay at 17 *gr.* whose complement is 73 *gr.* and the double of 73 *gr.* is 146 *gr.* the angle opposite to 110 *gr.* which was required.

But because this division is somewhat troublesome I have therefore added this line of *versed Sines* that having found the seuenth Sine you might looke over against it and there finde the angle. And so in this example having found the seventh sine to be 4 *gr.* 54. *m.* over against this sine you shall finde 146 *gr.* in the line of *versed Sines* for the angle required as before.

CHAP.

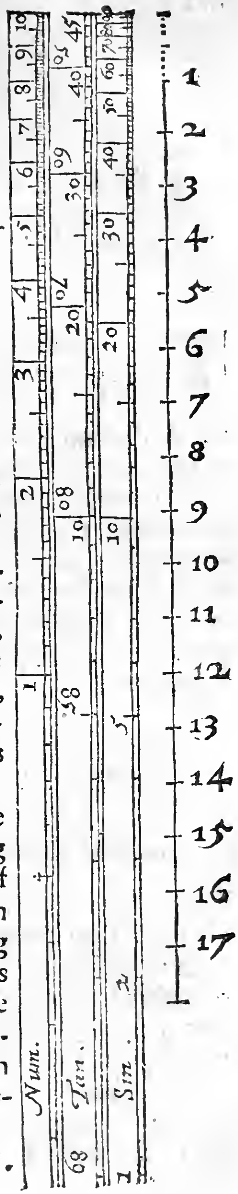
THE SECOND BOOKE OF THE *CROSSE-STAFFE.*

Of the uſe of the former lines of proportion more particularly exemplified in ſeverall kind.

THe former Booke containing the generall uſe of each line of proportion, may bee ſufficient for all thoſe which know the rule of *Three*, and the doctrine of triangles.

But for others, I ſuppoſe it would bemore difficult to finde either the declination of the Sunne, or his amplitude, or the like, by that which hath beene ſaid in the uſe of the line of *Sines*, unleſſe they may haue the particular proportions, by which ſuch propoſitions are to be wrought.

And therefore for their ſakes I have adjoyned this ſecond booke, containing ſeverall proportions for propoſitions of ordinary uſe, and ſet them downe in ſuch order, that the Reader conſidering which is the firſt of the three numbers giuen, may eaſily apply them to the Sector, and alſo reſolue them by Arithmetique, beginning with thoſe which require helpe onely of the line of *Numbers*.



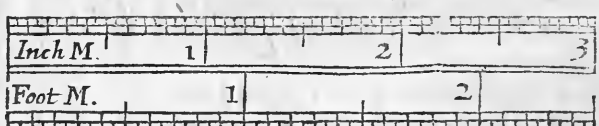
CHAP.

CHAP. I.

The use of the line of Numbers in broade measure, Such as boord, glaße, and the like.

THe ordinary measure for bredth and length are feete and inches, each foote divided into 12 inches, and euery inch into halues and quarters, which being parts of severall denominations, doth breed much trouble both in Arithmeticke and the use of instuments..

For the auoiding whereof, where I may prevaile I give this cousell, that such as are delighted in measure would use seueral lines, first a line of inchmeasure, wherin euery inch may be divided into 10 or 100 parts; secondly a line of foot measure, wherein every foote may be divided into 100 or .1000 parts, both which lines may be set on the same side of a two foote ruler, after this or the like manner.



Then if they be to give the content of any superficies or solid in inches, they may measure the sides of it by the line of inches and parts of inches; but if they be to give the content in feete, it would be more easie for them to measure those sides by the foote line and his parts.

For example, let the length of a plane be 30 inches, and the bredth 21 inches and $\frac{6}{10}$ of an inch; this length multiplied into the bredth, would give the content to bee 648 inches:

inches: but if I were to finde the content of the same plane in feet, I would measure the sides of it by the foote line and his parts; so the length would proue to bee 2 feete $\frac{50}{100}$, and the bredth 1 foote $\frac{80}{100}$, and the length multiplied by the bredth, cutting off the foure last figures, for the foure figures of the parts, would give content to bee 4. 5000, which is 4 foote and 5000 parts of a foote, divided into 10000 parts.

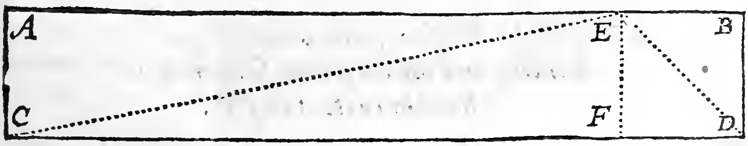
21.6	2. 50
30.0	1. 80
648.00	20000
	250
	4.5000

The like reason holdeth for yards and elnes, and all other measures divided into 10, 100, or 1000 parts.

This being presupposed, the worke will be more easie both by Arithmeticke and the line of *Numbers*, as may appeare by these propositions

1 *Having the bredth and length of an oblong superficies given in inch-measure, to finde the content in inches.*

As 1 inch unto the bredth in inches.
So the length in inches unto the content in inches.



Suppose in the plane A D, the bredth A C to be 30 inches,

 and

and the length *A B* to be 183 inches; extend the compasses from 1 unto 30, the same extent will reach from 183 unto 5490; or extend them from 1 unto 183, the same extent will reach from 30 unto 5490. So both wayes the content required is found to be 5490 inches.

As 1 unto 30: so are 183 unto 5490.

2 Having the breadth and length of any oblong superficies given in inches, to finde the content in feete.

As 144 inches vnto the breadth in inches:
So the length in inches unto the content in feet.

And thus in the former plane A D, working as before, the content will be found to bee 38. 125, which is 38 foote and ⅛ of a foote.

As 144 unto 30: so are 183 unto 38. 125.

3 Having the length and breadth of any oblong superficies given in foote measure, to finde the content in feet.

As 1 foote unto the bredth in foote measure:
So the length in feete unto the content in feet.

And thus in the former plane A D, the bredth will be 2 feete 50 parts, and the lengt 15 foote 25 parts; then working as before, the content will be found to be 38. 125.

As 1 unto 2. 50: so are 15. 25 unto 38. 125.

4 Having the bredth of any oblong superficies given in inches and the length in foote measure, to find the content in feet.

As 12 inches to the bredth in inches:
So the length in feete to the content in feet.

So also in the former plane, the content will be found to be 38. 125.

As the 12 unto 30: so are 15. 25 unto 38. 125.

5 Having the breadth of an oblong superficies given in inches, to finde the length of a foot superficiall in inch measure.

As the breadth in inches, unto 144 inches:

So 1 foote vnto the length in inch measure.

So the bredth being 30 inches, the length of a foote will be found to be 4 inches 80 parts, the length of two feet 9 inches 60 parts.

As 30 vnto 144: so are 1 unto 4. 80.

6 Having the bredth of an oblong superficies given in feet, to find the length of a foote superficall in foot measure.

As the bredth in foote measure to 1 foote:

So the number of feet to the length in foot measure.

So the breadth being 2 foote 50 parts, the length of a foot will be found to be 40 parts, the length of 2 feet 80 parts, and the length of 3 feete 1 foot 20 parts, &c.

As 250 unto 1: so are 1 unto 0. 40.

7 Having the length and breadth of an oblong superficies, to finde the side of a sqare equall to the oblong.

Divide the space betweene the length and the bredth into two equall parts, and the foote of the compasses will stay at the side of the square.

So the length being 183 inches, and the bredth 30 inches, the side of the square wil be found to be almost 74 inches and 10 parts of 100.

Or the bredth being 2 foote and 50 parts, the length 15 foote and 25 parts, the side of the square will be found to be about 6 feet and 17 parts.

As 30 unto 74. 10. so are 74. 10 unto 183. 027.
And as 2. 50 unto 6. 174: so are 6. 174 unto 15. 247.

8 *Having the diameter of a circle, to find the side of a square equall to that circle.*

As 10000 to the diameter:
So 8862 unto the side of the square.

So the diameter of a circle being 15 inches, the side of the square will be found about 13 inches and 29 parts.

As 10000 unto 8862: so are 15 unto 13. 29.

9 *Having the circumference of a circle to finde the side of a square equall to the same circle.*

As 10000 to the circumference:
So 2821 to the side of the square.

So the circumference of a circle being 47 inches 13 parts, the side of the square will be about 13 inches 29 parts.

As 10000 unto 2821: so are 47. 13 unto 13.29.

10 *Having the diameter of a circle, to finde the circumference.*

11 *Having the circumference of a circle, to finde the diameter.*

As 1000 to the diameter:
So 3142 to the circumference.

[illegible]

A Table for the vſe of the Chaine:

Long:

	Inch.	Cent:	Foot:	Pace:	Perch:	Chain	Acre	Mile.
Inch.	1	7. 92	12	60	198	792	7920	63360
* Cent	62. 7264	1	1.515	7.575	25	100	1000	8000
Foot.	144	2. 295	1	5	26.5	66	660	5280
Pace.	3600	57.485	25	1	3.3	13.2	132	1056
Perch.	39204	625	272.25	10.89	1	4	40	320
Chain.	627264	10000	4356	17424	16	1	10	80
Acre.	6272640	100000	43560	1742.4	160	10	1	8
Mile.	4014489600	64000000	27878400	1115136	102400	6400	640	1

Square:

* Centeſm of a Chain

Pag: 45 Croſſe-ſtaffe:

So the diameter being 15 inches, the circumference will be found about 47 inches 13 parts: or the circumference being 47. 13, the diameter will be 15.

CHAP. II.

The use of the line of Numbers in the measure of land by pearch and acres.

1 *Having the bredth and length of an oblong superfices, given in perches, to finde the content in perches.*

As 1 perch to the bredth in perches:
So the length in perches to the content in perches.

So in the former plane A D, if the bredth *A C* be 30 perches, and the length *A B* 183 perches, the content will be found to be 5490 perches.

2 *Having the length and breadth of an oblong superficies given in perches, to finde the content in acres.*

As 160 to the bredth in perches:
So the length in perches to the content in acres.

So in the former plane *A D*, the content will be found to be 34 acres, and 31 centesms or parts of an 100.
As 160 unto 30: so are 183 unto 34. 31.

To augment a superficies in a proportion,
To diminish a superficies in a proportion given.

3 *Having the length and bredth of an oblong superficies given in chaines, to finde the content in acres.*

It being troublesome to divide the content in perches by 160, we may measure the length and breadth by chaines, each chaine being 4 perches in length, and divided into 100 linkes, then will the worke be more easie in Arithmetique. For

As 10 to the bredth in chaines :
So the length in chaines to content in acres.

And thus in the former plane A D, the breadth *A C* will be 7 chaines 50 linkes, and the length *A B* 45 chaines 75 links; then working as before, the content will bee found as before, 34 acres 31 part.

4 *Having the perpendicular and base of a triangle given in perches, to find the content in acres.*

If the perpendicular goe for the bredth, and the base for the length, the triangle will be the halfe of the oblong, as the triangle C E D is the halfe of the oblong A D, whose content was found in the former *Prop.* Or without halfing,

As 320 to the perpendicular :
So the base to the content in acres.

So in the triangle C E D, the perpendicular being 30, and the base 183, the content will be found to be about 17 acres and 15 parts.

5 *Having the perpendicular and base of a triangle given in chaines, to find the content in acres.*

As 20 to the perpendicular :
So the base to the content in acres.

And

And so the triangle C E D, the perpendicular *E F* being 7.50, and the base C D 45·75, the content will be found as before to be about 17 acres 15 parts.

6 Having the content of a superficies after one kind of perch, to finde the content of the same superficies according to another kind of pearch.

As the length of the second perch
to the length of the first perch:
So the content in acres to a fourth number;
and that fourth to the content in acres required.

Suppose the plane A D measured with a chaine of 66 feete, or with a pearch of 16 feete and an halfe, contained 34 acres 31 parts; and it were demanded how many acres it would containe if it were measured with a chaine of 18 foot to the perch: these kind of propositions are wrought by the backward rule of *three*, after a duplicated proportion. Wherefore I extend the compasses from 16. 5 unto 18. 0, and the same extent doth reach backward, first from 34. 31 to 31 45, and then from 31. 45 to 28. 84, which shewes the content to be 28 acres 84 parts.

7 Having the plot of a plaine with the content in acres, to finde the scale by which it was plotted.

Suppose the plane, A D contained 34 acres 31 centesmes; if I should measure it with a scale of 10 in the inch, the length *A B* would be 38 chaines and about 12 centesmes, and the bredth *A C* 6 chaines and 25 centesmes; and the content would be found by the third *Prop.* of this Chapter, to be about, 23 acres 82 parts, wheras it should be 34 acres 31 parts.

Where-

Wherefore I divide the distance betweene 23. 82, and 34. 31, upon the line of *numbers* into two equall parts; then setting one foote of the compasses upon 10, my supposed scale, I find the other to extend to 12, which is the scale required.

8 *Having the length of the furlong to finde the breadth of the acre.*

As the length in perches to 160.
So 1 acre to the bredth in perches.

So the length of the furlong being 40 perches, the bredth of an acre will be found to be 4 perches. If the length be 50 the bredth for one acre must be 3. 20. the bredth for two acres 6, 40.

Or if the length be measured by chaines.

As the length in chaines unto 10
So 1 acre to his bredth in chaine measure.

So the length of the furlong being 12 Chaines 50 Linkes, the bredth for one acre will bee found to be 80 Links, the bredth for two acres 1 Chaine 60 Links.

As 12. 50 unto 10: so 1 unto 0.80.

Or if the length be measured by feet measure.

As the length in feete unto 43560.
So 1 acre to his bredth in foot measure.

So the length of the furlong being 792 feet, the breadth for one acre will be found to be 55 feet, the bredth for two acres 110 feet.

CHAP.

CHAP. III.

The use of the line of Numbers in solid measure, such as stone, timber, and the lke.

1 *Having the side of a sqare equall to the base of any solid given in inch measure to find the length of a foot solid in inch measure.*

THe side of a square equall to the base of a solid, may bee found by dividing the space betweene the length and bredth into two equall parts,as in the 7 *Prop.* of broad measure. Then

As the side of the square in inches to 41. 57:
So is 1 foote to a fourth number;
and that fourth to the length in inches.

So in the solid *A H*, the side of the squaie equall to the base *E C*, being about 25 inches 45 parts, the length of a foot solid will be found about 2 inches 67 parts, and the length of two foot solid 5 inches 33 parts.

As 25.45 vnto 41 57; so 1.00 unto 1. 63:
and so are 1. 63 unto 2. 67.

2 *Having the side of a square equall to the base of any solid given in foote measure, to find the length of a foot solid in foot measure.*

As the side of the square in feet unto 1:
So is 1 unto a fourth number;
And that fourth to the length in foot measure.

So in the solid *AH*, the side of the square equall to the base *E C*, being about 2 foote 120 parts, the length of a foot solid will be found about 222 parts of a foote.

As 2.120 unto 1.000: so 1.000 vnto 0.471:
and so are 471 unto 222.

3 *Having the bredth and depth of a squared solid given in foot measure, to finde the length of a foot solid in foote measure.*

As 1 unto the bredth in foote measure:
So the depth in feet to a fourth number;
which is the content of the base in foot measure. Then

As this fourth number unto 1:
So 1 unto the length in foote measure.

So in the solid *A H*, the bredth being 2 foote 50 parts, the depth 1 foot 80 parts, the content of the base *E C* will bee found 4 foote 50 parts, and the length of one foot solid about 222 parts, the length of two foot solid about 444 parts of 1000.

As 1. 00 unto 2. 50: so are 1. 80 unto 4. 50.
As 4, 50 unto 1, 00: so 1. 000 unto 0. 222.

4 *Ha-*

4 *Having the bredth and depth of a squared solid given in inches, to finde the length of a foot solid in inch measure.*

As 1 hath to the breadth in inches :
So the depth in inches to a fourth number ;
which is the content of the base in inches. Then

As this fourth number unto 1728 :
So 1 unto the length of a foot in inch measure.

So in the solid *A H*, the breadth *A C* being 30 inches, and the depth *A E* 21 inches 60 parts, the content of the base *EC* will be found to be 648 inches, and the length of a foote solid about 2 inches 67 parts, the length of a foot solid 5 inches 33 parts.

As 1 unto 21. 6 : so 30 unto 648 ;
As 648 unto 1728 ; so 1 unto 2 667.

Or as 12 to the bredth in inches ;
So the depth in inches to a fourth number.

As this fourth number to 144 ;
So 1 unto the length of a foote solid in inch measure.

So in the solid *A H*, the breadth being 30 inches, the depth 21 inches 6 parts, the fourth number will be found to be 54, and the depth o foote solid 2 inches 67 parts.

As 12 unto 21. 6 ; so 30 unto 54.
As 54 unto 144 ; so 1 unto 2.667.

5 Having the side of a square equall to the base of any solid, and the length thereof given in inch measure, to find the content thereof in feet.

As 41. 57 to the side of the square in inches:
So the length in inches to a fourth number;
and that fourth to the content in foot measure.

So in the solid *A H*, the length *A B* being 183 inches, and the side of the square equall to the base *E C* about 25 inches 45 parts, the fourth number will be found about 112, and the whole solid content about 68 feet 62 parts.

As 41. 57 unto 25. 45: so 183 unto 112:
and so are 112 unto 68. 62.

6 Having the side of a square equall to the base of any solid, and the length thereof given in foot measure, to find the content thereof in feet.

As 1 to the side of the square in foot measure:
So the length in feet to a fourth number;
and that fourth to the content in foot measure.

So in the former solid *A H*, the side of the square equall to the base *A E*, being about 2 foot 12 parts, and the length *A B* 15 foot 25 parts, the content will be found to bee about 68 foot 62 parts.

As 1 unto 2. 12; so 15. 25 unto 32. 35:
and so are 32. 35 unto 68. 62.

7 Ha-

7 *Having the side of a square equall to the base of any solid given in inch measure, and the length of the solid given in foote measure, to find the content thereof in feet.*

As 12 to the side of the square given in inches:
So the length in feet to a fourth number;
And that fourth to the content in foot measure.

So in the former solid *A H*, the side of the equall square being 25 inches 45 parts, the content will be found to bee about 68 feet 62 parts.

As 12 unto 25.45: so 15.25 unto 32.35:
and so are 32.35 vnto 68.62.

8 *Having the length, bredth and depth of a squared solid given in inches, to find the content in inches.*

As 1 unto the bredth in inches:
So the depth in inches unto the base in inches. Then

As 1 unto the base:
So the length in inches unto the solid content in inches.

So in the solid *A H*, whose bredth *A C* is 30 inches, the depth *A E* 21 inches and 6 parts of 10, and length *A B* 183, the content of the base *E C* will be found 648 inches, and the whole solid content about 118500 inches.

As 1 unto 21.6: so are 30 unto 648:
As 1 unto 648: so are 183 to 118584.

 9 Ha-

9 *Having the length, bredth and depth of a squared solid given in inches, to finde the content in feete.*

As 1 to the bredth in inches:
So the depth in inches to the base in inches.

As 1728 to that base:
So the length in inches to the content in feet.

So in the solid *AH*, the content will be found to be about 68 feete 62 parts.

As 1 unto 21. 6: so 30 unto 648:
As 1728 unto 648: so 183 to 68. 62.

Or as 12 to the bredth in inches:
So the depth in inches to a fourth number.

As 144 to that fourth number:
So the length in inches to the content in feet.

And so also in the same solid AH, the content will bee found to be about 68 feet 62 parts.

As 12 unto 21. 6: so 30 unto 54:
As 144 unto 54: so 183 unto 68. 62.

10 *Having the length, bredth and depth of a squared solid given in foot measure, to finde the content in feete.*

As 1 unto the bredth in foote measure:

So

So the depth in feet to the base in feet.

As 1 unto that base:
So the length in feet to the content in feet.

And thus in the former solid A H, the bredth *A C* will be 2 foot 50 parts, the depth *A E* 1 foot 80 parts, and the length *A B* 15 foot 25 parts; then working as before, the content of the base *A F* will be found 4 feet 50 parts, and the whole solid content about 68 foot 62 parts, whch of all others may uery easily be tried by Arithmetique.

As 1 unto 2.50: so 1.80 unto 4.50.
As 1 unto 4.50; so 15.25. unto 68.625.

11 *Having the bredth and depth of a squared solid given in inches, and the length in foot measure, to find the content thereof in feet.*

As 1 vnto the bredth in inches:
So the depth in inches unto a fourth number:
which is the content of the base in inches.

As 144 hath unto that fourth number:
So the length in feet to the content in feet.

And so in the same solid *A H*, the content will be found to be about 68 feet 62 parts.

As 1 unto 21. 6: so 30 unto 648.
As 144 vnto 15, 25. so 648, unto 68. 62.

Or as 144 unto the bredth in inches:
So the depth in inches unto a fourth number:

which

which is the content of the base in feet.

As 1 hath unto that fourth number:
So the length in feet to the content in feet.

And so in the same solid A H, the content will be found to be about 68 feet 62 parts.

As 144 unto 21. 6 : so 30 unto 4.50.
As 1 unto 4. 50 ; so 15. 25 unto 68. 62.

Or as 12 unto the bredth in inches:
So the depth in inches unto a fourth number.

As 12 unto this fourth number:
So the length in feet to the content in feet.

And so also in the same solid *A H*, the content will bee found to be about 68 feet 62 parts.

As 12 unto 21. 6 ; so 30 unto 54.
As 12 vnto 54 ; so 15. 25 unto 68. 62.

All these varieties (and such like not here mentioned) doe follow upon making of the base of the solid, to be *E C*; there would be as many more if any shall begin with the base E H, and so likewise if they make the base to be F D.

12 *Ha-*

12 Having the diameter of a Cylinder given in inch measure, to find the length of a foot solid in inches.

As the diameter in inches unto 46.90:
So is 1 unto a fourth number:
and that fourth to the length in inches.

So the diameter of a Cylinder being 15 inches, the fourth number will be about 3.12, and the length of a foote solid 9 inches 78 parts.

As 15 unto 46.90: so 1 unto 3.127:
and so are 3.127 unto 9.778.

13 Having the diameter of a Cylinder given in foote measure, to finde the length of a foote solid in foote measure.

As the diameter in feet unto 1.128:
So is 1 unto a fourth number;
and that fourth to the length in foote measure.

So the diameter being 1 foote 25 parts, the length of a foot solid will be found about 8.14 parts of 1000.

As 1.25 unto 1.128: so 1.00 to 0.9027:
and so are 9027 unto 8148.

14 Having the circumference of a Cylinder given in inches, to finde the length of a foot solid in inch measure.

As the circumference in inches to 147.36:
So is 1 to a fourth number;
and that fourth to the length in inches.

So the circumference being 47 inches 13 parts, the length of a foote solid will be found about 9 inches 78 parts.

As 47.13 unto 147.36: so 1.00 to 3.13.
and so are 3.13 unto 9.78.

15 Having the circumference of a Cylinder given in foot measure, to finde the length of a foot solid in foote measure.

As the circumference in feete to 3.545:
So is 1 to a fourth number;
and that fourth to the length in foote measure.

So the circumference being 3 foot 927 parts, the length of a foot solid will be found to be about 815 parts.

As 3.927 unto 3.545: so 1.000 unto 0.90.3:
and so are 903 unto 815.

16 Having the side of a square equall to the base of a Cylinder, to finde the length of a foot solid.

The side of a square equall to the circle, may bee found by the eighth *Prop.* of broad measure, and then this *Prop.* may be wrought by the first and the second *Prop.* of solid measure.

17 *Ha-*

17 *Having the diameter of a Cylinder, and the length given in inches, to finde the content in inches.*

As 1.128 unto the diameter in inches:
So the length in inches to a fourth number;
and that fourth number to the content in inches.

So the diameter being 15 inches, and the length 105, the content of the Cylinder will bee found to bee about 18560 inches.

As 1.1284 unto 15: so are 105 unto 1395.87:
and so are 1395.87 unto 18555.34.

18 *Having the diameter and length of a Cylinder in foote measure, to finde the content in feete.*

As 1.128 to the diameter in feet:
So the length in feet to a fourth number;
and that fourth to the content in feet.

So the diameter being 1 foote 25 parts, and the length 8 foot and 75 parts, the content of the Cylinder will bee found about 10 foote 74 parts.

As 1.128 unto 1.25: so 8.75 unto 9.69:
and so are 9.69 unto 10.737.

19 *Having the diameter of a Cylinder, and the length given in inches, to find the content in feet.*

As 46.90 to the diameter in inches:
So the length in inches to a fourth number;
and that fourth to the content in feet.

So the diameter being 15 inches, and the length 105, the content will be found about 10 foote 74 parts.

As 46. 906 unto 15: so 105 unto 33. 58:
and so are 33. 58 unto 10. 737.

20 *Having the diameter of a Cylinder, given in inches and the length in feete, to find the content in feete.*

As 3. 54 the diameter in inches:
So the length in feete to a fourth number;
and that fourth to the content in feete.

So the diameter being 15 inches, and the length 8 foote 75 parts, the content will be found about 10 foot 74 parts.

As 13. 54 unto 15: so 8. 75 unto 9. 69:
and so are 9. 69 unto 10. 74.

21 *Having the circumference and length of a Cylinder given in inches to find the content in inches.*

As 3. 545 to the circumference in inches:
So the length in inches to a fourth number;
and that fourth to the content in inches.

So the circumference being 47 inches 13 parts, and the length 105 inches, the content will bee found about 18560 inches.

As 3. 545 unto 47. 13: so 105 unto 1396:
and so are 1396 unto 18555.

22 *Hee*

22 Having the circumference and length of a cylinder given in inches, to find the content in feet.

As 147.36 to the circumference in inches:
So the length in inches to a fourth number;
and that fourth to the content in feet.

So the circumference being 47 inches 13 parts, and the length 105 inches, the content will bee found about 10 foote 74 parts.

As 147. 36 unto 47.13: so 105 unto 33. 58:
and so are 33. 58 unto 10. 74.

23 Having the circumference and length of a Cylinder given in foote measure, to find the content in feete.

As 3. 545 to the circumference in feet:
So the length in feet to a fourth number;
and that fourth to the content in feet.

So the circumference being 3 foote 927 parts, and the length 8 foot 75 parts, the content will be found to be 10 foot 74 parts.

As 3. 545 unto 3. 927: so 8. 75 unto 9. 69.
and so are 9.69 unto 10. 74.

24 Having the circumference of a Cylinder given in inches and the length in foot measure, to find the content in feete.

As 42. 54. to the circumference in inches:
So the length in feet to a fourth number;
and that fourth to the content in feet.

So the circumference being 47 inches 13 parts, and the length 8 foote 75 parts, the content will bee found as before, 10 foot 74 parts.

As 42. 54 unto 47. 13: so 8. 75 unto *9.69*:
and so are *9.69* unto 10. 74.

CHAP. IIII.

The use of the line of Numbers in gauge-ing of vessell.

THe vessels which are here measured, are supposed to be Cylinders, or reduced unto cylinders, by taking the mean betweene the diameter at the head and the diameter at the bongue, after the vsuall maner.

1 *Hauing the diameter and the length of a vessell with the content thereof, to finde the gavge point.*

Extend the compasses in the line of Numbers to halfe the distance betweene the content and the length of the vessell, the same extent will reach from the diameter to the gauge point.

I put this proposition first, because these kind of measures are not alike in all places.

Here

Here at London it is said that a wine vessell being *66* inches in length, and 38 inches the diameter, would containe 324 gallons. which if it be true, we may divide the space betweene 324 and *66* into two equall parts, and the middle will fall about 146, and the same extent which reacheth from 324 to 146, will reach from the diameter 38 unto 17. 15 the gauge point for a gallon of wine or oyle after London measure.

The like reason holdeth for the like measure in all other places.

2 *Having the meane diameter and the length of a vessell, to finde the content.*

Extend the compasses from the gauge point to the meane diameter, the same extent being being doubled, shall give the distance from the length to the content.

So the meane diameter of a wine vessell being 20 inches, and the length 25 inches, the content will be found to be 34 gallons after London measure.

For extend the compasses from 17. 15, unto 20, the same extent will reach from 25 unto *29*. 15, and from *29*. 15 unto 34.

In like maner if the meane diameter were 16 inches, and the length 23, the content would bee found to bee about 20 gallons.

For the same extent which reacheth backe from 17. 15 unto 16, will reach from 23 to 21. 45, and from 21. 45 unto 20.

So that if the meane diameter shall be 17 inches and 15 centesmes or parts of 100, the number of inches in the length of the vessell, will give the number of inches in the length of the vessell, will give the number of gallons contained in the same vessell: if the diameter shall be more or lesse then 17. 15, the content in gallons will bee accordingly more or lesse then the length in inches.

3 *Having the diameter and content, to find the length.*

Extend the compasses from the diameter to the gauge point, the same extent being doubled shall give the distance from the content to the lenghth of the vessell.

So the gauge point standing as before, if the diameter bee 38 inches, and the content 324 gallons wine measure, the length of the vessels will bee found about 66 inches.

4 *Having the length of a vessell and the content, to finde the diameter.*

Extend the compasses to halfe the distance betweene the length and the content, the same extent shall reach from the gauge point to the diameter.

So the length being 66 inches, and the content 324 gallons wine measure, the gauge point standing as before, the diameter of the vessell well bee found to be about 38 inches.

CHAP.

CHAP. V.

Containing such Astronomicall propositions as are of ordinary use in the practise of Navigation.

1 *To finde the altitude of the Sunne by the shadowes of a gnomon set perpendicular to to the horizon.*

As the parts of the shadow
are to the parts of the gnomon:
So the tangent of 45 *gr.*
to the tangent of the altitude.

Extend the compasses in the line of Numbers, from the parts of the shadow to the parts of the gnomon; the same extent will give the distance from the Tangent of 45 *gr.* to the Tangent of the Sunnes altitude.

So the gnomon being 36, and the shadow 27, the altitude will be found to be 36 *gr.* 52 *m.* Or the gnomon being 27, and the shadow 36, the altitude will bee found to bee 53 *gr.* 8 *m.* Or the shadow being 20, and the gnomon 9, the altitude will be found to be 24 *gr.* 14 *m.* as in the eighth *Prop.* of the use of the Tangent line. Pag. 12.

If the gnomon be 22 and the shadow 135 the altitude is 9 *gr.* 15 *m.* as I shewed before Pag. 24.

2 *Having the distance of the Sunne, from the next equinoctiall point, to find his declination.*

As the Radius is in proportion

to the ſine of the Sunnes greateſt declination:
So the ſine of the Sunnes diſtance from the next equinoctiall point,
to the ſine of the declination required.

Extend the compaſſes in the line of *ſines*, from 90 *gr.* to 23 *gr.* 30 *m.* the ſame extent will give the diſtance from the Sunnes place unto his declination.

So the *S*unne being either in 29 *gr.* of ♉, or 1 *gr.* of ♒, or 1 *gr.* of ♌, or 29 *gr.* of ♏, that is 59 *gr.* diſtant from the next equinoctiall point, the declination will be found about 20 *gr.*

If the Sunne be ſo neare the equinoctiall point, that his declination fall to be under 1 *gr.* it may be found by the line of *numbers*. As if the Sunne were in 2 *gr.* 5 *m.* of ♈, that is 125 *m.* from the equinoctiall point, the former extent of the compaſſes from the ſine of 90 *gr.* to the ſine of 23 *gr.* 30 *m.* will reach in the line of *numbers* from 125 unto 50, which ſhewes the declination to be about 50 *m.*

3. *Having the latitude of the place, and the declination of the Sun, to find the time of the Suns riſing and ſetting.*

As the cotangent of the latitude
to the tangent of the Suns declination:
So is the Radius
to the ſine of the aſcentionall difference betweene the houre of 6 and the time of the Suns riſing or ſetting.

Extend the compaſſes from the tangent of the complement of the latitude, to the tangent of the declination: the ſame extent will reach from the ſine of 90 *degr.* to the ſine of the aſcentionall difference.

Or extend the compaſſes from the cotangent of the latitude to the ſine of 90 *gr.* the ſame extent will reach from the

the tangent of the declination, to the sine of the ascentionall difference.

So the latitude being 51 *gr.* 30 *m.* Northward, and the declination. 20 *gr.* the difference of ascention will be found to be 27 *gr.* 14 *m.* which resolved into houres and minutes, doth give 1 houre and almost 49 *m.* for the difference betweene the Sunnes rising or setting, and the houre of 6, according to the time of the yeare.

4 *Having the latitude of the place, and the distance of the Sun from the next equinoctiall point, to find his amplitude.*

As the cosine of the latiude
to the sine of the Sunnes greatest declination:
So the sine of the place of the Sun,
to the sine of the amplitude.

So the latitude being 51 *degree* 30 *minutes*, and the place of the Sunne in 1 *degree* of ♒, that is 59 *degrees* distant from the next equinoctiall point, the amplitude will bee found about 33 *degrees* 20 *m.* For extend the compasses in the line of *sines*, from 38 *degrees* 30 *m.* the sine of the complement of the latitude, unto 23 *degrees* 30 *m.* the sine of the Sunnes greatest declination; the same extent will reach from 59 *degrees* unto 33 *degr.* 20 *m.* Or extend them from 38 *degrees* 30 *min.* unto 59 *degrees*, the same extent will reach from 23 *gr.* 30 *m.* unto 33 *gr.* 20 *m.* as before.

5 *Having the latitude of the place, and the declination of the Sun, to find his amplitude.*

As the cosine of the latitude
is to the Radius:
So the sine of the declination,
to the sine of the amplitude.

Extend the compasses from the cosine of the latitude to the sine of 90 *gr* the same extent will reach from the sine of the Sunnes declination to the sine of the amplitude.

Or extend them from the tangent of the latitude to the sine of the declination, the same extent will reach from the sine of 90 *gr.* to the sine of the amplitude.

So the latitude being 51 *gr.* 30 *m.* and the declination 20 *gr.* the amplitude will be found to bee 33 *gr.* 20 *m.*

6 *Having the latitude of the place, and the declination of the Sun, to finde the time when the Sun commeth to be due East or West.*

As the tangent of the latitude,
is to the tangent of the decination:
So the Radius
to the cosine of the houre from the meridian.

Extend the compasses from the tangent of the latitude to the tangent of the declination; the same extent will reach from the sine of 90 *gr.* to the sine of the complement of the houre.

Or extend them from the tangent of the latitude to the sine of 90 *gr*; the same extent will reach from the tangent of the declination to the sine of the complement of the houre.

So the latitude being 51 *gr.* 30 *m.* and the declination 20 *gr.* the Sunne will bee 73 *gr.* 10 *m*: that is 4 houres. and 53 *m.* from the meridian, when he cometh to be in the East or West.

7 *Having the latitude of the place, and the declination of the Sunne, to find what altitude the Sun shall have, when he commeth to be due East or West.*

As

As the ſine of the latitude
is to the ſine of the declination:
So the Radius
to the ſine of the altitude.

Extend the compaſſes in the line of *Sines* from the latitude to the ſine of the declination, the ſame extent will reach from the ſine of 90 *gr.* to the ſine of the altitude.

Or extend them from the ſine of the latitude to the ſine of 90 *gr*; the ſame extent will reach from the ſine of the declination to the ſine of the altitude.

So the latitude being 51 *gr.* 30 *m.* and the declination 20 *gr.* the altitude will be found about 25 *gr.* 55 *m.*

8 *Having the latiude of the place, and the declination of the Sunne, to find what altitude the Sunn ſhall have at the houre of ſix.*

As the Radius is in proportion
to the ſine of the Suns declination:
So the ſine of the latitude.
to the ſine of the altitude.

Extend the compaſſes in the line of *Sines*, from 90 *gr.* to the declination; the ſame extent will reach from the latitude to the altiude.

Or extend them from 90 *gr.* to the latitude, the ſame extent will hold from the declination to the altitude.

So the latitude being 51 *gr.* 30 *m.* and the declination of the Sunne 20 *gr.* the altitude of the Sunne will be found to be about 15 *gr.* 30.

9 *Having the latitude of the place, and the declination of the Sun, to find what Azimuth the Sun shall haue at the houre of six.*

As the cosine of the latitude
is to the Radius:
So the cotangent of the Suns declination,
to the tangent of the Azimuth from the North part
of the meridian.

So the latitude being 51 *gr.* 30 *m.* and the declination 20 *gr.* the Azimuth will be found to be 77 *gr.* 14 *m.* For extend the compasses in the line of *sines,* from 38 *gr.* 30 *m.* to 90 *gr.* the same extent will reach from the tangent of 70 *gr.* to the tangent of 77 *gr.* 14 *m.*

10 *Having the latitude of the place, and the declination of the Sun, and the altitude of the Sun, to find the Azimuth.*

First consider the declination of the Sunn, whether it be toward the North or the South; so have you his distance from your pole: then adde this distance, the complement of his altitude, and the complement of your latitude, all three together, and from halfe the summe subtract the distance from the pole, and note the difference.

1 As the Radius is in proportion
to the cosine of the altitude:
So the cosine of the latitude,
to a fourth sine.
2 As this fourth sine
is to the sine of the halfe summe:

So

So the sine of the difference,
to a seventh sine.

Then find a meane proportionall betweene this seventh sine and the Radius, this meane shall be the sine of the complement of halfe the Azimuth from the North part of the meridian.

Suppose the declination of the Sun being knowne by the time of the yeare to be 20 *degrees* Southward, the altitude aboue the horizon found by obseruation 12 *degrees*, and the latitude Northwards 51 *degrees* 30 *m.* it were required to find the Azimuth.

The declination is Southward, and therefore the distance from the pole 110 *degrees*; then turning the altitude and latitude unto their complements, I adde them all three together, and from halfe the summe subtract the distance from the pole, noting the difference after this maner.

Declin. South	20 *gr.*	0 *m.*	The distance	110 *gr.*	0 *m.*
Altitude	12	0	The complement	78	0.
Latitude N.	51	30	The complement	38	30.
			The summe of all three	226	30
			The halfe summe	113	15
			The difference	3	15

This done, I come to the *Staffe*, and extend the compasses from the sine of 90 *gr.* to the sine of 78 *gr.* and find he same extent to reach from the sine of 38 *gr.* 30 *m.* unto 37 *gr*; 30 *m*, Or if I extend them from 90 *gr.* to 38 *gr.* 30 *m.* the same extent doth reach from 78 *gr.* unto 37 *gr.* 30 *m.* which is the fourth sine required.

Then I extend the compasses againe, from this fourth sine of 37 *gr.* 30 *m.* unto the sine of the halfe summe 113 *gr.* 15 *m.*

that

that is to the sine of 66 *gr.* 45 *m.* (for after 90 *gr.* the sine of 80 *gr.* doth stand for a sine of 100 *gr.* and the sine of 70 *gr.* for a sine of 110 *gr.*) and so the rest for those which are their complements to 180 *gr.*) and this second extent doth reach from the sine of the difference 3 *gr.* 15. *m.* to the sine of 4 *gr.* 54 *m.* Or if I extend them from the fourth sine of 37 *gr.* 30 m. to the sine of the difference 3 *gr.* 15 *m.* the same extent will reach from the sine of the halfe summe 113 *gr.* 15 *m.* unto 4 *gr.* 54 *m.* which is the seventh sine required.

Lastly, I divide the space betweene this seventh sine of 4 *gr.* 54 *m.* and the sine of 90 *gr.* into two equall parts, and I finde the meane proportionall sine to fall on 17 *gr.* whose complement is 73 *gr*; the double of 73 *gr.* is 146 *gr.* and such is the Azimuth required.

Or having found the seventh sine to be 4 *gr.* 54 *m.* I might looke over against it, in the line of *versed sines*, and there I should finde 146 *gr.* for the azimuth from the North part of the meridian; and the complement of 146 *gr.* to a semicircle being 34 *gr.* will give the azimuth from the South part of the meridian.

But if it were required to find the azimuth in the same latitude of 51 *gr.* 30. Northward, with the same altitude of 12 *gr.* and like declination of 20 *gr.* to the Northward, it would be found to be onely 72 *gr.* 52 *m.* though the maner of worke be the same as before.

Declin. North	20 *gr.*	0 *m.*	The distance is	70 *gr.*	0 *m.*
Altitude	12	0	The complement	78	0
Latitud. North	51	30	The complement	38	30
			The summe of all three	186	30
			The halfe summe	93	15
			The difference	23	15

Here as the Radius is to the sine of 78 *gr*: so the sine of 38 *gr.* 30 *m.*:

gr. 30 *m.* to the sine of 37 *gr.* 30 *m.* which is the fourth sine, and the same as before.

Then as this fourth sine of 37 *gr.* 30 *m.* is to the sine of 93 *gr.* 15 *m.* so the sine of 23 *gr.* 15 *m.* to the sine of 40 *gr.* 20 *m.* which is the seventh sine.

The halfe way betweene this seventh sine and the sine of 90 *gr.* doth fall at 53 *gr.* 34 *m.* whose complement is 36 *gr.* 26 *m.* and the double of that is 72 *gr.* 52 *m.* the Azimuth required.

Or I may find this same Azimuth in the line of *versed sines*, over against the seventh sine of 40 *gr.* 20 *m.*

11 *Having the latitude of the place, the declination of the Sun, and the altitude of the Sun, to find the houre of the day.*

Adde the complement of the Sunnes altitude, and the distance of the Sunne from the pole, and the complement of your latitude, all three together, and from halfe the summe subtract the complement of the altitude, and note the difference.

1 As the Radius is in proportion
 to the sine of the Suns distance from the pole
 So the sine of the complement of the latitude,
 to a fourth sine.

2 As this fourth sine
 is to the sine of the halfe summe:
 So the sine of the difference
 to a seventh sine.

The meane proportionall betweene this seventh sine and the sine of 90 *gr.* will be the sine of the complement of halfe the houre from the meridian.

Thus in our latitude of 51 *gr.* 30 *m.* the declination of the Sunne being 20 *gr.* Northward, and the altitude 12 *gr.* I might find the Sunne to be 95 *gr.* 52 *m.* from the meridian.

Altitude 12 *gr.* 0 *m.* The complement is 78 *gr.* 0 *m.*

Declin. North	20	0	the dist. from the pole	70	0
Latitude	51	30	the complement is	38	30
			The summe of all three	186	30
			The halfe summe	93	15
			The difference	25	15

Here as the Radius, is to the sine of 70 *gr.*
So the sine of 38 *gr.* 30 *m.* to the sine of 35 *gr.* 48 *m.*
As this sine of 35 *gr.* 48. *m*, is to the sine of 93 *gr.* 15 *m.*
So the sine of 15 *gr.* 15 *m*, to the sine of 26 gr. 40 *m.*

The halfe way between this seventh sine of 26 gr. 40 *m*, and the sine of 90 gr. doth fall at 42 gr. 4 m, whose complement is 47 gr. 56 *m.* and the double of that, 95 gr. 52 m. which conuerted into houres, doth give 6 houres and almost 24 *m.* from the meridian.

Or I might find these 95 gr. 52 *m.* in the line of *versed sines*, ouer against the seuenth sine of 26 gr. 40 m.

12 *Hauing the azimuth, the Suns altitude, and the declination, to find the houre of the day.*

As the cosine of the declination
is to the sine of the azimuth:
So the cosine of the altitude
to the sine of the houre.

Thus the declination being 20 gr. Southward, the altitude 12 gr. and the azimuth found by the tenth *Prop.* 146 gr. I might finde the time to be 35 gr. 36 *m.* that is 2 houres 22 *m.* from the meridian.

13 *Having the houre of the day, the Sunnes altitude, and the declination, to find the azimuth.*

As the cosine of the altitude
is to the sine of the houre:

So

So the cosine of the declination,
to the sine of the azimuth.

So the altitude of the Sun being 12 gr. and the declination 20 gr. Southward, and the angle of the houre 35 gr. 36 m. I should find the azimuth to be 34 gr. And so it is if it be recconed from the South; but 146 gr. if it be taken from the North part of the meridian.

14 *Having the distance of the Sun from the next equinoctiall point, to find his right ascension.*

As the Radius
to the cosine of the greatest declination:
So the tangent of the distance,
to the tangent of the right ascension.

So the Sun being in the first degree of ♒, that is 59 gr. distant from the next equinoctiall point, and the greatest declination 23 gr. 30 m. the right ascension will be found to be 56 gr. 46 m. short of the beginning of ♈, and therefore 303 gr. 14 m.

15 *Having the declination of the Sun, to find his right ascention.*

As the tangent of the greatest declination
is to the tangent of the declination giuen:
So the Radius
to the sine of the right ascension.

So the greatest declination being 23 *gr.* 30 *m.* and the declination of the Sun giuen 20 *gr.* the right ascension will be found about 56 *gr.* 50 *m.*

16 *Having the longitude and latitude of a starre*
To finde the right ascension of that starre
17 *To finde the declination of that Starre.*

 The

The ſtarres have little or none alteration in their latitude, in therir longitude they moue forward, about 1 *gr.* 25 *m.* in an hundred yeares. These being knowne;

As the Radius
to the ſine of the ſtarres longitude from the next equinoctiall point:
*S*o the cotangent of the ſtarres latitude
to the tangent of a fourth arke.

Compare this fourth arke, with the arke of diſtance betweene the poles of the world and of the ecliptique. If the longitude and latitude of the ſtarre be both a like, as when the longitdde falleth to bee amonge the Northerne ſines ♈ ♉ ♊ ♋ ♌ ♍, and the latitude is North from the ecliptique: or the longitude among the *S*outherne ſignes ♎ ♏ ♐ ♑ ♒ ♓, and the latitude Southward, then ſhall the difference betweene this fourth arke and the diſtance of poles, be your fifth arke.

But if the longitude and latitude ſhall be unlike, as the longitude in a Northerne ſigne, and the latitude *S*outh, or the longitude in a Southerne ſine, and the latitude North, then adde this fourth arke to the diſtance of both poles, the ſume of both ſhall be your fith arke. And

As the ſine of the fourth arke:
to the ſine of the fifth arke,
*S*o the tangent of the ſtarres longitude
to the tangent of the ſtarres right aſcention,
from the next equinoctiall point.

As the coſine of the fourth arke
to the coſine of the fifth arke,
*S*o the ſine of the ſtarres latitude,
to the ſine of the ſtarres declination.

Then for proofe of the worke, if there bee no former errour, the proportion will hold.

As

As the Cosine of the latitude
to the Cosine of the right ascention:
So the Cosine of the declination
to the Cosine of the longitude.

For example, take the vpper of the two former starres in the square of the little Beare, which sea-men call the *Former Guard.* This in the yeare 1625, will be in 7 *degr.* 38 *m.* of ♌. and so his longitude from the beginning of ♎ 52 *degr.* 22 *m.* But his latitude is still the same 72 *gr.* 51 *m.* Northwards. Wherefore

As the sine of 90 *gr.* is to the sine of 52 *gr.* 22 *m.*
So the cotangent of 72 *gr.* 51 *m.*
to the tangent of 13 *gr.* 44 *m.*

Which is the fourth arke. Then because the longitude and latitude are both Northward, the difference betweene this fourth arke and 23 *gr.* 31 *m.* the distance of both poles will give you 9 *gr.* 47 *m.* for the fifth arke. And

As the sine of 13 *gr.* 44 *m.*
to the sine of 9 *gr.* 47 *m.*
So the tangent of 52 *gr.* 22 *m.*
to the tangent of 42 *gr.* 53 *m.*

Which is the right ascention of this starre, from the beginning of ♎ but 222 *gr.* 53 *m.* from the beginning of ♈.

As the cosine of 13 *gr.* 44 *m.*
to the cosine of 9 *gr.* 47 *m.*
So the sine of 72 *gr.* 51 *m.*
to the sine of 75 *gr.* 46 *m.*

Which is the declination of this starre from the æquator.

As the cosine of 72 *gr.* 51 *m.*

to the cosine of 42 *gr.* 53 *m.*
So the cosine of 75 *gr.* 46 *m.*
to the cosine of 52 *gr.* 23 *m.*

Which agreeing so well with the longitude of the starre proposed is a good proofe, that the right ascension and declination were truly found.

These are such Astronomicall propositions as I take to be vsefull for Sea-men. For the first and second will help them to find their latitude; the third to find the Suns rising and setting; the 4.5.6.7.8.9.10.13. *Prop.* to finde the variation of their compasse; the 11 and 12 *Prop.* to find the houre of the day; and the rest toward the finding of the houre of the night. For hauing the latitude of the place, with the declination and altitude of any starre, they may find the houre of the starre from the meridian, as in the 11 *Prop.* Then comparing the right ascension of the starre with the righ ascension of the Sunne, they may haue the houre of the night.

All these propositions and such others may be wrought also by the tables of *sines* and *tangents*. For where foure numbers do hold in proportion; as the first to the second, so the third to the fourth; there if we multiply the second into the third, and diuide the product by the first the quotient will giue the fourth required. As in the example of the 15 *Prop.* where the declination being giuen, it was required to find the right ascension. The tangent of 20 *gr.* the declination giuen is 3639702, whith being multiplied by the Radius, the product is 36397020000000, and this diuided by 4348124 the tangent of 23 *gr.* 30 *m.* the quotient is 8370741 the sine of 56 *gr.* 50 *m.* for the right ascension required.

Or if any will vse my tables of *artificiall sines* and *tangents*, they may adde the second and the third together, and from the summe subtract the first, the remainder will giue the fourth required. And so my tangent of 20 *gr.* is 9561.0658, which being added to the Radius, makes 19561.0658; from this if they subtract 9638.3019 the tangent of 23 *gr.* 30 *m.* they

they ſhall find the remainder to be 9922.7639, which in my *Canon* is the ſine of 56 *gr.* 49 *m.* 56 *ſeconds*; & ſuch is the right aſcenſion required, if it be reckoned from the next equinoctiall point.

The like reaſon holdeth for all other Aſtronomicall propoſitions, as I will farther ſhew by thoſe two examples which I gaue before for the finding of the azimuth in the 10 *Prop.* becauſe they are thought to be harder then the reſt, and require three operations.

In the firſt *example.*

Declin. *S*outh	20 *gr.*	0 *m.*	The diſtance	110 *gr.*	0 *m.*
Altitude	12	0	the complement	78	0
Latitude Nor.	51	30	the complement	38	30
			The ſumme of all three	226	30
			The halfe ſumme	113	15
			The difference	3	15

The firſt operation will be to finde the fourth ſine; and that is done by adding the ſine of the complement of the altitude to the ſine of the complement of the latitude, and ſubtracting the Radius: ſo adding 9990.4044 the ſine of 78 *gr.* vnto 9794.1495 the ſine of 38 *gr.* 30 *m.* the ſumme will be 19784.5539. And the Radius being ſubtracted, the remainder 9784.5539 is the fourth ſine, and belongeth to 37 *gr.* 30 *m.*

The ſecond operation will be to find the ſeuenth ſine; and that is done by adding the ſine of the halfe ſumme to the ſine of the difference, and ſubtracting the fourth ſine. So the halfe ſumme being 113 *gr.* 15 *m.* I take his complement to a ſemicircle, and ſo find his ſine to be 9963.2168, to which I adde 8753.5278, the ſine of the difference 3 *gr.* 15 *m.*; and the ſumme is 18716 7446. From this I take the fourth ſine 9784.5539, and the remainder will be 8932.1907, which is the ſeuenth ſine, and belongeth to 4 *gr.* 54 *m.*

The third operation will be to finde the meane proportionall ſine betweene the ſeuenth ſine and the Radius. This in

common

common Arithmetique is done by multiplying the to extremes, and taking the square roote of the product. As in finding a meane proportionall betweene 4 and 9, we multiply 4 into 9, and the product is 36, whose square root is 6, the meane proportionall betweene 4 and 9. But here it is done by adding the sine and the Radius, and taking the halfe of them. So the summe of the last seventh sine and the Radius is 18932. 1907 and the halfe of that 9466.0953, which is the meane proportionall sine required, and belongeth to 17 *gr.* whose complement is 73 *gr.* and the double of that 146 *gr.* the same Azimuth as before.

In the second example.

Declin. North	20 *gr.*	0 *m.*	The distance	70 *gr.*	0 *m.*
Altitude	12	0	the complement	78	0
Latitud. North	51	30	the complement	38	30
			The summe of all three	186	30
			The halfe summe	93	15
			The difference	23	15

The first operation will be to find the fourth sine; and that is here 9784.5539, as in the former example.

The second operation will be to find the seventh sine; and so here the sine of the halfe summe 93 *gr.* 15 *m.* being the same with the sine of 86 *gr.* 45 *m.* his complement to 180 *gr.* I find it to be 9999.3009, to whioh I adde 9596.3153 the the sine of the difference 23 *gr.* 15 *m.* and the summe is 19595.6162. From this I take the fourth sine 9784.5539, and the remainder will be 9811.0623 for the seventh sine, and belongeth to 40 *gr*, 20 *m*.

The third operation will be to find the meane proportionall sine betweene the seventh sine and the Radius. And so here the Radius being added to the seventh sine, the summe will be 19811.0623, and the halfe of that 9905. 5311, doth give the meane proportionall sine belonging to about

53

gr. 34 *m.* whoſe complement is 36 *gr.* 26 *m.* & the double of that 72 *gr* 52 *m.* the ſame Azimuth as before.

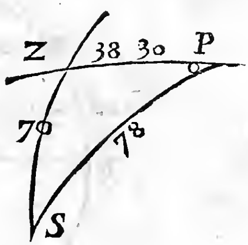
I haue ſet downe theſe three examples thus particularly, that I might ſhew the agreement between the *Staffe* and the *Canon.* But otherwiſe I might deliuer both the precept and the worke, for the two laſt, more compendiouſly. For generally in all ſphericall triangles, where three ſides are knowne, and an angle required, make that ſide which is oppoſite to the angle required, to be the baſe; and gather the ſumme, the halfe ſumme, and the difference as before.

As the rectangle contained vnder the ſines of the ſides,
is to the ſquare of the whole ſine:
So the rectangle contained vnder the ſines of the halfe ſumme and the difference,
to the ſquare of the coſine of halfe the angle.

Then for the worke, we may for the moſt part leaue out the two laſt figures; and if they be aboue 50, put an vnitie to the ſixt place, after this maner.

The ſecond example.

70 *gr.*	0 *m*		
78	0	9990	40
38	30	9794	15
186	30	19784	55
93	15	9999	30
23	15	9596	32
		20000	00
		39595	62
		19811	07
36	26	9905	53
72	52		



53 *gr.* 34 *m.*
107 8

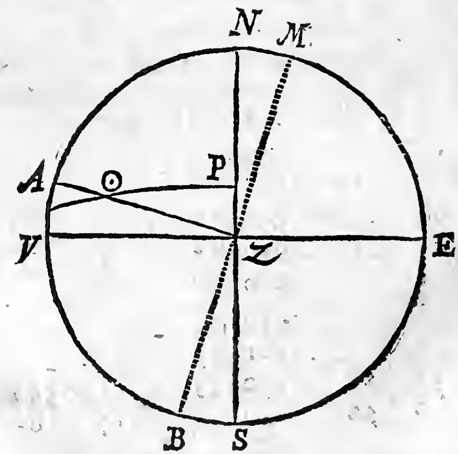
Or for ſuch numbers as are to be ſubtracted, I may take

them

them out of the Radius, and write downe the residue, and then adde them together with the rest. As in the same second example, the sines of 78 *gr.* and of 38 *gr.* 30 *m.* being the numbers to be subtracted; if I take 9990. 4044 the sine of 78 *gr.* out of the Radius 10000.0000, the residue is 9. 5956: and so the residue of 9794. 1495 is 205.8505. Wherefore in head of subtracting those sines, I may adde these residues after this maner:

70 *gr.*	0 *m.*				
78	0	9	59		
38	30	205	85		
186	30				
93	15	9999	30		
23	15	9596	32		
		19811	06		
36	26	9905	53	53 *gr.*	34 *m.*
72	52			107	8

Hauing these meanes to find the Sunnes azimuth, we may compare it with the magneticall azimuth, and so finde the variation of the needle.



For let the circle *AMB*, drawne by the center *Z*, be a plane

plane, parallell to the horizon; *A* the point whereon the Sun beareth from vs, *M* the North point of the magneticall needle, and the angle *AZM* the magneticall Azimuth. If we find the *Sunnes* Azimuth as before, to be 72 *gr.* 52 *m.* from the North to the Westward, we may allow so many degrees from *A* vnto *N*, and so we haue the true North point of the meridian, and consequently the East, South, & West points of the horizon; and the distance betweene *N* and *M* shall be the variation of the needle. So that if the magneticall Azimuth *AZM* shall be 84 *gr.* 7, *m.* and the Suns azimuth *AZN* 72 *gr.* 52 *m.* then must *NZM* the difference betweene the two meridians, giue the variation to be 11 *gr.* 15 *m.* as Mr. *Bourough* heretofore found it by his obseruations at *Limhouse* in the yeare 1580. But if the magneticall Azimuth *ZM* shall be 79 *gr.* 7 *m.* and the Suns Azimuth *AZN* 72 *gr.* 52 *m.* then shall the variation *NZM* be only 6 *gr.* 15 *m.* as I haue sometimes found it of late. Herevpon I enquired after the place where Mr. *Bourough* obserued, and went to *Limehouse* with some of my friends, and tooke with vs a quadrant of 3 foote semidiameter, and two needles, the one aboue 6 inches, and the other 10 inches long, where I made the semidiameter of my horizontall plane *AZ* 12 inches: and toward night the 13 of Iune 1622, I made obseruation in seuerall parts of the ground, and found as followeth.

Alt. ☉		*AZM*		*AZN*		*Variat*	
Gr.	*M.*	*Gr.*	*M.*	*Gr.*	*M.*	*Gr.*	*M.*
19	0	82	2	75	52	6	10
18	5	80	50	74	44	6	6
17	34	80	0	74	6	5	54
17	0	79	15	73	20	5	55
16	18	78	12	72	32	5	40
16	0	77	50	72	10	5	40
10	10	71	2	64	49	6	13
9	52	70	12	64	25	5	47

CHAP. VI.

Containing such nauticall questions, as are of ordinary vse, concerning longitude, latitude, Rumb, and distance.

1 *To keepe an account of the ships way*

THe way that the ship maketh, may be knowne to an old sea-man by experience, by others it may be found for some small portion of time, either by the logge line, or by the distance of two knowne markes on the ships side. The time in which it maketh this way may be measured by a watch, or by a glasse, or by the pulse or by repeating a certaine number of words. Then as long as the wind continueth at the same stay it followeth by proportion,

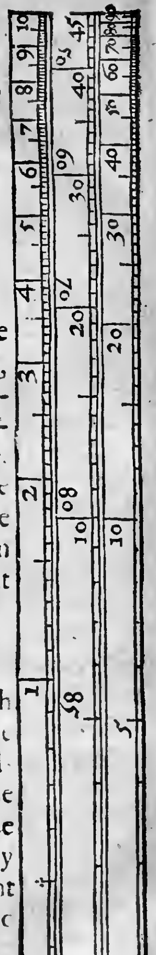
As the time giuen is to an houre:
So the way made, to an houres way.

Suppose the time to be 15 seconds, which make a quarter of a minute, and the way of the ship 88 feet: then because there are 3600 seconds in an houre, I may extend the compasses in the line of *numbers*, from 15 unto 3600, and the same extent will reach from 88 unto 21120. Or I may extend them from 15 unto 88, and this extent will reach from 3600 unto 21120; according to the ordinary worke in Arithmetique,

As 15 vnto 3600
So 88 vnto 21120

which shewes that an houres way came to 21120 feete.

But this were an vnnecessary businesse, to hearken after feet or fadoms. It sufficeth our sea-men to find the way of their ship in leagues or miles.



And

And they say that there are 5 feet in a pace, 1000 paces in a mile, and 60 miles in a degree, and therefore 300000 feete in a degree. Yet comparing severall obseruations, and their measures with our feete vsuall about *London*, I finde that we may allow 352000 feete to a degree; and then if I extend the compasses in the line of *numbers* from 352000 vnto 21120, I shall find the same extent to reach from 20 leagues the measure of one degree, to 1.2, and from 60 miles to 3.6; according to Arethmetique which shewes the houres way to be 1 league and 2 tenths of a league, or 3 miles and 6 tenths of a mile.

As	352000	vnto	21120
So	20-00	vnto	1-20
and	60-00	vnto	3-60

But to auoid these fractions and other tedious reductions, I suppose it would be much better to keepe this account of the ships way (as also of the difference of latitude, and the difference of longitude) by *degrees* and parts of *degrees* alowing in 100 parts to each *degree*, which we may therfore call by the name of *centesmes*. For so doing there would be some agreement betweene the account and the dayes sayling. Ordinarily the ship goes a degree in a day, as it may appeare by comparing severall Iournalls to the east and west *Indies*. The time of passage betweene the lizard and the southermost Cape of *Africa* is commonly said to be about three moneths and the distance is not much different from 90 degrees.

Againe this account by degrees and Centesmes would be more exact and the addition, subtraction, multiplication, division of them more easie. Neither would this be hard to conceaue. For,

	Centesm's,		*Minutes,*		*leagues,*
If	100	do equall	60	and	20
then	50	shall equall	30	and	10
and	5	be equall	3	and	1

And so in the former example of 88 feet in 15 seconds hauing firſt found that the houres way is about 21120 feet.

If I extend the compaſſes from 352000 vnto 21120 as before I ſhall find the ſame extent to reach from 100 vnto 6 as before, which ſhewes that the houres way required is 6 *cent.* ſuch as 100 do make a degre, & 5 do make an ordinary league.

This might alſo be done at one operation. For vpon theſe ſuppoſitions, diuide 44 feet into 45 lengths, and ſet as many of them as you may conueniently betweene two markes on the ſhips ſide, and note the ſeconds of time in which the ſhip goeth theſe lengthes: ſo the proportion will hold,

As the ſeconds, to the lengths
So 1 houre, vnto the Centeſmes

The lengths diuided by the time, ſhall giue the *cent.* which the ſhip goeth in an houre.

Suppoſe the diſtance betweene the two markes to be 60 lengths (which are 58 feet and 8 inches) & let the time be 12 ſeconds: extend the compaſſes from 12 to 1, in the line of *numbers*; ſo the ſame extent will reach from 60 vnto 5. Or extend them from 12 vnto 60, & the ſame extent will reach from 1 vnto 5. This ſhewes that the ſhips way is according to 5 *Cent.* in an houre.

This may be found yet more eaſily, if the logg line ſhal be fitted to the time. As if the time be 45 ſeconds, the log line may haue a knot at the end of euery 44 feete; then doth the ſhip run ſo many *cent.* in an houre, as there are knots vered out in the ſpace of 45 ſeconds. If 30 ſeconds do ſeeme to be a more conuenient time, the loggline may have a knot at the end of euery 29 feet and 4 inches; and then alſo the *centeſmes* will be as many as the knots. Or if the knots be made to any ſet number of feet, the time may be fitted vnto the diſtance. As if the knots be made at the end of euery 24 feet, the glaſſe may be made 24 ſecond & ſomewhat more then an halfe of a ſecond, and ſo theſe knots will ſhew the *cent.* If there be 5 knots vered out in a glaſſe, thē 5 *cent*; if 6 knots, then the ſhip goeth 6 *cent.* in the ſpace of an houre; & ſo in the reſt. For vpon this ſuppoſition the proportiō between the time & the feet will be as 45 vnto 44. But according to the common ſuppoſition it ſhould ſeeme to be as 45 vnto 37 $\frac{1}{2}$, or in leſſer termes as 6 vnto 5. Thoſe which are vpon the place, may make proofe of both, and follow that which agrees beſt with their experience.

2

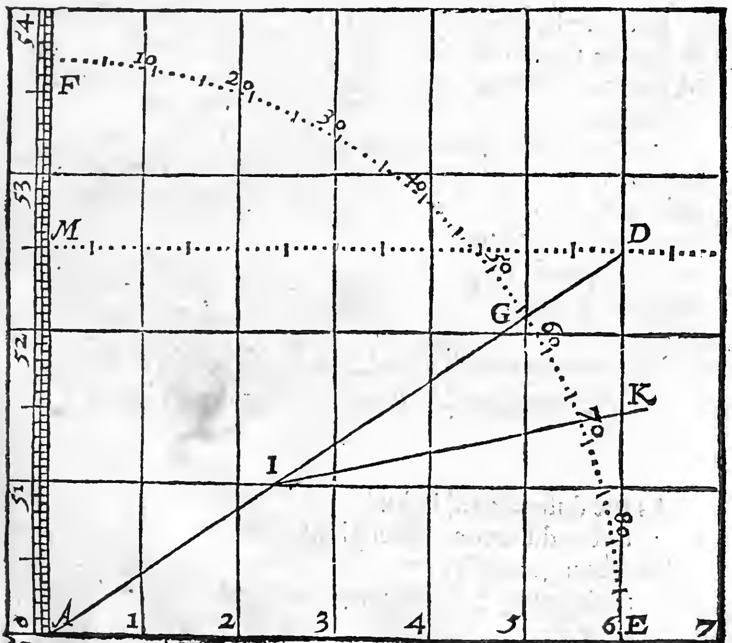
2 *By the latitude and difference of longitude, to find the distance vpon a course of East and West.*

As the sine of 90 *gr.*
to the cosine of the latitude
So the difference of longitude at the æquator
to the distance required on the parallell.

Extend the compasses from the sine of 90 *gr.* vnto the sine of the complement of the latitude; the same extent shall reach in the line of *numbers* from the difference of longitude to the distance.

So the measure of one degree in the æquator, being 100 *cent.* the distance belonging to one degree of longitude in the latitude of 51 *gr.* 30 *m.* will be found about 62 *cent.* and $\frac{1}{4}$.

Or if the measure of a degree be 60 miles, the distance will be found about 37 miles and $\frac{1}{3}$. If the measure be 20 leagues, then almost 12 leagues and $\frac{1}{2}$. If the measure be 17 $\frac{1}{2}$, as in



the Spanish charts, them somewhat lesse then 11 leagues sailing vpon this parallell, will giue an alteration of one degree of longitude.

3 *By the latitude and distance vpon a course of East or West, to find the difference of longitude.*

If the distance be giuen in leagnes or miles reduce them into centesmes, then will the proportion holde.

As the cosine of the latitude
 to the sine of 90 *gr*.
So the distance on the parallell
 to the difference of longitude.

Extend the compasses from the sine of the complement of the latitude, to the sine of 90 *gr*; the same extent will reach in the line of *numbers* from the distance to the difference of longitude.

So the distance vpon a course of East or West, in the latitude of 51 *gr*. 30 *m*. being 100 *cent*. the difference of longitude will be found 1.60, which make one degree and 60 *centesmes* or 1 *gr*. 36 *m*.

Or if it be 60 miles, the difference of longitude will be 96, which also make 1 *gr*. 36 *m*. as before.

4 *The longitude and latitude of two places being giuen, to find the Rumb leading from the one to the other.*

As the difference of latitude
 to the difference of longitude
So the tangent of 45 *gr*.
 to the tangent of the common Rumb.

Extend

Extend the compaſſes in the line of *numbers* from the difference of latitudes to the difference of longitudes; the ſame extent will giue the diſtance from the tangent of 45 *gr.* vnto the tangent of the Rumb, according to the proiection of the common ſea-chart.

So the latitude of the firſt place being 50 *degree* the latitude of the ſecond 52 *degree* 30 *m.* and the difference of longitude 6 *gr.* the Rumb will be found to be about 67 *gr.* 23 *m.* which is neare the inclination of the ſixth Rumb to the meridian. But this Rumb ſo found, is alwayes greater then it ſhould be, and therefore to be limited; which may be done ſufficiently for the *Sea-mans* vſe, after this maner:

As the ſine of 90 *gr.*
to the coſine of the midle latitude
So the tangent of the common Rumb
to the tangent of the Rumb required.

Extend the compaſſes either from the ſine of 90 *degree* vnto the ſine of the complement of the midle latitude, the ſame extent will reach from the tangent of the Rumb before found, to the tangent of the Rumb limited.

Or elſe extend them from the ſine of 90 *degree* vnto the tangent of the Rumb before found; the ſame extent will reach from the ſine of the complement of the middle latitude, vnto the tangent of the Rumb limited.

So the middle latitude between 50 *gr.* and 52 *gr.* 30 *m.* being 51 *gr.* 15 *m.* and the Rumb before found 67 *gr.* 23 *m.* the Rumb limited will be found to be about 56 *gr.* 20 *m.* which is but fiue minutes more then the inclination of the fift Rumb to the meridian.

If any pleaſe to worke by the *Canon* he may ioine both theſe in one operation.

As the difference of latitude
to the difference of longitude
So the cosine of the midle latitude
to the tangent of the Rumb required.

2 This Rumb may be found by the helpe of the *meridian line* vpon the Staffe. For if I take the difference of latitude out of the *meridian line* from 50 *degree* vnto 52 *degree* 30 *m.* and measure it in his equinoctiall, or at the beginning of the *meredian line*, I shall find it there to be equal to 4 *degree* with may be called the difference of latitude in larged. Wherefore I work as if the difference of latitude were 4 *gr*.

As the difference of latitude in larged
to the difference of longitude
So the tangent of 45 *gr*.
to the tangent of the Rumb required.

And extend the compasses in the line of *numbers* from 4 vnto 6: so shall I finde the same extent to reach from the tangent of 45 *degree* vnto the tangent of 56 *degree* 20 *m.* and this is the inclination of the Rumb required.

6 *By the Rumb and both latitudes, to find the distance vpon the Rumb.*

As the cosine of the Rumb from the meridiall
to the sine of 90 *gr*.
So the difference between both latitudes
to the distance vpon the Rumb.

Extend the compasses from the sine of the complement of the Rumb, vnto the sine of 90 *gr*. the same extent in the line

line of *numbers* ſhall reach from the difference of latitude vnto the diſtance vpon the Rumb.

So the latitude of the firſt place being 50 *gr.* the latitude of the ſecond 52 *gr.* 30 *m.* and the Rumb the fift from the meridian. If I extend the compaſſes from 33 *gr.* 45 *m.* vnto the ſine of 90 *gr.* I ſhall find the ſame extent in the line of *numbers* to reach from 2 *gr.* 50 *cent.* to 4 *gr.* 50 *cent.* and ſuch is the diſtance required.

7 *By the diſtance and both latitudes to find the Rumb.*

As the diſtance on the Rumb
to the difference between both latitudes
So the ſine of 90 *gr.*
to the coſine of the Rumb from the meridian.

Extend the compaſſes in the line of *numbers* from the diſtance vnto the difference of latitudes; the ſame extent will reach in the line of *ſines*, from 90 *gr.* vnto the complement of the Rumb.

☞ So the one place being in the latitude of 50 *degree* the other in the latitude of 52 *degree* 30 *m.* and the diſtance between them 4 *degres* 50 *cent.* If I extend the compaſſes from 4. 50 vnto 2. 50. in the line of *numbers*, I ſhall find the ſame extent to reach from the ſine of 90 *degree* vnto the complement of 56 *degree* 15 *m.* and ſuch is the inclination of the Rumb required.

8 *By one latitude, Rumb, and diſtance, to find the difference of latitudes.*

As the ſine of 90 *gr*
to the coſine of the Rumb from the meridian

So the distance vvon the Rumb
to the difference between both latitudes.

Extend the compasses in the line of *sines*, from 90 *gr.* vnto the complement of the Rumb; the same extent in the line of *numbers*, will reach from the distance, vnto the difference of latitudes.

So the lesser latitude being 50 *degres* and the distance 4 *degres* 50 *cent.* vpon the fifth Rumb from the meridian: if I extend the compasses from the sine of 90 *gr.* to 33 *gr.* 45 *m.* I shall finde the same extent to reach from 4.50 in the line of *numbers*, vnto 2.50; and therefore the second latitude to be 52 *gr.* 30 *m.*

9 *By the Rumb and both latitudes, to find the difference of longitudes.*

As the tangent of 45 *gr.*
to the tangent of the Rumb from the Meridian;
So the difference of latitude
to the difference of longitude in the common sea-chart.

Extend the compasses from the tangent of 45 *gr.* vnto the tangent of the Rumb; the same extent will reach in the line of *numbers* from the difference of latitudes vnto the difference of longitude, according to the proiection of the common sea chart.

So the first latitude being 50 *gr*, and the second 52 *gr.* 30 *m.* and the Rumb the fifth from the meridian: if I extend the compasses from the tangent of 45 *gr.* vnto 56 *gr.* 15 *m.* I shall find the same extent to reach from 2.50 in the line of *numbers* to about 3.75, which make 3 *gr.* 45 *m.* But this difference of longitude so found, is alwayes lesser then it should be, and therefore to be enlarged, which may be done sufficiently for the sea-mens vse, after this maner:

A

As the cosine of the middle latitude
  to the sine of *90 gr.*
So the difference of longitude in the common sea-chart
  to the difference of longitude inlarged.

Extend the compasses from the sine of the complement of the middle latitude, vnto the sine of 90 *gr.* the same extent will reach in the line of *numbers* from the difference of longitude before found, vnto the difference of longitude inlarged.

So the middle latitude in this example being 51 *gr.* 15 *m.* and the difference of longitude before found 3 *gr.* 75 *cent.* the difference of longitude inlarged will be found about 5 *gr.* 99 *cent.* which are neare 6 *gr.*

If any please to worke by the *Canon* he may ioyne both these in one operation.

As the cosine of the middle latitude
  to the tangent of the Rumbe from the meridian:
So the difference of latitude
  to the difference of longitude required.

E. 2 This difference of longitude may be found by helpe of the *meridian line* vpon the Staffe. For if I take the proper difference of latitude out of the meridian line, and measure it in his equinoctiall, or at the beginning of the meridian line, I shall find the latitude inlarged to be equall to foure of those degrees.

As the tangent of 45 *gr.*
  to the tangent of the Rumb from the meridian
So the difference of latitude inlarged
  to the difference of longitude required.

Wherefore hauing extended the compasses as before from the tangent of 45 *gr.* vnto the tangent of 56 *gr.* 15 *m.* the

the ſame extent will reach from 400 in the line of *numbers*, vnto 5. 99; which ſhewes the difference of longitude to be about 5 *gr.* 99 *cent.* or about halfe a minute ſhort of ſix degrees.

10 *By the Rumb and both latitudes, to finde the diſtance belonging to the chart of* Mercators *projection.*

Take the proper difference of latitudes out of the meridian line of the chart, and meaſure it in his equinoctiall, or one of the parallels, and it will there giue the difference of latitudes inlarged.

As the coſine of the Rumb from the meridian
to the ſine of 90 *gr.*
So the difference between both latitudes
to the diſtance vpon the Rumb.

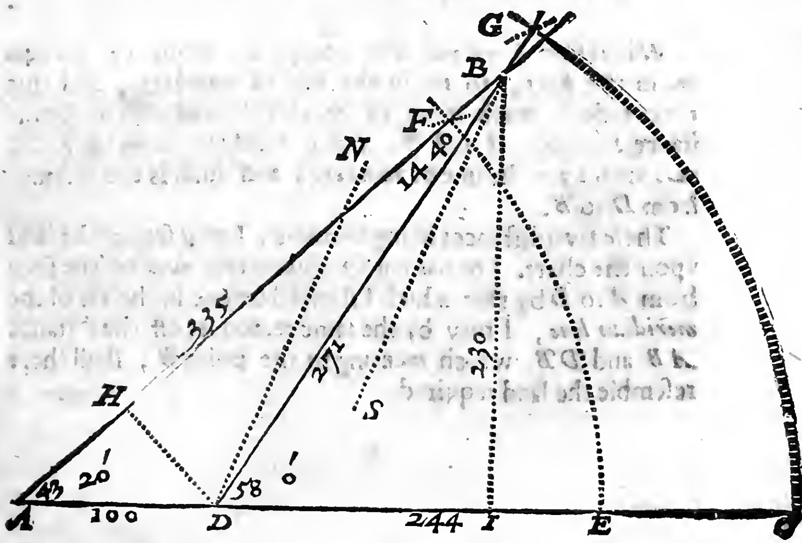
Then extend the compaſſes from the ſine of the complement of the Rumb vnto the ſine of 90 *gr.* the ſame extent will reach in the line of *numbers*, from the latitude inlarged, vnto the diſtance required. Or extend them from the complement of the Rumb to the latitude inlarged, the ſame extent will reach from 90 *gr.* vnto the diſtance.

For example, let the place giuen be *A* in the latitude of 50 *gr.* *D* in the latitude of 52 *gr.* 30 *m.* *A M* the difference of latitudes, and the Rumb *M A D* the fifth from the meridian. Firſt I take out *A M* the difference of latitudes, and meaſure it in *A E* one of the parallels of the æquinoctiall; I find it to be very neare 4 *gr.* this is the difference of latitudes inlarged. Then if I extend the compaſſes from the ſine of 33 *gr.* 45 *m.* the complement of the fifth Rumb vnto the ſine 90 *gr.* I ſhall find the ſame extent to reach in the line of *numbers* from 400 vnto 7.20. And this is the diſtance belonging to the chart. Wherefore I take out theſe 7 *gr.* 20 *cent.* out of the

the scale of the parallell *A E*, and pricke it downe vpon the Rumb from *A* vnto *D*, where it meeteth with the parallell of the second latitude. Lastly, I measure it in the *meridian line*, setting one foote of the compasses as much below the lesser latitude as the other aboue the greater latitude, and find it to be 4 *gr.* 50 *cent.* which is the same distance that I found before in the 5. *Prop.*

11 *By the way of the ship, and two angles of position, to find the distance betweene the ship and the land.*

The way of the ship may be knowne as in the first *Prop.* The angles may be obserued either by the Staffe, or by a needle set on the Staffe. For example, suppose that being at *A*,



I had sight of the land at *B*, the ship going East Northeast from

from *A* toward *C*, and the angle of the ships position *B A C* being 43 *gr*. 20 *m*: and after that the ship had made 10 *cent*. or 2 leagues of way from *A* vnto *D*, I obserued againe, and found the second angle of the ships position *B D C* to be 58 *degree* or the inward angle *B D A* to be 112 *degree* then may I finde the third angle *A B D* to be 14 *degree* 40 *m*. either by subtraction or by complement vnto 180 *gr*.

In this and the like cases, I haue a right line triangle, in which there is one side and three angles knowne, and it is required to finde the other two sides and the *Canon* for it, is this:

As the sine of the angle opposite to the knowne side,
is to that knowne side:
So the sine of the angle opposite to the side required,
is to the side required.

Wherefore I extend the compasses from 14 *gr*. 40 *m*. in the *sines*, to 10 in the line of *numbers*, and this extent doth reach from 58 *gr*. to 33$\frac{1}{5}$, and such is the distance between *A* and *B*, and it reacheth from 43 *gr*. 20 *m*. vnto 27 in the line of *numbers*; and such is the distance from *D* to *B*.

These two distances being knowne, I may set out the land vpon the chart. For hauing set downe the way of the ship from *A* to *D* by that which I shewed before in the vse of the *meridian line*, I may by the same reason set off the distance *A B* and *D B*, which meeting in the point *B*, shall there resemble the land required.

11 *By knowing the distance between two places on the land, and how they beare one from the other, and hauing the angles of position at the ship to find the distance betweene the ship and the land.*

If it may be conueniently, let the angle of position be obserued at such time as the ship cometh to be right ouer against one of the places: As if the places be East and West, seeke to bring one of them South or North from you, and then obserue the angle of position: so shall you haue a right line triangle, with one side and three angles, whereby to find the two other sides. First you haue the angle of position at the ship; then a right angle at the place that is ouer against you; and the third angle at the other place is the complement to the angle of position. Wherefore

As the sine of the angle position,
is to the distance betweene the two places:
So the cosine of the angle of position,
to the distance betweene the ship and the nearer place.
And so is the sine of *90 gr.*
to the distance from the ship to the farther place.

So the places being 15 *cent.* or three leagues one from the other, and the angle of position 29 *gr*, the nearer distance will be found about 27 *cent.* and the farther distance about 31 *cent.*

Or howsoeuer the angle of position were obserued, the distance betweene the ship and the land may be found generally as in this example:

Suppose *A* and *D* were two head lands knowne to be East Northeast, and West Southwest, 10 *cent.* or two leagues

Nn one

one from the other; and that the ship being at *B*, I obserued the angle of the ships position *D*B*A*, and found it to be 14 *gr.* 40 *m.* and that *D* did beare 9 *gr.* 30 *m.* and *A* 24 *gr.* 10 *m.* from the meridian B *S*, this example would be like the former. For if the angle S B *D* be 9 *gr.* 30 *m.* from the South to the Westward, then shall NDB be 9 *gr.* 30 *m.* from the North to the Eastward. Take these 9 *gr.* 30 *m.* out of the angle NDE which is 67 *gr.* 30 *m.* because the two head lands lie East Northeast, and there will remaine 58 *gr.* for the angle BDE, and the inward angle BDA shall be 122 *gr.* Take these two angles ABD and BDA out of 180 *gr.* and there wil remaine 43 *gr.* 20 *m.* for the third angle BAD. Wherefore here also are three angles and one side, by which I may find the two other sides, as in the last *Prop.*

These propositions thus wrought by the *Staffe*, are such as I thought to be vsefull for sea-men, and those that are skilfull may apply the example to many others. Those that begin, and are willing to practise, may busie themselues with this which followeth.

Suppose foure ports, L, N, O, P; of which L is in the latitude of 50 *degrees* N is North from L 200 leagues or 1000 *centesmes*; O West from L 1000 *centesmes* and P West from N 1000 *centesmes* so that L and O will be in the same latitude of 50 *gr.* N and P both in the latitude of 60 *gr*, Then let two ships depart from L, the one to touch at O, the other at *N*, and then both to meet at P, there to lade, and from thence to returne the nearest way vnto L. Here many questions may be proposed.

1 What is the longitude of the port at O?

2 What is the longitude of P? And why O and P should not be in the same longitude?

3 What is the Rumb from O vnto P?

4. What is the distance from O vnto P? And why the way should be more from L vnto P, going by O, then by N?

5 What

5 What is the Rumb from P vnto L?
6 What is the diſtance from P vnto L?
7 What is the Rumb from N vnto O?
8 What is the diſtance from N vnto O? And why it ſhould not be the like Rumb and diſtance from N vnto O, as from P vnto L?

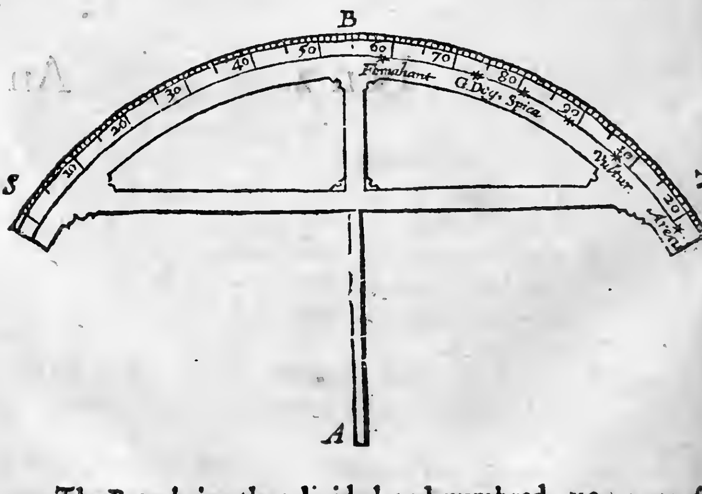
These queſtions well conſidered, and either reſolued by the Staffe, or pricked downe on the Chart, and compared with the globe and the common *S*ea-chart, ſhall giue ſome light to the direction of a courſe, and reduction of places to their due longitude, which are now fouly diſtorted in the common Sea-charts.

An Appendix concerning

The description and vse of an instrument, made in forme of a Crosse-bow, *for the more easie finding of the latitude at Sea.*

THe former *Prop.* suppose the latitude to be knowne I will here shew how it may be easily obserued.

Vpon the center *A* and semidiameter *A B*, describe an ark of a circle *S B N*. The same semidiameter will set of 60 *gr.* from *B* unto *S* for the South end, and other 60 *gr.* from B vnto *N* for the North end of the Bow: so the whole Bow will containe 120 *gr.* the third part of a circle. Let it therefore be diuided into so many degrees, and each degree subdiuided into six parts, that each part may be ten minutes: but let the numbers set to it be 5. 10. 15. vnto 90 *gr.* and then againe 5. 10. 15, vnto 25. that 55 may fall in the middle, as in this figure.



The Bow being thus diuided and numbred, you may set the

the moneths and dayes of each moneth upon the backe, and such starres as are fit for obseruation vpon the side of the Bow.

If you desire to make vse of it in North latitude, you may number 23 gr. 30 *m.* from 90 towards the end of the Bow at N, and there place the tenth day of Iune. And 23 *gr.* 30 *m.* from 90 towards *S*; and there at *66 gr.* 30 *m.* place the tenth day of December. And so the rest of the dayes of the yeare, according to the declination of the Sunne at the same dayes.

The starres may be placed in like maner according to their declinations.

Arcturus	21 *gr.*	10 *m.*
The Buls eye	15	42
The Lions heart	13	45
The Vultures heart	7	58
The little dog	6	9

from 90 toward the North end of the Bow at N. Then for Southerne starres, you may number their declination from 90 toward the South end of the Bow at S. As first the three starres in *Orions* girdle,

In *Orions* girdle the first at	0 *gr.*	37 *m.*
In *Orions* girdle the second	1	28
In *Orions* girdle the third	2	11
The Hydra's heart	7	5
The virgins spike	9	10
The great dog	16	12
Aquaries leg	20	
The Whales taile	18	
The Scorpions heart	25	30
Fomahant	31	30

And so the South crowne, the triangle, the clouds, the crosiers, or what other starres you think fit for obseruation. This I call the fore side of the Bow.

If you desire to make vse of it in South latitude, you may turne the Bow, and divide the backe side of it, and number it in like maner; and then put on the months and dayes of the yeare, placing the tenth of December at the *South* end, and the tenth of Iune toward the middle of the Bow, and the rest of the dayes according to the Sunnes declination as before.

The chiefest of the Northerne starres may here be placed in like maner according to their declination, Anno 1625.

The pole starre at	87 *gr.*	20 *m.*
The first guard	75	45
The second guard	73	25
The great Beares backe	63	45
In the great Beares taile first	58	2
In the great Beares taile second	57	55
In the great Beares taile third	51	15
The side of Perseus	48	28
The goate	45	33
The taile of the swan	44	0
The head of Medusa	39	30
The harp	38	30
Castor	32	38
Pollux	28	52
The North crowne	28	0
The Rams head	21	40
Arcturus	21	10
The Buls eye	15	42
The Lions heart	13	45
The Vultures heart	7	58
Orions right shoulder	7	17
Orions left shoulder	5	57

And so any other starre, whose declination is knowne vnto you, which being done. The vse of this Bow may be.

1 *The day of the moneth being knowne, to finde the declination of the Sunne.*

2 *The declination being giuen, to finde the day of the moneth.*

These two *Prop.* depend on the making of the Bow. If the day be knowne, looke it out in the backe of the Bow: so the declination will appeare in the side. Or if the declination be knowne, the day of the moneth is set ouer against it. As if the day of the moneth were the 14 of Iuly: looke for this day in the backe of the Bow, and you shall find it ouer against 20 *gr.* of North declination. If the declination giuen be 20 *gr.* to the Southward, you shall find the day to be either the eleuenth of November, or the eleuenth of Ianuary.

3 *To find the altitude of the Sunne or starres.*

Here it is fit to haue two running sights, which may be easily moued on the backe of the Bow. The vpper sight may be set either to 60 *gr.* or to 70 *gr.* or to 80 *gr.* as you shall find to be most conuenient: the other sight may be set on, to any place betweene the midle and the other end of the Bow. Then with the one hand hold the center of the Bow to your eye, so as you may see the Sunne or starre by the vpper sight, and with the other hand moue the lower sight vp or downe vntill haue you brought one of the edges of it to be euen with the horizon (as when you obserue with the Crossestaffe:) so the degrees contained betweene that edge and the vpper sight, shall shew the altitude required.

Thus

Thus if the vpper sight shall be at 80 *gr.* and the lower sight at 50 *gr.* the altitude required is 30 *gr.*

6 *To find any North latitude, by the meridian altitude of the Sun at a forward obseruation knowing either the day of the moneth, or the declination of the Sunne.*

As oft as you are to obserue in North latitude, place both the sights on the fore side of the Bow, the vpper sight to the declination of the Sunne, or the day of the moneth at the North end, and the lower sight toward the South end. Then when the Sunne cometh to the meridian, turne your face to the South, and with the one hand hold the center of the Bow to your eye, so as you may see the Sunne by the vpper sight; with the other hand moue the lower sight, vntill you haue brought one of the edges of it to be euen with the horizon: so that edge of the lower sight shall shew the latitude of the place in the fore side of the Bow.

Thus being in North latitude vpon the ninth of October: if I set the vpper sight to this day, at the fore side and North end of the Bow, I shall find it to fall to the Southward of 90 vpon 80 *gr.* and therefore at 10 *gr.* of South declination. Then the Sunne coming to the meridian, I may set the center of the Bow to mine eye, as if I went to find the altitude of the Sunne, holding the North end of the Bow vpward, with the vpper sight betweene mine eye and the Sunne, and mouing the lower sight, vntill it come to be euen with the horizon. If here the lower sight shall stay at 50 *gr.* I may well say, that the latitude is 90 *gr.* For the meridian altitude of the Sunne is 30 *gr.* by the third *Prop.* and the Sunne hauing 10 *gr.* of South declination, the meridian altitude of the æquator would be 40 *gr*; and therefore the obseruation was made in 50 *gr.* of North latitude.

By the same reason, if the lower side had stayed at 51 *gr.* 30 *m.* the latitude must have been 51 *gr.* 30 *m.* and so in the rest.

8 *To*

5 *To find any North latitude, by the meridian altitude of the starres to the Southward.*

Let the vpper sight be set to the starre, which you intend to obserue, here placed in the fore side of the Bow. Then hold the North end of the Bow vpward, and turning your face to the *South*, obserue the meridian altitude as before: so the lower sight shall shew the latitude of the place in the fore side of the Bow.

Thus if in obseruing the meridian altitude of the great Dog-starre, the lower sight shall stay at 50 *gr.* it would shew the latitude to be 50 *gr.* For this starre being here placed at 73 *gr.* 48 *m.* if we take thence 50 *gr.* his meridian altitude would be 23 *gr.* 48 *m.* to this if we adde 16 *gr.* 12 *m.* for the South declination of this starre, it would shew the meridian altitude of the equator to be 40 *gr.* and therefore the latitude to be 50 *gr.*

6 *To find any North latitude, by the meridian altitude of the starres to the Northward.*

If the Bow be intended onely for north latitude it may suffice to haue the degrees diuided onely on the foreside, and then the starres to the northward may be placed either on the backside or the inside of the Bow by these degrees: the pole starre at 87 *gr.* 20 *m.* neere the 20 day of *September*, the formost guard at 75 *gr.* 45 *m.* the hindmost guard at 73 *gr.* 25 *m.* and the rest according to their declinations before mentioned so the 90 *degree* shall represent the north pole of the world.

When any of these starres come to be in the meridian and vnder the pole set the vpper sight to that starre, hold the north end of the Bow vpward and turning your face to the north obserue his altitude as before so the degrees contained between the 90 degree and the lower sight shall shew the altitude of the pole.

Thus the former guard coming to be in the meridian vnder the pole if you obserue and find the lower sight to stay at 40 *gr.* the eleuation of the pole is 50 *gr.* according to the distance betweene 40 and 90.

If you would obserue any of these starres at such time as they come to be in the meridian and aboue the pole, you may place these starres in the Bow aboue 90 *gr.* the north starre at 2 *gr.* 40 *m.* neere the fourth day of *September* the formost guard at 14 *gr.* 15 *m.* the hindmost guard at 16 *gr.* 35 *m.* and such others as you thinke fittest according to their distance from the pole: then setting the vpper sight to the place of the starre aboue the pole, the rest of the obseruation will be the same as before.

But if the Bow be made to serue at large both in South and north latitude then these northerne starres would be let placed on the backside of the Bow by the degrees on that side according to the complement of their declinations, that the north starres may answer to the north sun in south latitude in such sort as the southerne starres did to the south sun in north latitude in the former *Prop.* This being done let the vpper sight be set to the stare which you intend to obserue, here placed on the backe side of the Bow. Then hold the North end of the Bow vpward. and turning your face to the North, obserue the altitude of the starre when he cometh to be in the meridian and vnder the pole: so the lower sight shall shew the altitude of the pole in the back side of the Bow.

Thus the former guard coming to be in the meridian vnder the pole, if you obserue and find the lower sight to stay at 50 *gr.* such is the eleuation of the pole, and the latitude of the place to the Northward. For the distance betweene the two sights will shew the altitude to be 35 *gr.* 45 *m.* & the star is 14 *gr.* 15 *m.* distant from the North pole. These two do make vp 50 *gr.* for the eleuation of the North pole, and therefore such is the North latitude.

10 *To find any South latitude, by the meridian altitude of the ſun at a forward obſeruation, knowing either the day of the moneth, or the declination of the Sunne.*

When you are come into South latitude, turne both your ſights to the backſide of the Bow: the vpper ſight to the declination of the Sun, or the day of the moneth at the *S*outh end, and the lower ſight toward the North end of the Bow. Then the Sun coming to the meridian, turne your face to the north, and holding the South end of the Bow vpward, obſerue the meridian altitude as before: ſo the lower ſight ſhall ſhew the latitude of the place in the backe ſide of the Bow.

Thus being in South latitude, vpon the tenth of May if you obſerue and find the lower ſight to ſtay at 30 *gr.* on the back ſide of the Bow, ſuch is the latitude. For the declination is 20 *gr.* northward, the altitude of the *S*unne betweene the two ſights 40 *gr.* the altitude of the equator 60 *gr.* and therefore the latitude 30 *gr.*

11 *To find any South latitude, by the meridian altitude of the Starres to the Northward.*

Let the vpper ſight be ſet to the ſtarre which you intend to obſerue, here placed on the backe ſide of the Bow. Then hold the *S*outh end of the Bow vpward, and turning your face to the north, obſerue the meridian altitude as before: ſo the lower ſight ſhall ſhew the latitude of the place in the back ſide of the Bow.

Thus being in South latitude, and the former guard comming to be in the meridian ouer the pole. If you obſerue and finde the lower ſight to ſtay at 5 *gr.* ſuch is the latitude. For this ſtarre is 14 *gr.* 15 *m.* from the north pole, the altitude of the ſtarre betweene the two ſights 9 *gr.* 15 *m.* the north pole depreſſed 5 *gr.* and therefore the latitude 5 *gr.* to the Southward.

9 *To obserue the altitude of the Sunne by the Bow or with an Astrolabe.*

Here it is fit to haue a third sight (like to the horizontall sight belonging to the staffe) which may be set to the center of the Bow.

If the sun be neere to the zenith, hold the Bow as when you obserue with the *Astrolabe*, so as the center being downward the line *A B* may be verticall and the line S *N* parallel to the horizon, then turning one end of the Bow toward the sun you may moue one of the sights on the back of the Bow, vntill the shadow thereof fall on the middle of the horizontall sight so the degrees contained betweene the verticall line *A* B and that vpper sight shall shew the distance of the Sunne from the zenith.

If the sunne be neerer to the horizon, you may hold the Bow so as the line S *N* may be verticall and the line *A* B parallell to the horizon, then obseruing as before the degrees contained between the line *A* B and the vpper sight shall shew the altitude of the sun aboue the horizon.

10 *To find a south latitude by the meridian altitude of the starres to the Southward.*

Let the vpper sight be set to the starre which you intend to obserue which might be here placed on the fore side of the Bow by the complement of their declinations if we knew the true place of such as neere to the south pole.

Then hold the south end of the Bow vpward and turning your face to the south, obserue the altitude when he commeth to be in the meridian and vnder the pole so the

lower

lower ſight ſhall ſhew the altitude of the pole in the fore ſide of the Bow.

11 *To obſerue the altitude of the Sunne backward.*

Set the vpper ſight either to 60, or 70, or 80 *gr.* as you ſhall find it to be moſt conuenient, the lower ſight on any place betweene the middle and the other end of the Bow, and haue an horizontall ſight to be ſet to the center. Then may you turne your backe to the Sunne, and the back of the Bow toward your ſelfe, looking by the lower ſight through the horizontall ſight, and mouing the lower ſight vp & downe, vntill the vpper ſight doe caſt a ſhadow vpon the middle of the horizontall ſight: ſo the degrees contained betweene the two ſights on the Bow, ſhall giue the altitude requi-red.

Thus if the vpper ſight ſhall be at 80 *gr.* and the lower ſight at 50 *gr.* the altitude required is 30 *gr.* as in the third *Prop.*

Or if you tourne the other end of the bowe vpward and ſet the vpper ſight to the beginning of the quadrant and then obſerue as before, the lower ſight will ſhew the alti-tude.

12 *To find any North latitude by the meridian alti-tude of the ſun at a backe obſeruation, kno-wing either the day of the moneth, or the declination of the Sunne.*

Place your three ſights as before on the fore ſide of the Bow: the vpper ſight to the declination of the *Sun*, or to day of the moneth, at the North end; the lower ſight to-ward the South end of the Bow; and the horizontall ſight to the center. Then the Sunne coming to the meridian, turne your face to the North, & holding the North end of the Bow vpward, the South end downeward, with the back of it toward your selfe, obserue the shadow of the vpper sight as in the former part of the, 5 *Prop.* so the lower sight shall shew the latitude of the place in the fore side of the Bow.

Thus being in North latitude vpon the ninth of October, if you obserue and find the lower sight to stay at 50 *gr.* on the fore side of the Bow, such is the latitude. For the declination is 10 *gr.* Southward, and the altitude of the Sunne betweene the two sights 30 *gr.* the altitude of the equator 40 *gr.* and therefore the latitude 50 *gr.* as in the sixth *Prop.*

13 *To find any South latitude by the meridian altitude of the sun at a back obseruation, knowing either the day of the moneth, or the declination of the Sunne.*

When you obserue in South latitude, place your three sights on the backe side of the Bow: the vpper sight to the declination of the Sunne, or the day of the moneth at the South end; the lower sight toward the North end of the Bow, and the horizontall sight to the center. Then the Sun coming to the meridian, turne your face to the South, and holding the South end of the Bow vpward, with the backe of it toward your selfe, obserue the shadow of the vpper sight as before: so the lower sight shall shew the latitude of the place in the back side of the Bow.

Thus being in the South latitude vpon the tenth of May, if you obserue and find the lower sight to stay at 30 *gr.* on the backe of the Bow, such is the latitude of the Sunne betweene the

the two sights 40 *gr.* the altitude of the equator 60 *gr.* and therefore the latitude 30 *gr.* as in the seuenth *Prop.*

14. *To find the day of the moneth, by knowing the latitude of the place, and obseruing the meridian altitude of the Sunne.*

Place your three sights according to your latitude; the horizontall sight to the center, the lower sight to the latitude, and the vpper sight among the moneths. Then when the Sunne cometh to the meridian, obserue the altitude, looking by the lower sight through the horizontall, and keeping the lower sight still at the latitude, but mouing the vpper sight vntil it giue shadow vpon the middle of the horizontal sight: so the vpper sight shall shew the day of the moneth required.

Thus in our latitude if you set the lower sight to 51 *gr.* 30 *m.* and obseruing finde the altitude of the Sunne betweene that and the vpper sight to be 28 *gr.* 30 *m.* this vpper sight will fall vpon the ninth of October, and the twelfth of Februarie. And if yet you doubt which of them two is the day, you may expect another meridian altitude; and then if you find the vpper sight vpon the tenth of October, and the eleuenth of Februarie, the question will be soone resolued.

15 *To find the declination of any vnknowne starre, and so to place it on the* Bow, *knowing the latitude of the place, and obseruing the Meridian altitude of the Starre.*

When you find a starre in the Meridian that is fit for obseruation. Set the center of the Bow to your eye, the lower sight

ſight to the latitude, and moue the vpper ſight vp or downe vntill you ſee the horizon by the lower ſight, and the ſtarre by the vpper ſight, then will the vpper ſight ſtay at the declination and place of the ſtarre.

Thus being in 20 *gr.* of North latitude, if you obſerue and find the meridian altitude of the head of the Croſier to be 14 *gr.* 50 *m.* The vpper ſight will ſtay at 34 *gr.* 50 *m.* and there may you place this ſtarre. For by this obſeruation the diſtance of this ſtarre from the South pole ſhould be 34 *gr.* 50 *m.* and the declination from the equator 55 *gr.* 10 *m.* And ſo for the reſt:

The ſtarres which I mentioned before, do come to the meridian in this order, after the firſt point of *Aries.*

16 *To find any north latitude on land by obſeruation with thread and plummet.*

Set the ſight to the day of the moneth at the fore ſide and ſouth end of the Bow: then when the ſun cometh to the meridian turning the north end in your left hand toward the ſouth, ſo as the ſight at the center may ſhadow the ſight at the day, obſerue where the thread falleth and abate 20 *gr.* If it fall on 70 *gr.* the latitude is 50 *gr.* If on 71 *gr.* 30 *m.* in the latitude is 51 *gr.* 30 in. And ſo in the reſt

If the Bow had ben made onely for finding the latitude on land I might then haue ſet ſuch numbers to it as needed no allowance.

17 *To find any ſouth latitude on land by obſeruation with thread and plummett.*

Set the ſight to the day of the moneth at the back ſide and north end of the Bow, and when the ſun cometh to the meridian turning the ſouth en di your left hand toward the north obſerue as before, and abate 20 degrees.

Or

Or you may set the sight to the day of the moneth at the fore side and north end of the Bow, and so obseruing as before, the thread will fall on the complement of the latitude.

	Ho,	Mi.		Ho.	Mi.
The pole starre at	0	29	The lionshart	9	48
The rams head	1	46	The great bearesbacke	10	40
The head of Medusa	2	44	First in gr beares taile	12	27
The side of Perseus	2	58	The Virgins spike	13	5
The Bulseye.	4	15	Second in gr. beares taile	13	9
The goate	4	49	Third iu gr. beares taile	13	33
Orions left shoulder	5	5	Arcturus	13	58
Orions girdle the first	5	13	The formost guard	14	52
Orions girdle the second	5	17	The North crowne	15	19
Orions girdle the third.	5	21	The hindmost guard	15	25
Orions right shoulder	5	35	Scorpions hart	16	7
The great dog	6	29	The harpe	18	24
Castor	7	10	Vulturs hart	19	33
The little dog	7	20	Swans taile	20	29
Pollux	7	22	Fomahant	22	36
The Hydra'shart	9	9			

Anno 1625.	*R. Ascen.*		*Declin:*		*M.*
Pole starre	6	28	87	20	2
	294	28	86	35	4
Little beare.	261	56	82	33	4
	239	54	79	0	4
	246	44	76	38	5
First guard	222	57	75	45	2
Second guard	231	18	73	25	3
Gr. beare					
Snout	117	48	61	40	4
eye	120	30	64	30	4
Forehead {	124	15	68	30	
	126	40	68	0	
Eare	131	15	71	15	5
Necke {	131	30	63	50	
	142	10	62	15	
brest {	143	50	61	25	
	140	40	60	0	
Knee	135	30	53	40	3
Right foot {	129	00	48	50	3
	127	00	48	40	3
In the square {	160	3	63	46	2
	159	38	58	22	2
	173	24	55	48	2
	179	10	59	6	2
In the taile {	189	20	58	2	2
	197	10	56	55	2
	203	2	51	16	2
Cassi opea					
Head	4	10	51	50	4
Brest	4	57	54	30	3
Waste	6	45	55	50	4
Belly	8	44	58	40	3
Knee	15	50	58	15	3
Thygh	22	15	61	58	3
Foot	29	45	65	40	4
Chaire {	3	10	60	50	4
	357	25	57	0	3
Auriga					
Head	81	0	0	0	
Left. sh. Hircus	72	16	45	32	1
Right should	83	8	44	51	2

Cepheus	*R. Ascen*		*Dec li.*		*M.*
Girdle	320	52	68	56	3
lower					
R. should					
Left. should					
Head					
Thighe {					
Right foot					
Left foot					
Draco	454		55	.	.
Tongue					
Mouth					
Eye					
Cheeke					
Head	267	0	51	36	3
In the 1 winding {	2				
	2				
	2				
	2				
Vnd. that w.	2				
In the □ of the 2 winding {	2				
	.				
	.				
In the first △ {	.				
	.				
	.				
In the second △ {	.				
	.				
	.				
In the 3 winding nere the pole of the zodiac. {	.				
	.				
	.				
	.				
Before the fourth winding {					
After the winding in the taile {					

The end of the second booke of the crostaffe.

THE THIRD BOOKE.

of the vse of the lines of Numbers, *Sines and Tangents for the drawing of Houre-lines on all sorts of Planes.*

THERE are ten seuerall sorts of Planes, which take their denomination from those great circles to which they are parallels, and may sufficiently for our vse be represented in this one fundamentall Diagram and be knowne by their horizontall and perpendicular lines, of such as know the latitude of the place, and the circles of the sphære.

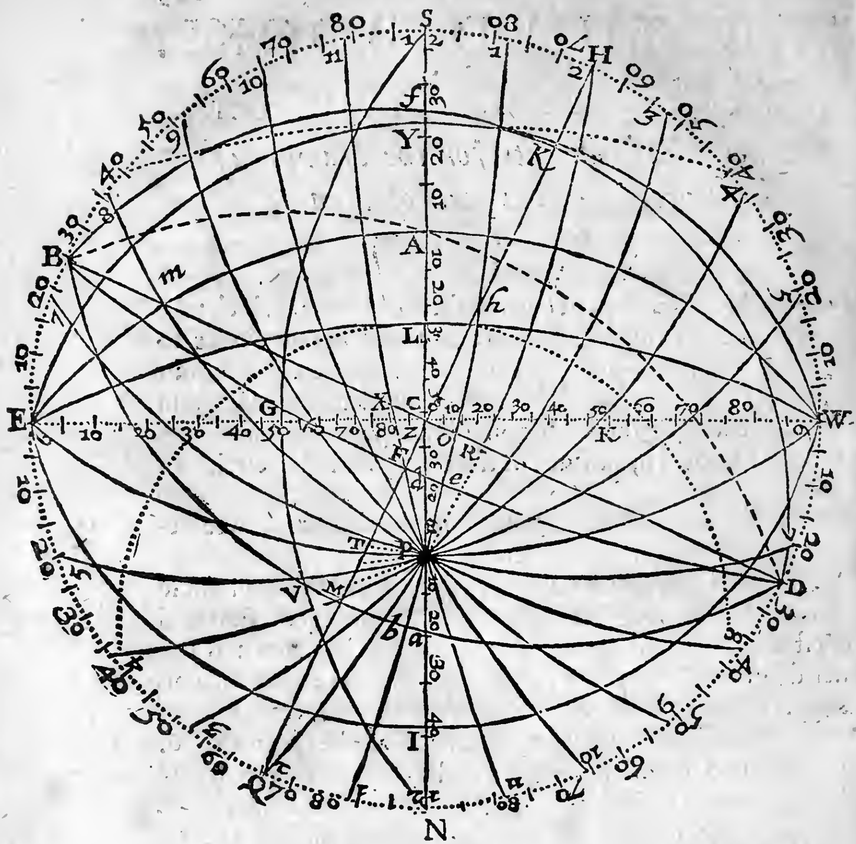
1 An horizontall plane parallell to the horizon, here represented by the outward circle *E S W N*.

2 A verticall plane parallell to the prime verticall circle which passeth through the zenith and the points of East and West in the horizon, and is right to the horizon and the meridian; that is, maketh right angles with them both. This is represented by *E Z W*.

3 A polar plane parallell to the circle of the houre of 6, which passeth through the pole and the points of East and West, being right to the Equinoctiall and the Meridian, but inclining to the horizon, with an angle equall to the latitude. This is here represented by *E P W*.

4 An æquinoctiall plane parallell to the Equinoctiall, which passeth through the points of East and West, being right to the Meridian, but inclining to the Horizon, with an angle equall to the complement of the latitude. This is here represented by *E A W*.

5 A verticall plane inclining to the horizon, parallell to any great circle, which passeth through the points of East and West, being right to the meridian, but inclining to the horizon, and yet not passing through the pole, nor parallell



to the æquinoctiall. This is here represented either by *E I W*, or *E Y W*, or *E L W*.

6 A meridian plane parallell to the meridian, the circle of the houre of 12, which passeth through the zenith, the pole, and the points of South and North, being right to the horizon, and the prime verticall. This is here represented by *S Z N*.

7 A meridian plane inclining to the horizon, parallell to any great circle, which passeth through the points of South and North, being right to the prime verticall, but inclining

clining to to the horizon. This is here represented by *S G N*.

8 A verticall declining plane, parallell to any great circle, which passeth through the zenith, being right to the horizon, but inclining to the meridian. This is represented by *B Z D*.

9 A polar declining plane, parallell to any great circle, which passeth through the pole, being right to the equinoctiall, but inclining to the meridian. This is here represented by *H P Q*.

10 A declining inclining plane, parallell to any great circle, which is right to none of the former circles, but declining from the prime verticall, and inclining both to the horizon and the meridian, and all the houre circles. This may be here represented either by *B M D*, or *B F D*, or *B K D*, or any such great circle, which passeth neither through the South and North, nor East and West points, nor through the zenith nor the pole.

Each of these planes (except the horizontall) hath two faces whereon houre-lines may be drawne; and so there are 19 planes in all. The meridian plane hath one face to the East, and another to the West: the other verticall planes haue one to the South, and another to the North, and the rest one to the zenith, and another to the nadir: but what is said of the one, may be vnderstood of the other.

To describe the fundamentall Diagram.

The description of this *diagram* is set downe at large in the vse of the *Sector* Pag. 65. but for this purpose it may suffice if it haue the verticall circle, the houre circles, the equator and the tropiques first drawne in it, other circles may be supplyed afterward as we shall haue vse of them. And those may be readily drawne in this maner.

Let the outward circle representing the horizon be drawne and diuided into foure equall parts with *SN* the meridian & *EW* the verticall and each fourth part into 90 *gr.* That done lay a ruler to the poynt S, and each degree in the quadrant *EN* and note the intersections where the ruler crosseth the verticall, so shall the semidiameter *EC* be diuided into other 90 *gr.* and from thence the other semidiameters may be diuided in the same sort. These may be numbered with 10. 20 30. &c. from *E* toward *C*, and for varietie with 10. 20. 30. &c. from *C* toward *W*. But for the meridian the South part would be best numbered according to the declination from the equator and the North part according to the distance from the pole.

Then with respect vnto the latitude which here we suppose to be 51 *gr.* 30 *m.* Open the compasses vnto 38 *gr.* 30 *m.* from *C* toward *W*, and prick them downe in the meridian from *C* vnto *P* so this point *P* shall represent the pole of the world, and through it must be drawne all the houre circles.

Hauing three points *E, P, W*, finde their center which will fall in the meridian a little without the point *S*, and draw them into a circle *EPW*, which will be the circle of the houre of 6.

Through this center of the houre of 6, draw an occult line at length parallell to *EW*, so this line shall containe the centers of all the other houre circles. Where the circle of the houre of 6 crosseth this occult line, there will be the centers of the houre circles of 9 and 3. The distance between these centers of 9 and 3, will be equall to the semidiameters of the houre circles of 10 and 2. And where these two circles of 10 and 2 shall crosse this occult line there will be the centers for the houre circles of 11 & 7 & 5 and 1. Againe diuide the distance between the centers of 10 and 2, into three equall parts, so the feet of the compasses will rest in two points: the one is the center of the houre circle of 8, and the other the center of the houre circle of 4. & the extent of the compasses to one of these third parts shall be the true semidiameter of these circles if there be no error committed in the finding of the other centers.

The

The houre circles being thus drawne, take 51 *gr.* 30 *m.* from *C* toward *W* and prick them downe in the South part of the meridian from *C* vnto *A*, and bring the third point *E, A, W*, into a circle this circle so drawne shall represent the equator.

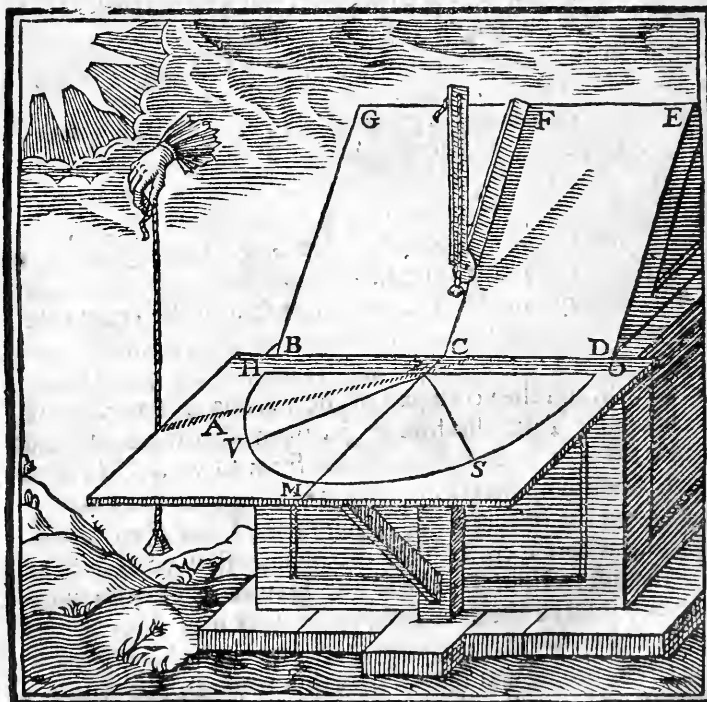
The tropique of ♋ is 23 *gr.* 30 *m.* aboue the equator, and 66 *gr.* 30 *m.* distant from the pole, and so in this latitude it will crosse the South part of the meridian at 28 *gr.* from the zenith, and the North part of the meridian at 15 *gr.* below the horizon. Take therfore 28 *gr.* frō *C* toward *W* & princk them downe in the meridian from *C* vnto *L*, so haue you the, South intersection. Then lay the ruler to the point *E* & 15 *gr.* in the quadrant *NE* numbered from *N* toward *E*, and note where it crosseth the meridian, so shall you haue the North intersection. The halfe way between these two intersections will fall in the meridian at the point a a a a, & the circle drawne on the center a, and semidiameter a *L*, shall represent the tropique of ♋, and here crosse the horizon before 4 in the morning & after 8 in the euening, about 40 *gr.* nortwhard from *E* and *W.* according to the rising and setting of the sun at his entrance into. ♋.

The tropique of ♑ is 23 *gr.* 30 *m.* below the equator, & 113 *gr.* 30 *m.* distant from the north pole, so that in this latitude it crosseth the South part of the meridian at 75 *gr.* from the zenith, and the north part of the meridian at 62 *gr.* below the horizon. Take therfore 75 *gr.* from *C* toward W, and pricke them downe in the meridian from *C* vnto *Y* so haue you the South intersection, then lay the ruler to the point *E* & 62 *gr.* in the quadrant *NE* numbered from *N* toward *E.* and note where it crosseth the meridian, so shall you haue the North niter section. The halfe way between these two intersections shall be the center whereon you may describe the tropique of ♑. and this tropique will crosse the horizon after 8 in the morning and before 4 in the euening, about 40 *gr.* southward from *E* and W. according to the rising and setting of the sun at his entrance into ♑.

To find the inclination of any Plane.

For the distinguiſhing of theſe Planes we may finde whether they be horizontall, or verticall, or inclining to the horizon, and how much they incline, either by the vſuall inclinatorie quadrant, or by fitting a thread and plummet vnto the *Sector*.

For let the *Sector* be opened to a right angle, the lines of *Sines* to an angle of 92 *gr.* the inward edges of the *Sector* to 90 *gr.* and let a thread and plummet be hanged vpon a line



parallell to the edges of one of the legs, ſo that leg ſhall be verticall, and the other leg parallell to the horizon.

If

If the plane seeme to be verticall (like the wall of an vpright building) you may trie it by holding the *Sector*, so that the thread may fall vpon his plumet line. For then if the verticall edge of the *Sector* shall lie close to the plane, the plane is erect, and therefore said to be verticall; and if you draw a line by that edge of the *Sector*, it shall be a verticall line.

If the plane seeme to be levell with the horizon, you may trie it by setting the horizontall leg of the Sector to the plane, and holding the other leg vpright: for then if the thread shall fall on his plummet line, which way soeuer you turne the *Sector*, it is an horizontall plane.

If the one end of the plane be higher then the other, and yet not verticall, it is an inclining plane, and you may find the inclination in this manner.

First hold the verticall leg of the Sector vpright, and turne the horizontall leg about, vntill it lie close with the plane, and the thread fall on his plummet line, so the line drawne by the edge of that horizontall leg, shall be an horizontall line.

Suppose the plane to be B *G E D*, and that B *D* were thus found to be the horizontall line vpon the plane then may you crosse the horizontall line at right angles with a perpendicular *C F*: that done, if you set one of the legs of the *Sector* vpon the perpendicular line *C F*, and make the other leg with a thread and plummet to become verticall, you shall haue the angle betweene the verticall line and the perpendicular on the Plane, as before in the vse of the *Sector*, pag. 50. and the complement of this angle is the inclination of the plane to the horizon.

To find the declination of a Plane.

The declination of a Plane is alwayes reckoned in the horizon betweene the line of East and West, and the horizontall line vpon the plane. As in the fundamentall Diagram, the prime verticall line (which is the line of East and West) is *E C W*; if the horizontall line of the plane propoſed ſhal be B *C D*. the angle of declination is *E C* B.

But becauſe a Plane may decline diuers wayes, that we may the better diſtinguiſh them, we conſider three lines belonging to euery Plane: the firſt is the horizontall line; the ſecond the perpendicular line, croſſing the horizontall at right angles; the third the axis of the plane, croſſing both the horizontall line, and his perpendicular, and the plane it ſelfe at right angles.

The perpendicular line doth help to find the inclination of the plane as before, the horizontall to finde the declination, the axis to giue denomination vnto the plane.

For example, in a verticall plane in the fundamentall diagram repreſented by *EZW*, the horizontall line is *ECW*, the ſame with the line of Eaſt & Weſt, & therefore no declination; the perpendicular croſſing it is *CZ*, the ſame with the verticall line, drawne from the center to the zenith, right vnto the horizon, and therefore no inclination. The axis of the plane is *S C* N, the ſame with the meridian line, drawne from the South to the North, and accordingly giues the denomination to the plane. For the plane hauing two faces, and the axis two poles, S and N; the pole *S* falling directly into the South, doth cauſe that face to which it is next to be called the South face; and the other pole at N, pointing into the North, doth giue the denomination to the other face, and make it to be called the North face of this plane.

In like manner in the declining inclining plane in the fundamentall diagram repreſented by B F D, the horizontall line is B C D, which croſſeth the prime verticall line *E C W*, & therfore it is called a declining plane, according to the angle of declination *ECB* or *W C D*. The perpendicular to this horizontall line is *C F*, where the point *F* falleth in the plane *Q Z H* perpendicular to the plane propoſed, betweene the zenith and the North part of the horizon, and therefore it is called a plane inclining to the Northward, according to the arke *F Q*, or the angle *F C Q*. The axis of the plane is here repreſented by the line C *K*, where the pole *K* is 90 *gr.* diſtant from

from the plane, and so is as much aboue the horizon at H, and the other pole as much below the horizon at Q, as the plane at F is distant from the zenith: and this pole K here falling betweene the meridian and the prime verticall circle into the Southwest part of the world, this vpper face of the plane is therefore called the Southwest face, and the lower the Northeast face of the plane.

The declination from the prime verticall may be found by the needle in the vsuall inclinatorie Quadrant, or rather by comparing the horizontall line drawne vpon the plane with the azimuth of the Sunne and the meridian line, in such sort as before we found the variation of the magneticall needle. For take any boord that hath one side straight, and draw as in the last diagram the line HO parallel to that side,& the line *Z M* perpendicular vnto it,and on the center Z make a semicircle H M O:this done, hold the boord to the plane,so as H O may be parallel to B D the horizontall line on the plane & the boord parallel to the horizon;then the Sun shining vpon it, hold out a thread and plummet, so as the thread being verticall, the shadow of the Sunne may fall on the center *Z*, and draw the line of shadow *A Z* representing the common section, which the Azimuth of the Sunne makes with the plane of the horizon, and let another take the altitude of the Sunne at the same instant: so by resoluing a triangle, as I shewed before pag. 65 you may find what Azimuth the Sun was in when he gaue shadow vpon *A Z*.

Suppose the azimuth to be (as before pag. 64.)72 *gr.* 52 *m.* from the North to the Westward, and therefore 17 *gr.* 8 *m.* from the West, we may allow these 17 *gr.* 8 *m.* from *A* vnto *V*, and draw the line *Z V*, and so we haue the true West point of the prime verticall line: then allowing 90 *gr.* from *V* vnto *S*, we haue the South point of the meridian line *Z S*, and the angle H Z V shall giue the declination of the plane from the verticall, and the angle O Z S the declination of the plane from the meridian.

Or we may take out onely the angle A *Z* H, which the line of shadow makes with the horizantall line of the plane, and compare it with the angle *A Z V*, which the line of shadow makes with the prime verticall. And so here if *A Z V* the Sunnes Azimuth shall be 17 *gr*. 8 *m*. past the West, and yet the line of shadow *A Z* 7 *gr*. 12 *m*. short of the plane, the declination of the plane shall be 24 *gr*. 20 *m*. as may appeare by the site of the plane and the circles.

If the altitude of the Sunne be taken at such time as the shadow of the thread falleth on B D or H O, and then a triangle resolued, the declination of the plane will be such as the Azimuth of the Sunne from the prime verticall.

If at such a time as the shadow falleth on *M Z*, the declination will be such as the Azimuth of the Sunne from the meridian.

If it be a faire Summers day you may first finde what altitude the Sunne will haue when he cometh to be due East or West, and then expect vntill he come to that altitude; so the declination of the plane shall be such as the angle contained betweene the line H O and the line of the shadow.

Hauing distinguished the Planes, the next care will be for the placing of the style and the drawing of the houre-lines.

The style will be as the axis of the world, sometimes parallel to the plane, sometimes perpendicular, sometimes cut the plane with oblique angles.

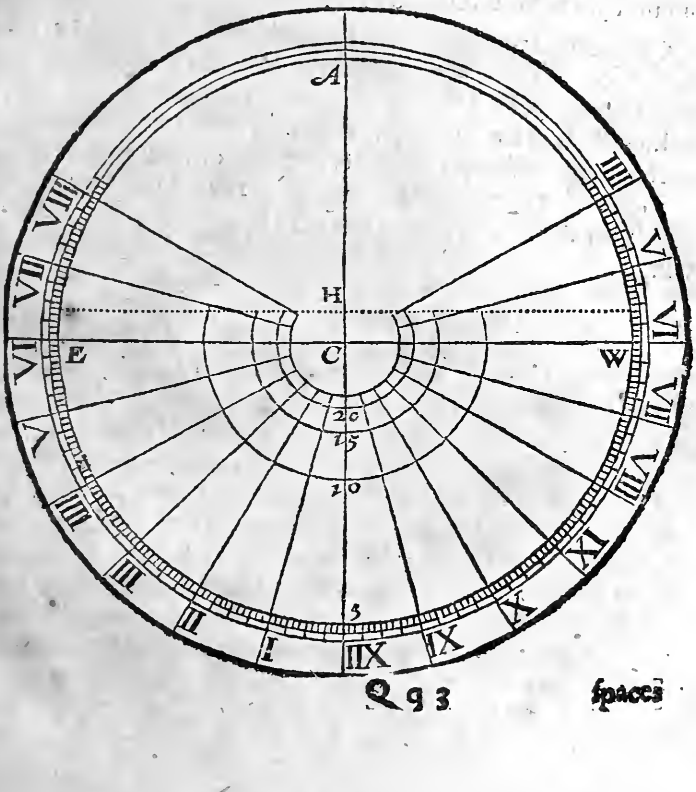
The houre-lines will be either parallell one to the other, or meete in a center with equall angles, or meete with vnequall angles. If the style be perpendicular to the plane, the angles at the center will be equall; and this falls out onely in the South and North face of an equinoctiall plane: if the style be parallel to the plane, the houre-lines will be also parallell one to another; and this falls out in all polar planes, as in the East and West meridian planes parallel to the circle of the houre of 12, in the vpper and lower direct polars parallell to the circles of the houre of 6; and in the vpper and lower declining polars which are parallel to any of the other houre circles.

But in the horizontall and all other planes, the ſtyle will cut the plane with an acute angle, and the houre lines will meet at the root of the ſtyle, and there make vnequall angles.

CHAP. I.

To draw the houre-lines in an equinoctiall Plane.

AN equinoctiall plane is that which is parallell to the equinoctiall circle here repreſented by *E A V*, wherein the



 ſpaces

ſpaces betweene the houre circles being equall, there is no need of further precept, but onely to draw a circle and to diuide it into 24 equall parts for the 24 houres, and ſubdiuide each houre into halues and quarters, and then to ſet vp the ſtyle perpendicular to the plane in the center of the circle. The help which theſe lines of proportion doe here affoord vs, is onely in the diuiſion of the circle, which may be done readily by that which I ſhewed before, *Pag.* 29.

For example, ſuppoſe the ſemidiameter of the equinoctiall circle to be ſix inches, and that it were required to know the diſtance of the houre-points each from other: here each houre being 15 *gr.* diſtant from other, I extend the compaſſes from the ſine of 30 *gr.* vnto the ſine of 7 *gr.* 30 *m.* the halfe of 15 *gr.* and I find the ſame extent to reach in the line of numbers from 6.00 vnto 1.56.

Or in croſſe worke I extend them from the ſine of 30 *gr.* vnto 6.00 in the line of numbers, the ſame extent will reach from the ſine of 7 *gr.* 30 *m.* vnto 1.56 in the line of numbers; which ſhewes that in a circle of ſix inches ſemidiameter, the diſtance of the houre-points each from other will be about 1 inch and 56 *centeſmes* or parts of 100. The like reaſon holds for the inſcribing of all other chords in the *Prop.* following.

CHAP

CHAP. II.

To draw the houre-lines in a direct polar plane.

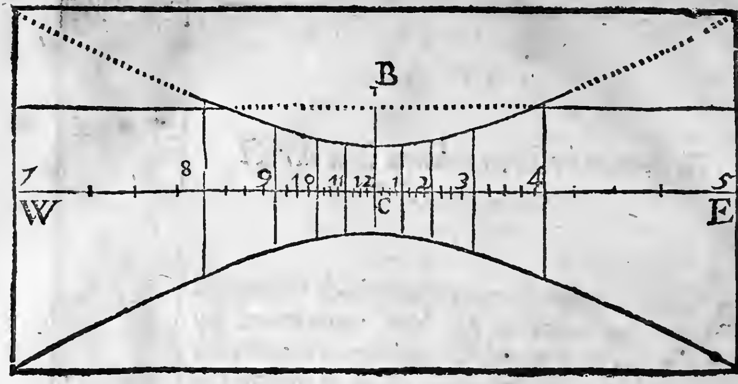
A Direct polar plane is that which is parallell to the houre of *6*, here represented by *E P* W, wherein the style will be parallell to the plane, and the houre-lines parallell one to the other, and therefore may be best drawne by that which I haue shewed in the vse of the *Sector.* They may be also drawne by the helpe of these lines of proportion, in this maner.

First draw a right line W E for the horizon and the æquator, and crosse it at the point C, about the midle of the line with C B another right line, which may serue for the meridian and the houre of 12, and must also be the substylar line wherein the style shall stand. Then, to proportion the style vnto the plane, consider the length of the horizontall line, and what houre-lines you would haue to fall on your plane.

For the distance of any one houre-line from the meridian being knowne, we may finde both the length of the style and the distance of the rest: because.

As the tangent of the houre giuen,
is to the distance from the meridian:
So the tangent of 45 *gr.*
to the height of the style.

Suppose



Suppose the length of the horizontall line to be 12 inches, and that it were required to put on all the houre-lines from 7 in the morning vnto 5 in the euening. Here we haue 5 houres and 6 inches on either side the meridian. Wherefore I allow 15 *gr.* for an houre, and extending the compasses from the tangent of 75 *degrees* I find the same extent to reach in the line of numbers from 6.00 to about 1.61. This shewes both the height of the style, and the distance of the houre-points of 9 and 3 from the meridian to be 1 inch, 61 parts.

To find the length of the Tangent betweene the substylar and the houre-points.

As the tangent of 45 *gr.*
 to the tangent of the houre:
So the height of the style
 to the length of the tangent line betweene the substylar and the houre-points.

Thus hauing found the length of the style in our example

ple to be 1. 61, if I extend the compasses from the tangent of 45 *gr.* vnto the tangent of 15 *gr.* the measure of the first houre from the substylar, I shall find the same extent to reach in the line of numbers from 1.61 vnto 0.43, for the length of the tangent betweene the substylar and the houre-points of 11 and 1. If I extend them from the tangent of 45 *gr.* vnto the tangent of 75 *gr.* the measure of the fift houre, I shall finde them to reach in the line of numbers from 1.61 vnto 6.00. for the length of the tangent from the substylar to the houre-points of 7 and 5. For howsoever it be the same distant in the line of tangents from 45 vnto 75, as from 45 vnto 15; yet because 75 are more, and 15 lesse then 45, the tangent lines that answer to them wil be accordingly more or lesse then the length of the style.

Ho.	An. Po Gr.	M.	Tang. In	Pa.
12	0	0	0	0
11. 1	15	0	0	43
10. 2	30	0	0	93
9. 3	45	0	1	61
8. 4	60	0	2	79
7. 5	75	0	6	00
6. 6	90	0	Infin.	

Againe, if I extend them from 45 *gr.* in the tangents vnto 30 *gr.* the measure of the second houre, I shall finde them to reach in the line of numbers from 1. 61 vnto 0. 93 for the houre of 10 and 2: if I extend them from the tangent of 45 *gr.* vnto the tangent of 60 *gr.* for the fourth houre, I shall find them to reach in the line of numbers from 1.61 vnto 2 79, and such is the length of the tangent line from the substylar vnto the houre of 8 and 4. And the like reason holdeth for the inscribing of all other tangent lines in the propositions following.

But for such tangents as fall vnder 45 *gr.* I may better vse crosse worke, and extend the compasses from the tangent of 45 *gr.* vnto 1.61 in the line of numbers, so shall I finde the same extent to reach from 30 *gr.* in the tangents, to 93 parts in the line of numbers, for the distance of the second houre, and from 15 *gr.* in the tangents to 43 parts for the distance of the first houre from the meridian.

Or if this extent from 45 *gr.* backward to 1.61 be too large for the compasses, I may extend them forward from the tangent of 5 *gr.* 43 *m.* to 1 61 parts in the line of numbers, & the same extent shall reach from 15 *gr.* in the tangents, to 43 parts in the line of numbers, for the distance of the first houre; and from 30 *gr.* to 93 parts, for the distance of the second houre, as before.

Hauing found the length of the tangent lines in inches and parts of inches, and pricked them in the æquator on both sides of the meridian, from the center *C*; if we draw right lines through each of those points, crossing the æquator at right angles, they shall be the houre-lines required; and if we set a style ouer the meridian, so as the edge of it be parallel to the plane, and the height of it be as much aboue the meridian as the distance between the meridian and the houre-points of 3 or 9, it shall represent the axis of the world, and be truly placed for the casting of the shadow vpon the houre-lines in a polar plane.

Hora	Ang Po Gr.	Ang Po M	Tang. In.	Tang. Pa.
6		0		0
	3	45	1	65
	7	30	1	32
	11	15	1	99
7	15	0	2	68
	18	45	3	39
	22	30	4	14
	26	15	4	93
8	30	0	5	77
	33	45	6	68
	37	30	7	67
	41	15	8	77
9	45	0	10	00
	48	45	11	40
	52	30	3	03
	56	15	14	97
10	60	0	17	32
	63	45	20	28
	67	30	24	14
	71	15	29	46
11	75	0	37	32
	78	45	50	27
	82	30	75	96
	86	15	152	57
12	90	0	Infin.	

CHAP. III.

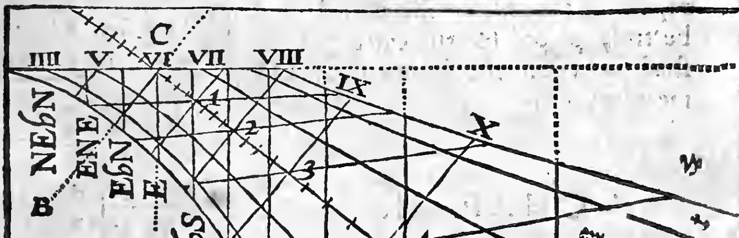
To draw the houre-lines in a meridian plane.

A Meridian plane is that which is parallell to the meridian circle in the fundamentall diagram represented by *S Z N*; it hath two faces, one to the East, and the other to the West; in each of them the style will be parallell to the

plane

plane, and the houre-line parallell one to the other, as in a polar plane, the difference being onely in the placing of the æquator and in numbring of the h ures.

For in these meridian planes hauing drawne on occult verticall line C Z, and an occult horizontall line C N, crossing one the other at right angles in the point C, the æquator A C will cut the verticall with an angle Z C A, equall to the latitude of the place: then may we crosse the æquator at right angles with the line C B for the houre of 6, and from this set off the houre-points in the æquator as in the former *Prop.*



For supposing the length of the style C B to be ten inches, the length of the tangent line belonging to the first houre wil be 2 *in.* 68 *p.* the length of the second 5 *in* 77 *p.* as in the Table. Then the tangent of 15 *gr.* being prickt downe in the æquator on both sides from 6, shal serue for the houres of 5 and 7, and the tangent of 30 *gr.* for the houres of 4 and 8, and so in the rest. This done, if we draw right lines through each of these points, crossing the æquator at right angles, they shal be the houre lines required: and if we set a style ouer the houre of 6, so as the edge of it may be parallell to the plane, and the height of it may be equall to the distance betweene the houres of 6 and 9 in the æquator, it shall represent the axis of the world, and be truly placed for the casting of the shadow vpon the houre-lines in a meridian plane.

Latitud	51			30
Ho.	Ang.Po		Arc.Pla	
	Gr.	M.	Gr.	M.
12	0	0	0	0
	3	45	2	56
	7	30	5	52
	11	15	8	51
1	15	0	11	50
	18	45	14	52
	22	30	17	57
	26	15	21	6
2	30	0	24	20
	33	45	27	36
	37	30	31	0
	41	15	34	28
3	45	0	38	3
	48	45	41	45
	52	30	45	34
	56	15	49	30
4	60	0	53	35
	63	45	57	47
	67	30	62	6
	71	15	66	33
5	75	0	71	6
	78	45	75	45
	82	30	80	25
	86	15	85	13
6	90	0	90	0

CHAP. III.

To draw the houre-lines in an horizontall plane.

AN horizontall plane is that which is parallell to the horizon, represented in the fundamentall diagram by the outward circle *ESWN*, in which the diameter *SN* drawne from the South to the North, may go both for the meridian line and the meridian circle, Z for the zenith, *P* for the pole of the world, and the circles drawne through *P* for the houre-circles of 1. 2. 3. 4. &c. as they are numbred from the meridian.

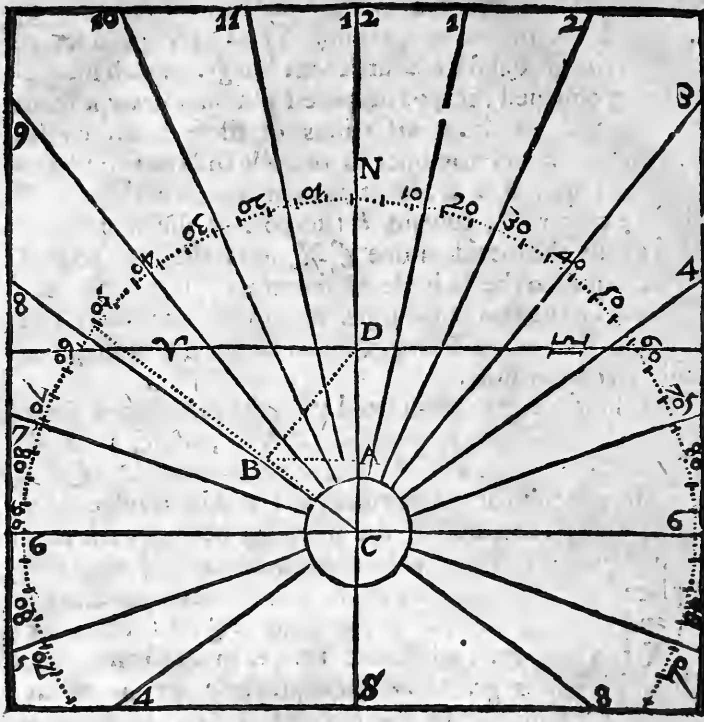
These

5 These are equall at the pole and at the æquator but vnequally distant at the horizon. the distance between the meridian and the first houre being not full 12 *gr.* the distance between the fift and sixth houre aboue 18 *gr.* which inequality bring obserued, if you suppose right lines drawne from the center *C* to the intersections of these houre-circles with the horizon, the lines so drawne shall be the houre-lines here inquired. And then if you can imagin a line drawne from the center *C*, toward *P* the pole of the world and raised aboue the meridian line *CN* so as the angle *PCN* may be equall to the latitude of the place, this right line *CP* shall be the axis of the style. And so you haue both style and houre-lines ready drawne to your hand. But more particularly to our purpose.

These houre-circles considered with the meridian and the horizon, doe make diuers triangles, *PN 1*, *PN 2*, *PN 3*, in which we haue knowne first the right angle at *N* the North intersection of the meridian and the horizon; secondly the side *PN*, the arke of the meridian between the pole and the horizon, which is alwayes equall to the latitude of the place; thirdly the angles at the pole, made by the meridian and the houre-circles, the angle *NP 1* being 15 *gr.* *NP 2* 30 *gr.* each houre 15 *gr.* more then other, each halfe houre 7 *gr.* 30 *m.* each quarter 3. *gr.* 45 *m.* as in the second columne of this table. And these three being known. we may finde the arks of the horizon between the meridian and the houre-circles *N 1*, *N 2*, *N 3*, &c. For.

As the sine of 90 *gr.*
  is to the sine of the latitude:
So the tangent of the houre
  to the tangent of the houre line
  from the meridian.

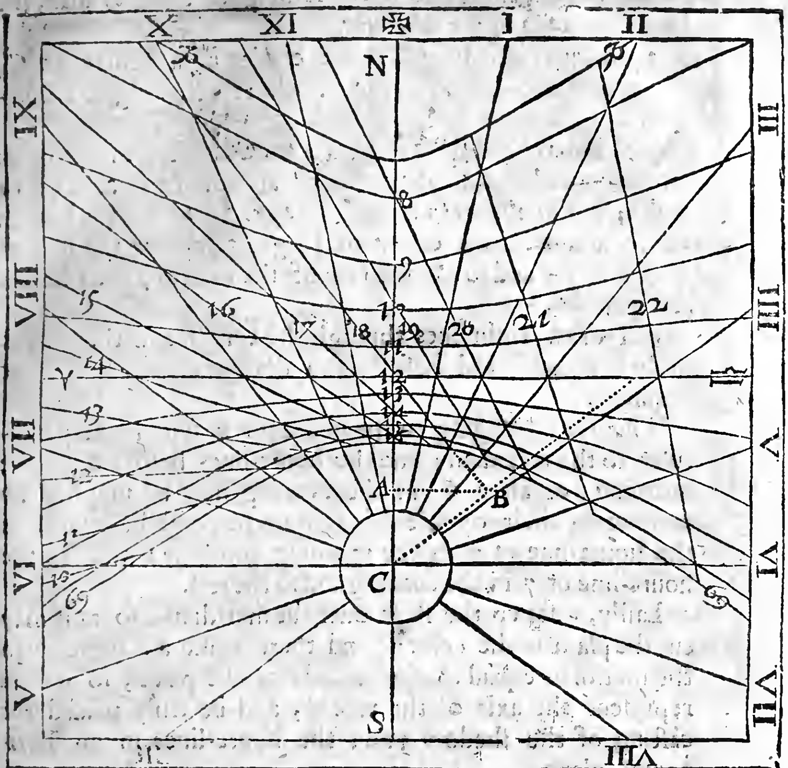
Extend the compasses from the sine of 90 *gr.* to the sine of the latitude, so the same extent shall reach from the tangent of the houre, to the tangent of the houre-line from the meridian.



meridian. Thus the latitude being 51 *gr.* 30 *m.* I extend the compasses from the sine of 90 *gr.* to the sine of 51 *gr.* 30 *m*, & find the same extent to reach from the tangent of 3 *gr.* 45 *m.* vnto the tangent of 2 *gr.* 56 *m.* for the distance of the first quarter from the meridian; and from the tangent of 7 *gr* 30 *m.* vnto the tangent of 5 *gr.* 52 *m.* for the halfe houre; and from the tangent of 11 *gr.* 15 *m.* to the tangent of 8 *gr.* 51 *m.* for the third quarter; and from the tangent of 15 *gr.* 0 *m.* vnto 11 *gr.* 50 *m.* for the first houre: and so the rest; as in the third columne of this table vnder the title of the arks of the plane.

Only when I come to set one foote of the compasses to

48 *gr.*



48 *gr.* 45 *m.* for the finding of a quarter past 3, the other foote will fall out of the line, and then I may either take out so much as is out of the line beyond 45 *gr.* and turne it backe into the line, and it will reach from 45 *gr.* to 41 *gr.* 45 *m.* or I may vse crosse worke, extending the compasses from the sine of 90 *gr.* to the tangent of 48 *gr.* 45 *m.* so the same extent wil reach from the sine of 51 *gr.* 30 m. to the tangent of 41 *gr.* 45 *m.* And such is the distance of the line of 3 houre $\frac{1}{4}$ from the meridian.

This done, I come to the Plane, and there according as the lines do fall in the fundamentall diagram, 1 I draw a right line *SN* seruing for the meridian, the houre of 12 and the substylar.

2 In this meridian I make choice of a center at *C*, and there describe an occult circle representing the horizon.

3 I find a chord of 11 *gr.* 50 *m.* and inscribe it into this circle on either side of the meridian for the houres of 11 and 1; in like maner, a chord of 24*gr.* 20 *m.* for the houres of 10 and 2; and a chord of 38 *gr.* 3. *m.* for the houres of 9 and 3; and so for the rest of the houres, their halues and quarters.

4 I draw right lines through the center and the termes of these chords, and these lines so drawne are the houre-line required.

The line belonging to the houre of 6 will be perpendicular to the meridian, and the houre-lines before 6 in the morning, or after 6 in the evening may be supplied by continuing their opposet houre-lines beyond the center. As the houre-line of 7 in the morning continued will be the houre-line of 7 in the euening and so the rest.

Lastly, I set vp the style ouer the meridian, so as it may cut the plane in the center, and there make an angle with the meridian equall to the latitude of the place, so it shall represent the axis of the world, and be truly placed for casting of the shadow vpon the houre-lines in an horizontall plane.

CHAP.

CHAP. V.

To draw the houre-lines in a verticall plane.

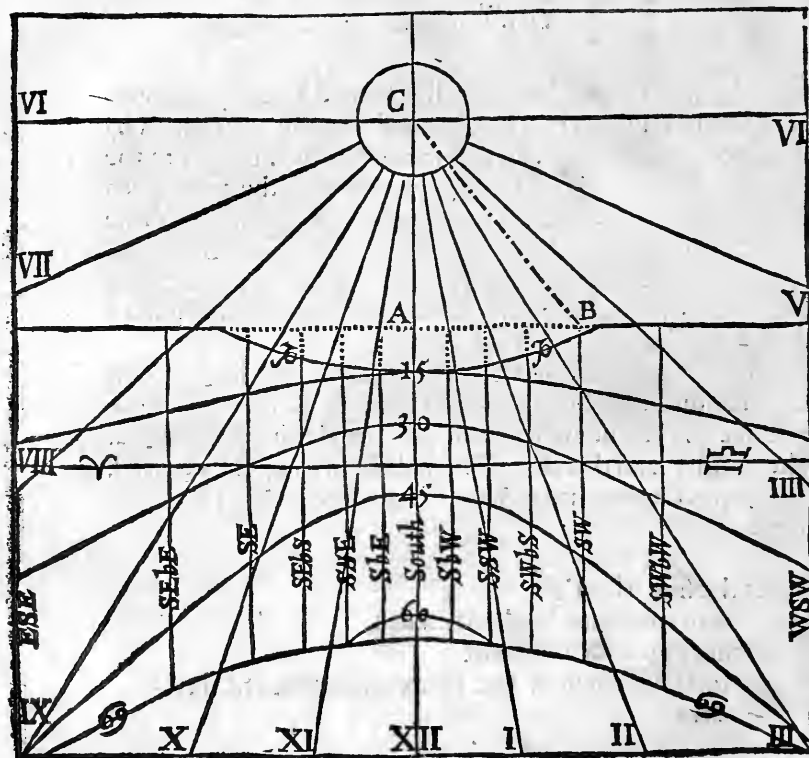
A Verticall plane is that which is parallel to the prime verticall circle in the fundamentall diagram represented by *E Z W*. It hath two faces, one to the North, the other to the South; in each of them the substylar will be the same with the meridian line, and the angle of the style aboue the plane will be equall to *Z P* the complement of the latitude and the houre-lines here inquired may be supplied by imagining right lines drawne from the center *C* to the intersections of the houre-circles with *E Z W*.

The triangles here considered are made by the verticall, the meridian, and the houre-circles, in which we know the side *Z P*, the angles at the pole, and the right angle at the zenith, and therefore may find the arks of the verticall, between the meridian and the houre-circles after this maner:

As the sine of 90 *gr*:
is to the cosine of the latitude:
So the tangent of the houre
to the tangent of the houre-line from the meridian.

Extend the compasses from the sine of 90 *gr*. to the sine of the complement of the latitude, so the same extent shal reach from the tangent of the houre, to the tangent of the houre-line from the meridian.

Thus in the latitude of 51 *gr*. 30 *m*. I extend the compasses from the sine of 90 *gr*. to the sine of 38 *gr*. 30 *m*. and find the same extent to reach from the tangent of 15 *gr.* to the tangent of 9 *gr.* 28 *m.* for the distance of the first houre from the meridian: and from the tangent of 75 *gr.* vnto the tangent of 66 *gr.* 42 *m* for the fift houre; and so in the rest as in the Table following.



These arks being knowne, I may come to the plane, and there by help of a thread and plummet draw a verticall line seruing both for the meridian and the houre of 12, and the substylar; then may I draw an occult verticall circle, and there in inscribe the chords of those former arks, and draw the

the houre-lines, and set vp the style, as before in the horizontall plane.

If it be the South face of the plane, the center will be vpward, and the style must point downward; if the North face, the center must be in the lower part of the meridian line, and the style-point vpward in all such places as are to the Northward of the equinoctiall line, as it may appeare by considering how the lines do fall in the fundamentall Diagram.

Latitud	51	30		
Ho.	Ang.Po		Arc.Pla.	
	Gr.	M.	Gr.	M.
12	0	0	0	0
	3	45	2	20
	7	30	4	41
	11	15	7	3
1	15	0	9	28
	18	45	11	56
	22	30	14	27
	26	15	17	4
2	30	0	19	54
	33	45	22	35
	37	30	25	32
	41	15	28	38
3	45	0	31	54
	48	45	35	22
	52	30	39	3
	56	15	42	58
4	60	0	47	9
	63	45	51	36
	67	30	56	20
	71	15	61	23
5	75	0	66	42
	78	45	72	17
	82	30	78	3
	86	15	84	0
6	90	0	90	0

CHAP. VI.

To draw the houre-lines in a verticall inclining plane.

ALl those Planes that haue their horizontall line lying East and West, are in that respect said to be verticall; if they be also vpright and passe through the zenith, they are direct verticals; if they incline to the pole-they are direct polars: if to the equinoctiall, they are properly called equinoctiall planes, and are described before: if to none of these three points, they are then called by the generall name of inclining verticals.

These may incline either to the North part of the horizon, or to the South; and each of them hath two faces,

 one

one to the zenith, the other to the nadir, in which we are first to consider the height of the pole aboue the plane, by comparing the inclination of the plane to the horizon, with the latitude of the place.

As in our latitude of 51 *gr.* 30 *m.* if the inclination of the plane *E I W* in the fundamentall diagram shall be 13 *gr.* Northward, that is, if *I N* the ark of the meridian between the plane and the North part of the horizon shall be 13 *gr.* we may take these 13 *gr.* out of *P N* 51 *gr.* 30 *m.* the eleuation of the pole aboue the horizon, and there wil remain *P I* 38 *gr.* 30 *m.* for the eleuation of the North pole aboue the vpper face of the plane, and therefore 38 *gr.* 30 *m.* for the height of the *S*outh pole aboue the lower face of the plane.

Or if the inclination of the plane shall be found to be 62 *gr.* to the Southward, we may number them in the meridian from *S* the South part of the horizon vnto *L*, and there draw the arke *E L W* representing this plaine; so the arke of the meridian *P L* shall giue the height of the North pole aboue the vpper face of this plane to be 66 *gr.* 30 *m.* and therefore the height of the South pole aboue the lower face of the plane is also 66 *gr.* 30 *m.*

In like maner if the inclination of the plane *E Y W* shall be 15 *gr.* Southward, that is, if *S Y* the arke of the meridian between the South part of the horizon and the plane, shall be 15. *gr.* The height of the North pole aboue the vpper face of the plane, and the height of the South pole aboue the lower face of the plane, will be also found to be 66 *gr.* 30. *m.*

But if the plane shall fall betweene the zenith and the North pole, then will the North pole bee eleuated aboue the lower face, and the South pole aboue the vpward face of the plane, as may appeare by the proiection of the spheare in the fundamentall Diagram.

Then in the triangles made by the plane, the meridian, and the houre-circles, we haue the side which is the height of the pole aboue the plane, together with the angles at the pole,

pole, and the right angle at the interſection of the meridian with the plane, by which we may find the arks of the plane betweene the meridian and the houre-circles, after this maner.

As the ſine of *90 gr.*
is to the ſine of the pole aboue the plane:
So the tangent of the houre
to the tangent of the houre-line from the meridian.

Thus in the former example, where *P I* the height of the pole aboue the plane was found to be 38 *gr.* 30 *m.* if you ſhall extend the compaſſes from the ſine of 90 *gr.* to the ſine of 38 *gr.* 30 *m.* the ſame extent will reach from the tangent of 15 *gr.* vnto the tangent of 9 *gr.* 28 *m.* for the diſtance of the firſt houre from the meridian, and from 30 *gr.* vnto 19 *gr.* 46 *m.* for the ſecond houre, and ſo forward as in the direct verticall.

And for the two laſt examples, you may extend the compaſſes from the ſine of 90 *gr.* vnto the ſine of 66 *gr.* 30 *m:* ſo the ſame extent ſhall reach in the line of tangents from 15 *gr.* vnto 13 *gr.* 48 *m.* for the firſt houre, from 75 *gr.* vnto 73 *gr.* 43 *m.* for the fift houre, from 30 *gr.* vnto 27 *gr.* 54 *m.* for the ſecond houre, from 60 *gr.* vnto 57 *gr.* 48 *m.* for the fourth houre, and from 45 *gr.* vnto 42 *gr.* 31. *m.* for the third houre from the meridian.

Theſe arkes being knowne, you may firſt draw the horizontall line, and croſſe it in the middle with a perpendicular that may ſerue both for the meridian and the houre of 12, and the ſubſtylar; then knowing which pole is eleuated aboue the plane, you may accordingly make choice of a fit point in the meridian for the center of your houre-lines, and thence deſcribe an occult arke of a circle, inſcribe the chords of thoſe former arkes, and draw the houre lines, and ſet vp the ſtyle, as I ſhewed before in the horizontall plane.

CHAP. VII.

To draw the houre-lines in an verticall declining Plane.

ALl vpright planes whereon a man may draw a verticall line, are in this respect said to be verticall; if they shall also stand directly East and West, they are direct verticals; if directly North and South, they are properly called meridian planes, and are described before: if they behold none of these foure principall parts of the wolrd, but shall stand between the prime verticall and the meridian, they are then called by the generall name of declining verticals.

These haue two faces, one to the South, the other to the Northward. which may be distinguished in these Northerne parts of the world after this manner. If the Sunne coming to the meridian shall shine vpon the plane. it is the South face; if not, it is the North face of that plane. Againe, if the Sunne shall shine vpon the plane at high noone, and yet longer in the forenone then in the afternoon, it is the Southeast face; if longer in the afternoone then in the forenoone, it is the Southwest face of the plane. But how much the declination cometh to, is best found as before.

When the declination is found, there be foure things more to be considered before we can come to the drawing of the houre-lines.

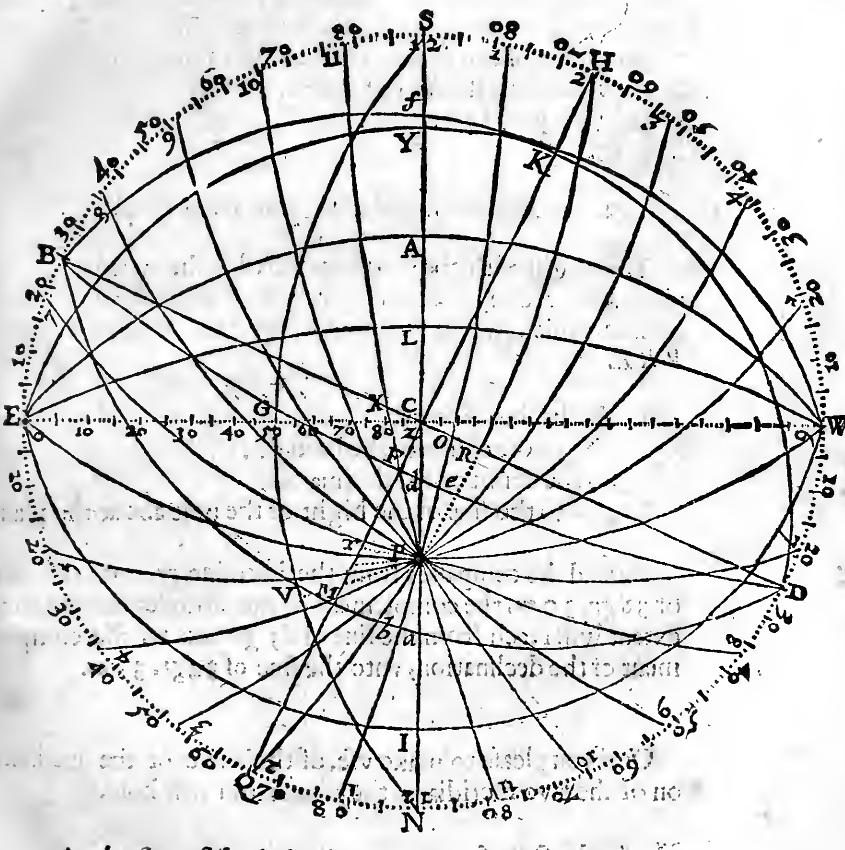
1 The meridian of the plane and his inclination to the meridian of the place.
2 The hight of the pole aboue the plane.
3 The distance of the substylar from the meridian line
4 The distance of each houre-line from the substylar.

And these foure may all be represented in the fundamentall *Diagram* as in this example.

Suppose that in our latitude of 51 *gr.* 30 *m.* northward the declination

declination of an vpright plane *vide Pag. 114. lin. 20.*

In the triangle P R Z we know the angle at R to be a right angle, and the angle at Z, for it is the complement of the declination, and the base P Z, for it is the complement of the latitude. And these three being knowne we may finde the other angle R P Z, which is the angle of inclination betweene both meridians.



As the sine of the latitude
is to the sine of 90 gr.

So the tangent of the declination
to the tangent of inclination of meridian.

Thus in our former example I extend the compasses from the sine of the latitude 51 *gr.* 30 *m.* vnto the sine of 90 *gr.* the same extent will reach in the line of tangents from 24 *gr.* 20 *m.* the declination giuen, to about 30 *gr.* and such is *Z P R* the angle of inclination between the meridian of the place and the meridian of the plane; and therefore the meridian of the plane will here fall vpon the circle of the second houre from the meridian of the place, (as it may also appeare by opening the compasses to the nearest extent, between the pole and the plane) and there I place the letter *R* to make this rectangle *P R* Z.

2 *To find the hight of the pole aboue the plane.*

The height of the pole is to measured in the meridian of the plane it is here represented by the arke *P* R, and may be found by that which we haue knowne in the former triangle *P R* Z.

As the sine of 90 *gr.*
to the cosine of the latitude:
So the cosine of the declination
to the sine of the hight of the pole aboue the plane

Extend the compasses from the sine of 90 *gr.* vnto the sine of 38 *gr.* 30 *m.* the complement of the latitude, and the same extent will reach from the sine of 65 *gr.* 40 *m.* the complement of the declination, vnto the sine of 34 *gr.* 33 *m.*

Or if you please to make vse, of the angle of the inclination of the two meridians, the proportion will hold.

As the sine of 90 *gr.*
to the cosine of the inclination of meridians:

So

So the cotangent of the latitude
to the tangent of the height of the pole aboue the plane.

And then you may extend the compaſſes from the ſine of 90 *gr.* vnto the ſine of 60 *gr.* the complement of the inclination of the meridians, and the ſame extent will reach from the tangent of 38 *gr.* 30 *m.* the complement of the latitude, vnto the tangent of 34 *gr.* 33 *m.* and ſuch is the arke *P R*, the hight of the pole aboue the plane.

3 *To find the diſtance of the ſubſtylar from the meridian.*

This is here repreſented by the arke Z R, and may be found by that which we haue knowne in the former triangle *P R* Z.

As the ſine of 90 *gr.*
to the ſine of the declination:
So the cotangent of the latitude
to the tangent of the ſubſtylar from the meridian.

Extend the compaſſes from the ſine of 90 *gr.* vnto the ſine of 24 *gr.* 20 *m.* the declination giuen, and the ſame extent will reach from the tangent of 38 *gr.* 30 *m.* the complement of the latitude, vnto the tangent of 18 *gr.* 8 *m.* and ſuch is the arke Z *R*, the diſtance of the ſubſtylar from the meridian.

4 *To find the diſtance of each houre-line from the ſubſtylar.*

The diſtances of the houre-lines from the ſubſtylar, are here repreſented by thoſe arks of the declining verticall belonging to the plane, which are intercepted betweene the proper meridian of the plane and the houre-circles.

To this purpoſe we haue diuers triangles made by the declining plane, together with his proper meridian and the houre-circles. In theſe we haue knowne, firſt the right angle at the interſection of the proper meridian with the plane; then the side which is the hight of the pole aboue the plane; and thirdly the angles at the pole. For knowing the angle of inclination betweene the meridian of the plane and the meridian of the place, which is alwayes the houre of 12, we may finde the angle betweene the meridian of the plane and the houre of 1, by allowing in 15 *gr.* and the angle betweene the meridian of the plane and the houre of 2 by allowing in 30 *gr.* and so for the rest, which being knowne and set down in a table we may find the arks of the plane from the substylar to the houre-circles, in this maner.

As the sine of *90 gr.*
to the sine of the hight of the pole aboue the plane:
So the tangent of the houre from the proper meridian,
to the tangent of the houre-line from the substylar.

Thus in our latitude of 51 *degrees* 30 *minutes*, if the declination of an vpright plane shall be found to be 24 *gr.* 20 *m.* from the prime verticall, the one face open to the Southwest, the other to the Northeast, I may number these 24 *gr.* 20 *m.* in the horizon of the fundamentall Diagram, from *E* vnto *B*, according to the situation of the plane, and there draw the verticall *B Z D*, which shall represent the plane proposed.

The two poles of this plane will fall in the horizon at *H* and *Q* and therefore the proper meridian drawne through the poles of the plane, and the pole of the world must be the circle *H P Q* which here crosseth the plane at right angles in the point *R*, and inclineth to *P Z S* the meridian of the place, according to the angle *R P Z*.

The quantity of this inclination may be readily found by the houre circle where the proper meridian falleth. As here it falleth on the second houre circle, and so the inclination is 30 *gr.*

The height of the pole above the plane which giueth the height of the stile aboue the substylar is here represented by the arke *P R*. For as in the Horizontall, so in this and all other

ther planes the line *C P* the axis of the world is alwaies the axis of the ſtile, and the neereſt line that can be drawne vpon the plane to the axis of the world is the fitteſt for the ſubſtylar, and that is the line *C R*, ſo the angle *P C R* is the angle betweene the axis and the plane, commonly called the height of the ſtyle and the meaſure of this angle is the arke *P R*, This arke is alwayes leſſe then the complement of the latitude, and may be eſtimated by taking the diſtance *P R* with the compaſſes, and meaſuring it in the Meridian from *P* toward Z. So in this example it will appeare to be about 34 *gr*. $\frac{1}{2}$.

The diſtance of the ſubſtylar from the meridian is here repreſented by the arke *Z* R. For the meridian line vpon the plane is *C Z*, the ſubſtylar line is *C R*, ſo the angle contained betweene them is *Z C R*, and the meaſure of this angle is the arke Z R, which taken with the compaſſes and meaſured in the ſemidiamiter *C W*, from *C* toward *W*, will be found about 18 *gr*.

The diſtances of each houre line from the ſubſtylar are here repreſented by the arks of the plane between the point R and the interſections of the houre circles. For the ſubſtylar line is *C* R, and the houre circle of 1 croſſing the plane in the point *O*, the houre line of 1 vpon the plane, muſt be *C O*, So the angle betweene the ſubſtylar and the houre line of 1 is R *C O*, and the meaſure of this angle is the arke R*O*. In like manner the houre line of 12 will be *C* Z, and the diſtance from the ſubſtylar R *Z*. The houre line of 11, will be *C X* and the diſtance from the ſubſtylar R *X* and ſo the reſt. Theſe diſtances R *O*, R Z, R *X*, &c. may alſo be taken with the compaſſes, and meaſured as before.

Beſides theſe foure repreſentations the diagrame will ſhew what pole is elevated above the plane, and what time the Sun ſhineth vpon the plane. If it be the North-Eaſt face of this plane, you may thinke P to be the North-pole, and the houre circles to be drawne on a convex hemiſphære, ſo *C* R the ſubſtylar, and C P the axis of the ſtile will both point vpward, and having drawne the tropique of ♋ you ſhall find by the meeting of the plane with the tropique,and the houre circles, that the Sun at the higheſt, may ſhine vpon the plane, from the time of the riſing untill it be paſt 9 in the morning, and from 7 in the Evenning unto the time of his ſetting. But if it be the South-weſt face of the plane,then you may either ſuppoſe the ſubſtylar,and the axis to be continued downe belowe the center, like unto the houres before and after 6 in an horizontall plane, or elſe you may turne the diagrame and thinke *P* to be the South pole, and the houre circles to be drawne in an horizontall concave ſo C R the ſubſtylar, C P the axis of the ſtile will both point downward, and ſo alſo the houre lines from 8 to the morning untill after 7 in the Evening, as it doth appeare by the meeting of the plane with the horizon, and the houre circles.

Thus with the drawing of one line in the diagram to repreſent the plane according to his declination, you may have the houre lines fitted to any declining verticall with the ſtyle and ſubſtilar in their due place, which may ſuffice to free you from groſſe error, but for more exactneſſe; wee conſider three triangles.

1 *To find the inclination of Meridians.*

The meridian of the place is a circle paſſing through the poles of the world, the Zenith and the nadir. The proper meridian of the plane is a circle paſſing through the poles of the world and the poles of the plane. The circle of the plane, and theſe two meridians doe make a triangle, ſuch as P R Z, wherein we know the angle at R.

I conſider the angle of inclination of the meridians R P Z,and there ſee how that P Z the meridian of the place,which is the houre of 12,being 30 *gr.* diſtant frō *P*R the meridian of

the

the plane, and that one face of the plane being open to the Southweſt, and the other to the Northeaſt, this meridian of the plane falleth to be the ſame with the houre of 2, (otherwiſe with the houre of 10:) therefore allowing 15 *gr.* for an houre, the houre of 1, *R P O* will be 15 *gr.* and *R P X* the houre of 11 will be 45 *gr.* diſtant from *P* R the proper meridian of the plane: and ſo I gather the incliation of the reſt of the hourecircles towards this meridian, according to their angles at the pole, as in the ſecond colume of this Table.

Latitude N.	51	30.
Declinatio:	24	20.
Diff. merid:	30	0.
Alt. Styl:	34	33.
Diſt. ſubſt.	18	8.

Then taking my compaſſes in my hand, I extend them from the ſine of 90 *gr.* vnto the ſine of 34 *gr.* 33 *m.* the hight of the pole aboue the plane, and find them to reach in the line of tangents from 15 *gr.* the inclination of the houre of 1, to 8 *gr.* 38 *m.* for the arke of 1, from the ſubſtylar, and from 30 *gr.* vnto 18 *gr.* 8 *m.* for the houre of 12, agreeable to the third *Prop.* & from 45 *gr.* vnto 29 *gr.* 33 *m.* for the houre of 11, and ſo the reſt, which I alſo ſet downe in the third columne of the Table.

Houre		Ang. Po		Ar. Pla.	
M.	E.	Gr.	M.	Gr.	M.
4	8	90	0	90	0
5	7	75	0	64	42
6	6	60	0	44	30
7	5	45	0	29	33
8	4	30	0	18	8
9	3	15	0	8	38
10	2	*Merid*		*ſubſtyl*	
11	1	15	0	8	38
	12	30	0	18	8
1	11	45	0	29	33
2	10	60	0	44	30
3	9	75	0	64	42
4	8	90	0	90	0

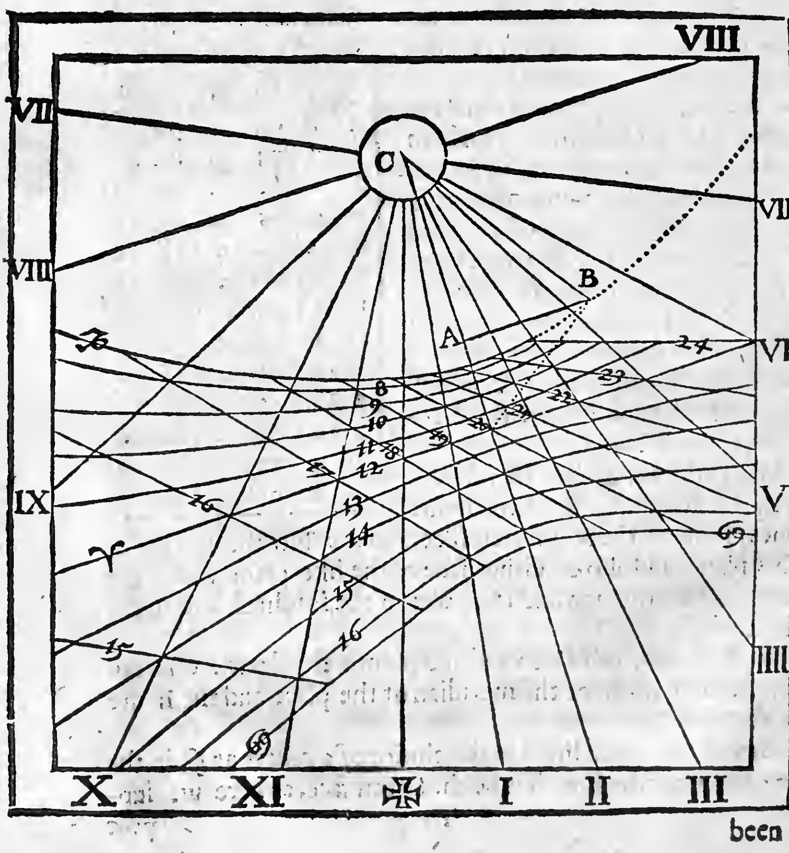
Theſe arks being thus found, will ſerue for the drawing of the houre lines, both on the Southweſt face, and the Northeaſt face of this plane, and alſo on either face of the like plane that hath the ſame declination and the poles in the ſoutheaſt and north weſt.

1 By the helpe of a thread and plummet I draw a verticall line, ſeruing both for the meridian of the place and the houre of 12.

In this meridian line I make choice of a center at *C*, in the vpper part of the line, if it be the South face, as here we ſuppoſe

poſe it, that the ſtyle may haue roome to point downward; but in the lower part of the line, if it be the North face of the plane; for there the ſtyle muſt point vpward: and vpon this center I deſcribe an occult circle, repreſenting the declining verticall belonging to the plane.

3 I find a chord of 18 *gr.* 8 *m.* the diſtance of the ſubſtylar from the meridian of the place, and inſcribe it into this circle, from the meridian vnto *A* toward the right hand, becauſe in this example the meridian of the plane falls among the houres after noone, (for otherwiſe it muſt have



been

been inſcribed toward the left hand) and there I draw the line *C A* ſeruing for the ſubſtylar.

4 According to the Table of the arkes of the plane from the ſubſtylar, I find a chord of 8 *gr.* 38 *m.* and inſcribe it into this circle, from the ſubſtylar toward the meridian, for the houre of 1. In like maner a chord of 29 *gr.* 23 *m.* for the houre of 11, and a chord of 44 *gr.* 30 *m.* for the houre of 10, and ſo for the reſt of the houres, their halues and quarters.

5 I draw right lines through the center and the termes of theſe chords, and theſe lines ſo drawne are the houre-lines required.

Laſtly, I ſet vp the ſtyle over the ſubſtylar, ſo as it may cut the plane in the center, and there make an angle with the ſubſtylar of 34 *gr.* 33 *m.* according to the height of the pole above the plane; ſo it ſhall repreſent the axis of the world, and be truely placed for caſting of the ſhadow vpon the houre lines in this declining plane.

A ſecond example.

Suppoſe another vpright plane in the ſame latitude to decline from the verticall 65 *gr.* 44 *m.* with one face open to the South-Eaſt, the other to the North-weſt. Theſe 65 *gr.* 40 *m.* would be numbred from E unto *Q*, and from *W* unto *H*, and the plane repreſented by *Q Z H*. For ſo the one pole will fall at *B* in the South-Eaſt, and the other at *D*, in the North-weſt according to the ſuppoſition. The proper meridian of this plane may be ſupplyed by the circle *B P D*, croſſing the plane in the point *T*, betweene the houre of 7 and 8, and there is the place of the ſubſtylar. The South-Eaſt face will containe all the houres from Sun riſing vnto two afternoone, and the Northweſt face all the houres from one after noone vnto Sunne ſetting. Then working as before.

1 The angle ZPT the inclination of the two meridians

ridians will be found to be abont 70 *degrees* 30 *minutes.*

2 The arke P T the measure of the angle P C T, the hight of the pole aboue the plane, and so the hight of the style aboue the substylar will be 14 *gr.* 51 *m.*

3 The arke Z T the measure of the angle *Z* C T, shewing the distance of the substylar from the meridian will be 35 *gr.* 56 *m.*

4 The arks of the plane betweene the substylar and the houre lines depending on the difference of meridians which is here 70 *gr.* 30 *m.* or 4 *Ho.* 42 *m.* short of the meridian I first draw a table with three columnes, one for the morning and euening houres, another for the angles at the pole and the third for the arks of the plane and there write 70 *gr.* 30 *m.* by the houre of 12 and place the meridian and substylar between the houres of 7 and 8 according as the poles of the plane do fall in the Diagram.

Latitude N.				51	30
Declination.				65	40
Diff. merid.				70	30
Altitude styl:				14	51
Diff. substyl:				35	56
Hours		An.Po		Ar. Pla.	
M.	E.	Gr.	M.	Gr.	M.
2	10	79	30	54	12
3	9	64	30	28	16
4	8	49	30	16	42
5	7	34	30	10	0
6	6	19	30	5	11
7	5	4	30	1	9
		Merid		*substy*	
8	4	10	30	2	43
9	3	25	30	6	58
10	2	40	30	12	21
11	1	55	30	20	28
12		70	30	35	56
1	11	85	30	72	56

Then will the angle at the pole betweene the proper meridian and the houre of 11 be 55 *gr.* 30 *m.* the houre of 10 will be 40 *gr.* 30 *m.* distant from that meridian and the rest in their order which being noted in the second columne, the arks of the plane will be found to be such as I haue noted in the third columne.

With this table thus made, you may draw the houre-lines and set vp the style on either face of this or the like plane, the difference being onely in the placing of the substylar and that is resolued by the sight of the Diagram.

A third example of a Plane falling neere the Meridian.

After the like manner if in our latitude an vpright plane shall decline 85.*gr.* from the prime verticall, the one face of it being open to the Northwest, and the other to the Southeast, we may in some sort represent it by the verticall *Q Z H*, and then working as before.

1 The angle *Z P T*, the inclination of the two meridians will be found to be 86 *gr.* 5 *m.* so that *P T* the meridian of this plane, will here fall betweene the houre-circles of 6 and 7 from the meridian.

2 The arke *P T* the measure of the angle *P C T*, the height of the pole aboue the plane will be onely 3 *gr.* 6 *m.*

3 The arke *Z T* the measure of the angle *Z C T*, the distance of the substylar from the meridian 38 *gr.* 23 *m.*

Latitude	51	30
Declination	85	0
Diff. Merid.	86	5
Altitude styl.	3	6
Dist. substy.	38	23

4 The Table of the angles at the pole will be also gathered, by comparing the meridian of the plane with the rest of the houre-circles. For the angle *T P Z* betweene *P T* the meridian of the plane, *P Z* the meridian of the place, and the houre of 12, being 86 *gr.*

 5 *m.* allow-

5 *m.* allowing 15 *gr.* for an houre, the houre of $11\frac{1}{2}$ will be 78 *gr.* 35 *m.* and the houre of 11 71 *gr.* 5 *m.* distant from the meridian of the plane; and so the rest of the houres. Or because the difference of meridians 86 *gr.* 5 *m.* resolved into time makes 5 houres, 44 *m.* and so the meridian of the plane falls betweene the houres of 6 and 7 from the meridian. I first place this meridian betweene these houres, and then taking 75 *gr.* the common measure for 5 houres out of 86 *gr.* 5 *m.* there remaine 11 *gr.* 5 *m.* for the angle at the pole betweene the meridian of the plane and the houre of 7. againe I take 86 *gr.* 5 *m.* out of 90 *gr.* the common measure for 6 houres, and there remaine 3 *gr.* 55 *m.* for the angle at the pole betweene the meridian of the plane and the houre of 6. To these angles so found I allow 15 *gr.* for euery houre, as in the second columne of this Table.

Hor.	An. Po		Ar. Pla.		C F		C G	
	Gr.	M.	Gr.	M.	In.	Par.	In.	Par
12	86	5	38	23	91	08	79	21
	78	35	15	3	30	92	26	89
11	71	5	9	6	18	42	16	02
	63	35	6	13	12	52	10	89
10	56	5	4	36	9	25	8	05
9	41	5	2	42	5	43	4	72
8	26	5	1	31	3	05	2	65
7	11	5	0	36	1	20	1	05
	Merid.		*Substy.*		0	0	0	0
6	3	55	0	13	0	44	0	38
5	18	55	1	4	2	15	1	86
4	33	55	2	5	4	18	3	64
3	48	55	3	33	7	13	6	20
2	63	55	6	20	12	77	11	10
	71	25	9	10	18	56	16	14
1	78	55	15	28	31	82	27	67
	86	25	40	55	99	67	86	68

Then hauing the height of the pole aboue the plane, and these angles at the pole; the arkes of the plane, betweene the substylar and the houre-circles, will bee found as in the third columne.

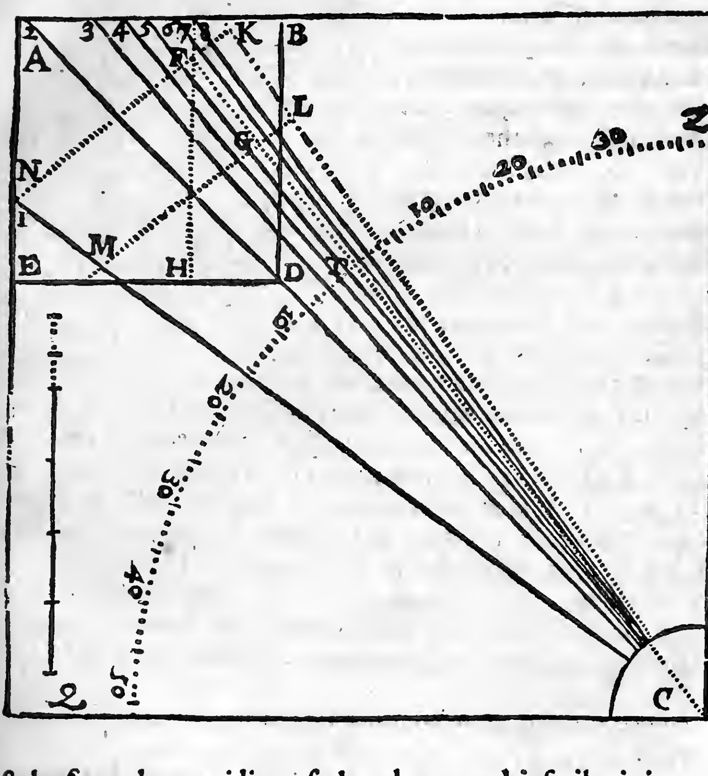
These arkes being found, will serue for the drawing of the houre-lines on either face of this or the like plane.

1 By the helpe of a thread and plummet I draw *Z C* a verticall line, seruing both for the meridian of the place and the houre of 12.

2 In this meridian line I make choice of a center in the vpper

vpper part of the line, if it had beene the Southerne face of the plane, but here in *C* the lower part of the line, because we supposed it to bee the Northwest face of the plane, and the style must point vpward; and vpon this center I describe an occult circle representing the declining verticall belonging to this plane.

3 I finde a chord of 38 *gr.* 23 *m.* the distance of the sub-



stylar from the meridian of the place, and inscribe it into this circle, from *Z* in the meridian, vnto *T* toward the left hand, according as the proper meridian *P T* falls in the fundamentall Diagram; and here I draw the line *C* T seruing for the substylar.

 4 The

4 The substylar being drawne, I may inscribe the chords of the arkes of the plane from the substylar, and draw the houre-lines, and set vp the style as in the former plane.

Or the arkes of the plane from the substylar being found as before, wee may draw the houre-lines vpon the plane otherwise then by chords. For hauing drawne the houre-lines as in the last figure, vpon paper or paist boord, we shall finde the most part of them, in this and such like planes that haue greater declination, to fall so close together, that they can hardly be discerned: wherefore to draw them at large to the best aduantage of the plane, I leaue out the center, and draw them by tangents, as in the polar plane.

1 I consider the length and bredth of the plane whereon I am to draw the houre-lines, which I suppose to be a square, whose side is 36 inches, and find that the little square *ABDE* will containe both the substylar and all those houre-lines which are required in the great square A Z C Q.

2 I draw two parallel lines F N, G M, crossing the substylar at right angles in the points F and G, so as they may best crosse all the houre-lines, and yet the one be distant from the other as farre as the plane will giue me leaue; and I finde by the sight of the figure that if *AB* the side of the lesser square sholl be 36 inches, the line C F will be about 115 inches, and the line C G about 100 inches, and therefore F G 15 inches. Againe, that the point F will fall about 6 inches below the vpper horizontall side *AB*, and about 12 inches from the next verticall side *BD*; for I need not here stand vpon parts.

3 Because these two parallel lines are tangent lines in respect of circles drawne vpon the semidiameters C F, C G, and such tangent as belong to the arkes of the plane, being tweene the substylar and the houre-lines, the proportion will hold,

As the tangent of 45 *gr.*
to the tangent of the arke of the plane:
So the length of the semidiameter
to the length of the tangent line.

As

As for example, the arke of the plane betweene the substylar and the houre of 1, is 15 *gr.* 28 *m.* in the former Table, the semidiameter *C* F 115 inches, and the semidiameter *C G* 100 inches: wherefore I extend the compasses from the tangent of 45 *gr.* vnto the tangent of 15 *gr.* 28 *m.* the same extent will reach from 115 in the line of numbers vnto 31, 82, which shewes the length of the tangent line betweene F in the substylar and the houre-line of 1, to be 31 inches, 82 *cent.* or parts of 100. Againe, the same extent will reach from 100 vnto 27, 67; and such is the length of the lesser tangent from *G* to the houre of 1.

The like reason holds for the length of the other tangents from the substylar to the rest of the houres, as in the Table; as also for the height of the style aboue these tangent lines; and so the angle of the style aboue the plane being 3 *gr.* 6 *m.* the height F K will be found to be 6 inches 23 *cent.* and the height G *L* 5 inches 42 *cent.*

Where the Reader may obserue, that if the extent from the tangent of 45 *gr.* to the tangent of 3 *gr.* 6 *m.* or to 115 in the line of numbers, be too large for his compasses, hee may vse the tangent of 5 *gr.* 43 *m.* in stead of the tangent of 45 *gr.* as I noted before *Pag.* 100.

4 Hauing found these lengths and heights, and set them downe in a Table, I come to the plane here resembled by the lesser square *A B D E*, where I begin with an occult verticall F *H*, about 12 inches from the side *B D*, and vpon the center F, abount 6 inches below the side *A B* describe an occult arke of a circle.

5 Into this arke I first inscribe a chord of 38 *gr.* 23 *m.* the distance of the substylar from the meridian, to make the angle *H* F G equall to the angle *Z C* T; so the line F G shall be the substylar: and then another chord of 51 *gr.* 37 *m.* the complement of this distance, to make vp the right angle G F N; so the line F N shall be the greater of the two tangent lines before mentioned.

6 I set off 15 inches from F vnto G, toward the center, and

and through *G* draw the lesser tangent line *G M* parallel to the former.

7 These two occult tangent lines being thus drawne, I looke vnto the former Table for the houre of 1, and there finde the arke of the plane betweene the substylar and the houre of 1, to be 15 *gr.* 28 *m.* and the length belonging to it in the greater tangent line to bee 31 inches, 82 *cent.* in the lesser tangent line 27 inches, 67 *cent*: wherefore I take out 31 inches 82 parts, and pricke them downe in the greater tangent from *F* to *N*, and then 27 inches 67 parts, and prick them downe in the lesser tangent from *G* to *M*, and draw the line *M N* for the houre of 1, which if it were produced would crosse the substylar *F G* in the center *C*, and there make the angle F *C N* 15 *gr.* 28 *m.* The like reason holdeth for the drawing of all the rest of the houre-lines.

Lastly, I set vp the style right ouer the substylar, so as the height *F K* may be 6 inches 23 *cent.* and the height *G L* 5 inches 42 *cent.* then shall *K L* represent the axis of the world, and if it were produced would crose the substylar F G in the center C, and there make the angle F C K to bee 3 *gr.* 6 *m.* and so be truly placed for casting of the shadow vpon the houre-lines in this declining plane.

CHAP. VIII.

To draw the houre-lines in a meridian inclining Plane.

ALl those planes wherein the horizontall line is the same with the meridian line, are therefore called meridian planes; if they be right to the horizon, they are called by the generall name of meridian planes without farther addition, and are described before; if they leane to the horizon, they are then called meridian incliners.

These

These may incline either to the East part of the horizon, or to the West, and each of them hath two faces, the vpper toward the zenith, the lower toward the Nadir, wherein knowing the latitude of the place, and the inclination of the plane to the horizon, we are to consider.

1 The inclination of the meridian of the plane to the meridian of the place.
2 The height of the pole about the plane.
3 The distance of the substylar from the meridian.
4 The distance of each houre-line from the substylar.

And all these foure are represented in the fundamentall Diagram, as in this example.

In our latitude of 51 *gr.* 30 *m.* a meridian plane inclineth Eastward 50 *gr.*; these 50 *gr.* I number in the verticall circle from *E* vnto *G*, according to the inclination of the plane, and there draw the arke *S G N* representing the plane proposed. Againe I number 50 from *Z* vnto *K*, so the point *K* (being 90 *gr.* from the plane at *G*) shall bee the pole of this plane and the proper meridian of this plane may bee supplied by a circle drawne through *K* and *P*. This meridian doth here fall betweene the houres of 4 and 5, and crossing the plane at right angles in the point *V*, in the right line C *V* shall be the substylar, and the angle *P* C *V* the height of the style aboue the plane and right lines drawne from the center C to the intersections of the houre-circles with *S G N* shall bee the houre-lines here inquired. The lower face of the plane will containe all the houre-lines from sunrising vnto 11 in the morning, and the vpper face the houres from 9 in the morning vnto sun-setting. Then haue I a rectangle triangle *P V N*, wherein the base *P N* is the height of the pole aboue the North part of the horizon, and the angle *P N V* the complement of the inclination to the horizon; and these being knowne,

1 I may finde the angle *N P V* of inclination of the two meridians. For

As

As the cosine of the latitude
is to the sine of 90 *gr.*
So the tangent of inclination to the horizon,
to the tangent of inclination of meridians.

Extend the compasses from the sine of 38 *gr.* 30 *m.* the complement of the latitude, vnto the sine of 90 *gr.* the same extent will reach from the tangent of 50 *gr.* 0 *m.* the inclination of the plane to the horizon, vnto the tangent of 62 *gr.* 25 *m.* and such is the inclination of the meridian of the plane to the meridian of the place; which being resolued into time, doth giue about 4 houres and 10 *m.* from the meridian, for the place of the substylar among the houre-lines.

2 The height of the pole aboue the plane is here represented by the quantity of the arke of the proper meridian *P V*, betweene the pole and the plane, and may bee knowne by that which wee haue giuen in the former triangle *P, V N*. For

As the sine of 90 *gr.*
to the sine of the latitude:
So the cosine of the inclination to the horizon,
to the sine of the height of the pole aboue the plane.

Extend the compasses from the sine of 90 *gr.* vnto 51 *gr.* 30 *m.* the sine of the latitude, the same extent will reach from the sine of 40 *gr.* the complement of the inclination of the plane to the horizon, vnto the sine of 30 *gr.* 12. *m.*

Or as the sine of 90 *gr.*
to the cosine of inclination of meridians:
So the tangent of the latitude
to the tangent of the height of the pole aboue the plane.

Extend the compasses from the sine of 90 *gr.* vnto the tangent of 51 *gr.* 30 *m.* the latitude of the place, the same extent will reach from the sine of 27 *gr.* 35 *m.* the complement

of

of the inclination of the two meridians, vnto the tangent of 30 *gr.* 12 *m.* And ſuch is *P* V the height of the pole aboue the plane, and ſuch muſt bee the height of the ſtyle aboue the ſubſtylar.

3 The diſtance of the ſubſtylar from the meridian is here repreſented by *N* V the arke of the plane betweene the two meridians, and may be found by that which we haue giuen at the firſt in the former triangle *P* V *N*. For

As the ſine of 90 *gr.*
  to the ſine of the inclination to the horizon:
So the tangent of the latitude
  to the tangent of the ſubſtylar from the meridian.

Extend the compaſſes from the ſine of 90 *gr.* vnto the tangent of 51 *gr.* 30 *m.* the latitude of the place, the ſame extent will reach from the ſine of 50 *gr.* the inclination of the plane to the horizon, vnto the tangent of 43 *gr.* 55 *m.* And ſuch is the arke *N V* the diſtance of the ſubſtylar from the meridian.

4 The diſtances of the houre-lines from the ſubſtylar, are here alſo repreſented by thoſe arkes of the plane, which are here intercepted betweene the proper meridian and the houre-circles, and may bee found by that which we haue giuen in the triangles made by the plane, with his proper meridian and the houre-circles. For the angle at V, betweene the plane and the proper meridian, is well knowne to bee a right angle, and the ſide P V is the height of the pole aboue the plane, and the angles at the pole betweene the proper meridian and the houre-circles are eaſily gathered into a Table. The angle V P N betweene V P the proper meridian of the plane, and P N the generall meridian of the place being 62 *gr.* 25 *m.* the angle betweene the proper meridian and the

 circle

circle of the houre of 11, will bee 77 *gr.* 25 *m.* and the angle belonging to the houre of 1, 47 *gr.* 25. *m*, and so the rest of the angles at the pole. Then

As the sine of 90 *gr.*
to the sine of the pole aboue the plane:
So the tangent of the angle at the pole,
to the tangent of the houre-line from the substylar.

Wherefore I extend the compasses from the sine of 90 *gr.* vnto the sine of 30 *gr.* 12 *m.* the height of the pole aboue the plane, and I finde the same extent to reach in the line of tangents from 77 *gr.* 25 *m.* vnto 66 *gr.* 4. *m.* for the distance belonging to the houre of 11; and from the tangent of 62 *gr.* 25. *m.* to 43 *gr.* 55 *m.* for the houre of 12, as when I found the the distance of the substylar from the meridian. And so for the rest of the arks of plane betweene the substylar and the houre-circles, as in the Table.

Latitude 51 30.				
Inclination 50 0.				
Diff. Merid. 62 25.				
Alt. styli 30 12.				
Dist. substy. 43 55.				
Hor.	Ang. Po		Arc. Pla.	
	Gr.	M.	Gr.	M.
11	77	25	66	4
12	62	25	43	55
1	47	25	28	41
2	32	25	17	43
3	17	25	8	58
4	2	25	1	13
	Merid		*Substyl*	
5	12	35	6	26
6	27	35	14	44
7	42	35	24	48
8	57	35	38	23
9	72	35	58	3
10	87	35	85	12

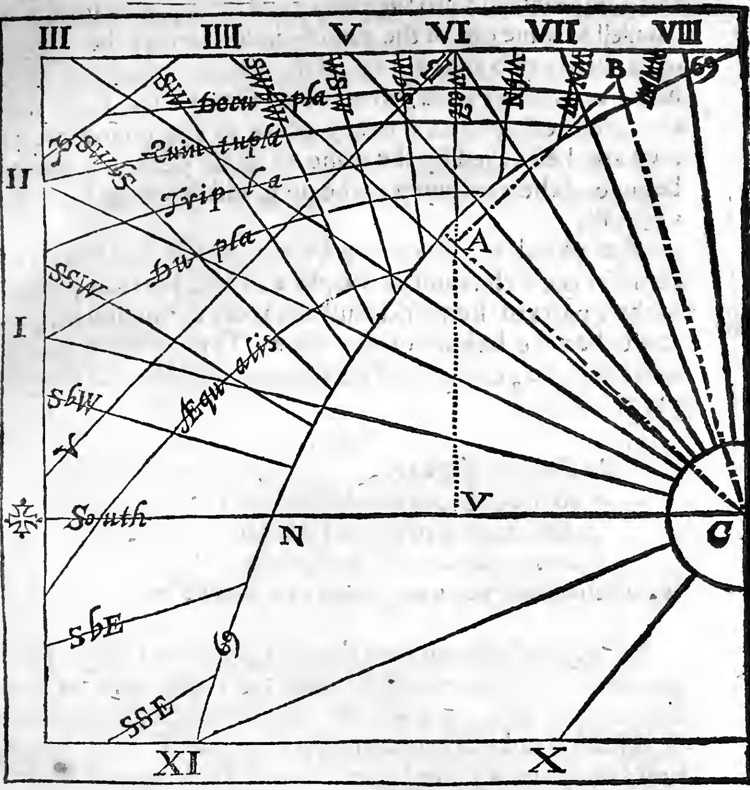
These arks being thus found, will serue to draw the houre-lines on either side of this plane: but supposing it to bee the vpper side,

1 I draw the horizontall line C N, seruing for the meridian and houre of 12.

2 In this line I make choice of a center at C, and thence describe an occult arke of a circle representing the plane proposed.

3 I find a chord of 43 *gr.* 55 *m.* the distance of the substylar from the meridian, and inscribe it into this circle from **N** vnto **A**, according as I finde the proper meridian P V to fall in the fundamentall diagram, and there I draw the line **C A**, seruing for the substylar.

4 The



4 The ſubſtylar being drawne, I may inſcribe the chords of the arkes of the plane from the ſubſtylar, and draw the houre-lines, and ſet vp the ſtyle, as in the former planes.

CHAP. IX.

To draw the houre-lines in a polar declining Plane.

THoſe planes wherein a line may be drawne parallell to the axis of the world, are called polar planes, becauſe that line pointeth vnto the poles, and these planes are always parallell to some one of the houre-circles. If they be parallell to the houre of 6, they are called direct polar planes; if to the houre of 12, they are called meridian planes; and both these are described before: if to any other of the houre-circles, they are then called by the name of polar declining planes, because of their inclining to the pole, and declining from the verticall.

These kind of planes may be knowne in this sort: First consider the inclination of the plane to the horizon, which in these parts of the world must alwayes be Northward, and more then the latitude of the place. Then find the declination from the verticall. These two being knowne, if the proportion hold,

As the sine of 90 *gr.*
to the cosine of the declination:
So the tangent of the inclination
to the tangent of the latitude;

it is then a polar declining plane, otherwise not.

For example, in our latitude of 51 *gr.* 30 *m.* a plane is proposed declining from the verticall 65 *gr.* 40 *m.* and inclining Northward 71 *gr.* 51 *m.* the vpper face being open to the Southeast, and the lower to the Northwest. If I number those 65 *gr.* 40 *m.* in the horizon of the fundamentall diagram from *E* vnto *Q*, and draw the line *H C Q*, it shall represent the horizontall line of the plane; then crossing it at right angles with the plane *B Z D* drawne through the zenith, I number 71 *gr.* 51 *m.* for the inclination from *D* vnto *R*, and there draw the circle *H R Q*, this circle so drawne shall represent the plane proposed; and because it also passeth through the pole, it is therefore a polar plane. But for farther triall I extend the compasses from the sine of 90 *gr.* to the sine of 24 *gr.* 20 *m.* the complement of the declination, and I find the same extent to reach from the tangent of 71 *gr.* 51 *m.* the inclination proposed, vnto the tangent of 51 *gr.* 30 *m.* which

is

is the true latitude of the place, and therefore it is a polar plane.

Againe I number the inclination 71 *gr.* 51 *m.* in the circle *B Z D* from Z vnto *M.* so this point *M,* will fall at the meeting of *B Z D* with the equator and being 90 *gr.* from the plane at *R,* it shall be the pole of this plane, and a circle drawnn through *M* and *P* will be the proper meridian of this plane. This meridian *M P* here falling on the houre of 8 doth giue *M P Z* the angle of inclination of meridians to be 4 houres or 60 degrees, then crossing the plane at the point *P* it shewes that the substylar should be *C P* and be placed at the houre of 8. But because *P* is the pole and *C P* the axis of the world, wherein all the houre circles doe meet, and so there would be no distinction betweene the axis, the substylar and the hourelines. I now suppose the plane in a parallell to the circle *H R Q* according to the distance that I would haue betweene the axis of the style and the substylar then will the style bee parallell to the plane pag. 128. lin. 1.

Here then the style will be parallell to the plane, and the houre-lines parallell one to the other, as in the meridian and direct polar planes. Yet that we may better know how to draw the houre-lines, and where to place the style, we are to consider

1 *The arke of the plane betweene the horizon and the pole.*

In a meridian plane the arke betweene the horizon and the pole which represents the arke betweene the horizon and the houre-lines, is alwayes equall to the latitude of the place; in a direct polar it is an arke of 90 *gr*; in these declining polars it is greater then the latitude, and yet lesse then 90 *gr.* This arke is here represented by *P Q* and may be knowne by resoluing the triangle *Q N P* or *P R Z.*

As the sine of 90 *gr.*
to the cosine of the latitude:
So the sine of the declination
to the cosine of the arke betweene the horizon and the Pole.

Extend the compasses from the sine of 90 *gr.* vnto the sine of 38 *gr.* 30 *m.* the complement of the latitude, the same extent will reach from the sine of 65 *gr.* 40 *m.* the declination proposed, vnto the sine of 34 *gr.* 34 *m.* whose complement is 55 *gr.* 26 *m.* the arke of the plane required betweene the horizon and the pole.

Or as the cosine of inclination to the horizon,
to the sine of 90 *gr.*
So the cotangent of the declination
to the tangent of the arke betweene the horizon and the pole.

And so extending the compasses from the sine of 18 *gr.* 9 *m.* the complement of the inclination to the tangent of 24 *gr.* 20 *m.* the complement of the declination the same extent doth reach from the sine of 90 *gr.* vnto the tangent of 55 *gr.* 26 *m.* And such is *QP* the arke of the plane betweene the horizon and the pole, the measure of the angle *QCP* betweene the horizontall line and the substylar.

2 *The inclination of the meridian of the plane, to the meridian of the place.*

The substylar in a direct polar plane is alwaies the same with the houre of 12. in a meridian plane it is the same with the houre-line of 6: in these declining polars it must be placed betweene 12 and 6, according to the inclination of the meridian of the plane to the meridian of the place, which is here

here represented by *M P Z* the complement of the angle *R P Z*, and thus knowne.

As the sine of 90 *gr.*
to the sine of the latitude :
So the tangent of the declination of the plane,
to the tangent of the inclination of meridians.

Extend the compasses from the sine of 90 *gr.* to the sine of 51 *gr.* 30 *m.* the latitude of the place, the same extent will reach from the tangent of 65 *gr.* 40 *m.* the declination proposed, vnto the tangent of 60 *gr.* and such is the angle of inclination betweene the meridian of the place and the proper meridian of the plane, which resolued into time doth make foure houres; and so the substylar must here be placed vpon the houre of 8 in the morning.

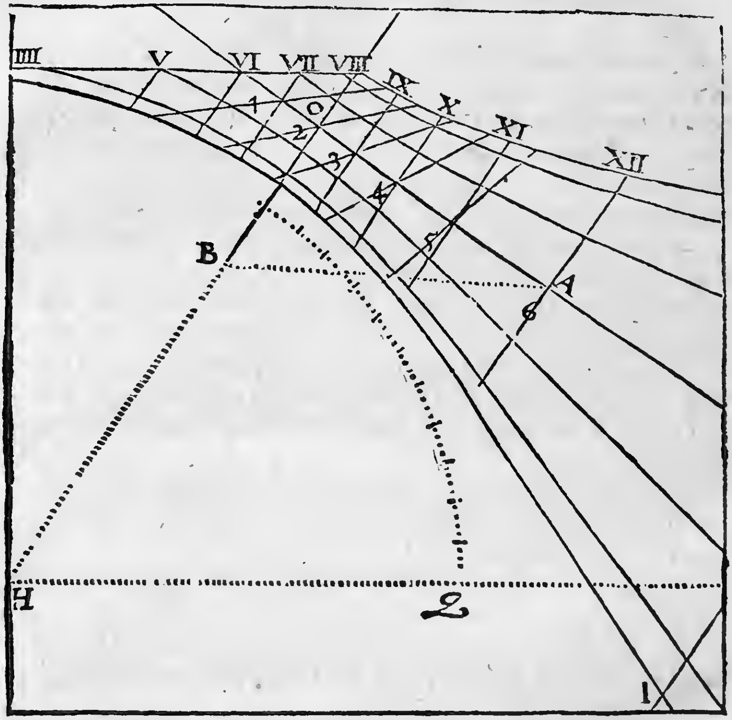
This angle being knowne, the rest of the angles at the pole are easily gathered. For if the houre of 12 be 60 *gr.* distant from the meridian of the plane, the houre of 1 will be 75 *gr.* and the houre of 11, will be 45 *gr.* distant, and the rest of the houres, as in the Table following. Then comming to the plane.

1 I draw an occult horizontall line *H Q*, wherein I make choice of a center *H*, and describe an occult circle for the horizon of the plane.

2 I find a chord of 55 *gr.* 26 *m.* and inscribe it into this circle, from *Q* vnto *B*, according to the situation of the plane; so the line *H B* shall be the meridian of the plane, and therefore the substylar: and the line *A C* crossing it at right angles, shall be the equator.

3 I consider the length of the plane, and how many houres I am to draw vpon it, that so I may proportion the height of the style; and I finde by the fundamentall diagram and the former table, that it will containe all the houres from Sun rising vntill it be past 1 afternoone: and therefore the meridian of the plane falling on the houre of 8 in the morning, there will be foure houres on the one side, and fiue on the

the other side of the substylar. But in all polar planes the height of the style aboue the substylar must be equall to the distance of the third houre from the substylar, or about $\frac{4}{7}$. of the fourth houre, or little more then $\frac{1}{4}$. of the fift houre, and thereupon I allow the height of this style to be equall to *C B*, which you may suppose to be ten inches.



4 Because the equator *A C* is a tangent line in respect of the Radius *B C*, and the parts thereof are such as belong to the angles betweene the meridian of the plane and the houre-lines, which angles are set downe in the table following, I may finde the length of each seuerall tangent in this manner.

As

As the tangent of 45 *gr.*
is to the tangent of the houre:
So the parts of the Radius,
to the parts of the tangent line.

The angle *A B C* betweene the meridian of the plane and the houre of 12, the meridian of the place is 60 *gr.* in the former table, and the Radius *B C* is supposed to be ten inches; whereupon I extend the compasses from the tangent of 45 *gr.* vnto the tangent of 60. *gr.* the same extent will reach from 10 in the line of numbers, vnto 17. 32, which shewes the length of the tangent *A C* betweene the substylar and the houre of 10, to be 17. 32 *cent.* The like reason holds for the rest of the houres.

Latitude		51		30
Declinat.		65		40
Inclinati.		71		51
Dist. Mer.		60		0
Dist. subst.		55		20
Hor.	An. Po.		Tangen	
	Gr.	M.	In.	Par.
4	60	0	17	31
5	45	0	10	00
6	30	0	5	77
7	15	0	2	68
8	*Merid*		*Substyl.*	
9	15	0	2	68
10	30	0	5	77
11	45	0	10	00
12	60	0	17	32
1	75	0	37	32
2	90	0	Infinit.	

5 These lengths being thus found and set downe in the table, I take out 17 inches 32 *cent.* and prick them in the equator from *C* vnto *A* for the houre of 12, and 37 inches 32 *cent.* and prick them downe for the houre of 1. And so the rest of the houre-points.

6 This done, if I draw right lines through each of these points, crossing the equator at right angles, they shall be the houre-lines required; and if I set the style ouer the substylar, so as the edge of it may be parallel to the plane, and the height of it be ten inches equall to the former Radius B C, it shall represent the axis of the world, and be truly placed for casting of the shadow vpon the houre-lines in this declining polar plane.

CHAP. X.

To draw the houre-lines in a declining inclining plane.

IF a plane ſhall decline from the prime verticall, and incline to the horizon, and yet not lie euen with the poles of the world, it is then called a declining inclining plane.

Of theſe there are ſeuerall ſorts; for the inclination being Northward, the plane may fall betweene the horizon and the pole, as the circle B *MD* in the fundamentall Diagram; or betweene the zenith and the pole, as B F D: or the inclination may be Southward, and ſo be repreſented by B K D, it may alſo fall either below the interſection of the meridian and the equator, or aboue it; and each of theſe haue two faces, the vpper toward the zenith, and the lower toward the nadir; wherein hauing the latitude of the place with the declination and inclination of the plane, we are farther to conſider,

1 The arke of the meridian betweene the pole and the plane.
2 The inclination of the plane to the meridian.
3 The arke of the plane betweene the horizon and the meridian.
4 The angle of inclination betweene both meridians.
5 The height of the pole aboue the plane.
6 The diſtance of the ſubſtylar from the meridian.
7 The diſtances of each houreline from the ſubſtylar.

And

And all these seuen may be represented in the fundamentall diagram, as in this example.

In our latitude of 51 *gr.* 30 *m.* a plane is proposed, declining from the verticall 24 *gr.* 20 *m.* and inclining Northward 36 *gr.* the vpper face lying open to the Southwest, the lower to the Northeast. If I number these 24 *gr.* 20 *m.* in the horizon from *E* to B, and there draw the line B C D, it shall represent the horizontall line of the plane: then crossing it at right angles with the plane *H* Z *Q* drawne through the zenith, I number 36 *gr.* for the inclination from *Q* vnto *M*, and there draw the circle B M D, crossing the meridian in the point *a*; this circle so drawne shall represent the plane proposed; and because it doth not passe through the pole, is therefore no polar, but an ordinary declining inclining plane.

1 The arke of the meridian of the place betweene the pole and the plane, is here represented by *P a*, and may be found by resoluing the triangle *D N a*, wherein the angle at *N* is knowne to be a right angle, the angle at *D* is the angle of inclination, the side D N the complement of the declination, which being knowne,

As the sine of 90 *gr.*
to the cosine of declination:
So the tangent of inclination to the horizon,
to the tangent of the meridian betweene the horizon and the plaine.

Extend the compasses from the sine of 90 *gr.* vnto the sine of 65 *gr.* 40 *m.* the complement of the declination, the same extent will reach from the tangent of 36 *gr.* the inclination proposed, vnto the tangent of 33 *gr.* 30 *m.* and such is the arke of the meridian *N a*, between the horizon and the plane. This arke *N a* being compared with the arke *N P*, which is the elevation of the pole aboue the horizon, and is here supposed to be 51 *gr.* 30 *m.* the difference *N a* commeth to 18. *gr.* and such is the of the meridian required betweene the pole and the plane.

2 The inclination of the plane to the meridian is here represented by the angle $N a D$, and may be found by that which we haue giuen in the former triangle $D N a$. For

As the sine of 90 *gr.*
to the sine of the declination from the verticall:
So the sine of inclination to the horizon,
to the cosine of inclination of the plane to the meridian.

Extend the compasses from the sine of 90 *gr.* vnto the sine of 24 *gr.* 20 *m.* the declination of the plane, the same extent will reach from the sine 36 *gr.* the inclination giuen, vnto the cosine of 76 *gr.* And such is $N a D$ the angle of inclination betweene the plane $D a$, and $N a$, the meridian of the place. Or

As the sine of the arke of the meridian betweene the horizon and the plane,
is to the sine of 90 *gr.*
So the cotangent of the declination
to the tangent of inclination of the plane to the meridian.

Extend the compasses from the sine of 33 *gr.* 30 *m.* the arke of the meridian betweene the horizon and the plane, vnto the sine of 90 *gr.* the same extent will reach from the tangent of 65 *gr.* 40 *m.* the complement of the declination vnto the tangent of 76 *gr.* And such is the inclination of the plane to the meridian, the same as before.

3 The arke of the plane between the horizon and the meridian, is here represented by $D a$, and may also be found by that which we have giuen in the former triangle $D N a$.

As the cosine of inclination to the horizon
is to the sine of 90 *gr.*

So the cotangent of the declination
to the tangent of the arke of the plane from the horizon to the meridian.

Extend the compasses from the sine of 54 *gr.* the complement of the inclination of the plane to the horizon, vnto the sine of 90 *gr.* the same extent will reach from the tangent of 65 *gr.* 40 *m.* the complement of the declination, vnto the tangent of 69 *gr.* 54 *m.* And such is *D a* the arke of the plane, betweene the horizon and the meridian of the place.

4 The inclination of meridians is here represented by the angle *a P b.* For hauing drawne the proper meridian *b P k*, or let down a perpendicular *P b* from the pole vnto the plane, this perpendicular shall be the meridian of the plane; and we shall haue another triangle *a b P*, wherein the angle at *b* is a right angle, because of the perpendicular, the angle at *a* is the inclination of the plane to the meridian of the place, and the side *P a*, is the arke of the meridian betweene the pole and the plane, which being knowne,

As the cosine of the arke of the meridian between the pole and the plane
is to the sine of 90 *gr.*
So the cotangent of the inclination of the plane to the meridian,
to the tangent of inclination of the meridian of the plane, to the meridian of the place.

Extend the compasses from the sine of 72 *gr.* the complement of the arke *P a*, betweene the pole and the plane, vnto the sine of 90 *gr.* the same extent will reach from the tangent of 14 *gr.* the complement of the inclination of the plane to the meridian, vnto the tangent of 14 *gr.* 41 *m.* And such is the angle *a P b* of inclination betweene the meridian of the place and the proper meridian of the plane, which resolued into time, doth make about 59 *minutes*, and so the substylar must here be placed neere the houre of 1, after noone.

5 The height of the pole aboue the plane is here represented by P *b*, the arke of the proper meridian betweene the pole and the plane, and may be found by that which we haue giuen in the triangle *a b* P. For

As for the sine of 90 *gr.*
to the sine of the meridian of the place betweene the pole and the plane:
So the sine of inclination of the plane to the meridian,
to the sine of the height of the pole aboue the plane.

Extend the compasses from the sine of 90 *gr.* vnto the sine of 18 *gr.* the arke P *a* of the meridian of the place from the pole to the plane, the same extent will reach from the sine of *b a* P the inclination of the plane to the meridian of the place, vnto the sine of 17 *gr.* 26 *m.* Or

As the sine of 90. *gr.*
to the cosine of inclination of meridians:
So the tangent of the meridian of the place betweene the pole and the plane,
to the tangent of the height of the pole aboue the plane.

Extend the compasses from the sine of 90 *gr.* vnto the sine of 75 *gr.* 19 *m.* the complement of *a P b* the inclination of the two meridians, the same extent will reach from the tangent of 18 *gr.* the arke *P a* of the generall meridian betweene the pole and the plane, vnto the tangent of 17 *gr.* 26 *m.* And such is *P b* the height of the pole aboue the plane; and such must be the height of the style aboue the substylar.

6 This distance of the substylar from the meritidian of the place, is here represented by *a b* the arke of the plane between the two meridians, and may be found by that which we had giuen at the first in the former triangle *a b P.* For

As

As the sine of 90 *gr.*
to the cosine of the inclination of the plane to the meridian:
So the tangent of the meridian of the place betweeme the pole and the plane,
vnto the tangent of the substylar from the meridian of the place.

Extend the compasses from the sine of 90 *gr.* vnto the sine of 14 *gr.* the complement of *b a P*, the inclination of the plane to the meridian, the same extent will reach from the tangent of 18 *gr.* the arke of the generall meridian betweene the pole and the plane, vnto the tangent of 4 *gr.* 30 *m.* And such is the arke of the plane betweene the two meridians; and such must be the distance from the houre of 12 to the substylar.

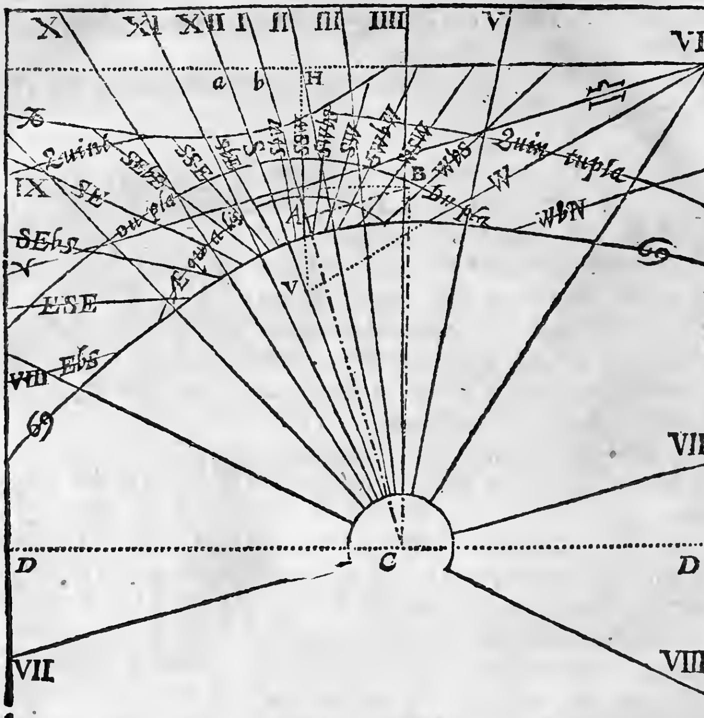
Latitude 51 30.				
Declina. 24 20.				
Inclin. N. 36 0.				
Alt. Merid. 69 54.				
Diff. Merid. 14 41.				
Alt. styli. 17 26.				
Dist. subst. 4 30.				
Hor.	Ang. Po		Arc. Pla.	
	Gr.	M.	Gr.	M.
7	89	41	88	57
8	74	41	47	35
9	59	41	27	9
10	44	41	16	31
11	29	41	9	41
12	14	41	4	30
	Merid		*Substyl*	
1	0	19	0	6
2	15	19	4	42
3	30	19	9	56
4	45	19	16	52
5	60	19	27	45
6	75	19	48	51

7 The distances of the houre-lines from the substylar, are here also represented by those arks of the plane, which are intercepted betweene the proper median and the houre-circles. For in these triangles the angle at *b* betweene the plane and the proper meridian is a right angle, the side P *b* is the height of the pole aboue the plane, and then the angles at the pole betweene the proper meridian and the houre-circles being gathered into a table.

As the sine of 90 *gr.*
to the sine of the pole aboue the plane:
So the tangent of the angle at the pole,
to the tangent of the houre-line from the substylar.

Extend

Extend the compasses from the sine of 90 *gr.* vnto the sine of 17 *gr.* 26 *m.* the height of the pole aboue the plane, the



same extent will reach from the tangent of 14 *gr.* 41 *m.* the angle at the pole belonging to the houre of 12, vnto the tangent of 4 *gr.* 30 *m.* for the arke of the plane betweene the substylar and the houre of 12; and from the tangent of 29 *gr.* 41 *m.* vnto the tangent of 9 *gr.* 41 *m.* for the houre of 11, and so for the rest of the arks of the plane between the substylar and the houre-lines, as in the former table.

These arkes being thus found, will serue for the drawing of the houre-lines on either side of the plane: but supposing it it to be the vpper side, I consider how the lines doe fall in the fundamentall diagram, and accordingly

1 I draw an occult horizontall line DD, wherein I make choice of the center *C*, and thence draw an occult circle for the horizon of the plane.

2 I finde a chord of *69 gr. 54. m.* the arke of the plane betweene the horizon and the meridian, and describe it into this circle from D vnto *a*, and there draw the line *Ca* for the houre of 12.

3 I finde a chord of *4 gr. 30 m.* the arke of the plane betweene the two meridians, and inscribe it into this circle from *a* vnto *b*, and there draw the line *Cb* for the substylar.

4 The substylar being drawne, I may inscribe the chords of the arkes of the plane from the substylar, and draw the houre-lines, and set vp the style as in the former planes.

A second example of a Plane falling betweene the pole and the zenith.

In like maner if in our latitude a plane be proposed declining from the verticall 24 *gr.* 20 m. as before, but inclining to the horizon 75 *gr.* 40 *m.* Northward, the vpper face being open to the Southwest, the lower to the Northeast, this plane shall be here represented by the circle B F D, crossing the meridian in the point *d*, betweene the pole and the zenith, and the proper meridian of this plane, by the perpendicular arke *P e.*

Then in this triangle D N *d* knowing the side D N the complement of the declination, with the angle of inclination to the horizon at D, and the right angle at N, these former Canons will giue *N d* the arke of the meridian betweene the horizon and the plane to be 74 *gr.* 20 *m*; and therefore *P d* the arke of the meridian betweene the pole and the plane will be 22 *gr.* 50 *m.* the angle *D d N* of the inclination of the plane to the meridian, will bee found to be *66 gr. 29 m.*

 and

and *D d* the arke of the plane betweene the horizon and the meridian 83 *gr.* 36 *m.*

Againe, in the triangle *P e d* knowing the ſide *P d* the arke of the meridian betweene the pole and the plane, with the angle of inclination to the meridian at *d*, and the right angle at *e*, the angle *d P e* of the inclination of the two meridians will be found to be 25 *gr.* 17 *m.* and *P e* the height of the pole aboue the plane to be 20 *gr.* 50 *m.* and *d e* the diſtance of the ſubſtylar from the meridian about 9 *gr.* 32 *m.*

Latitude	51	30
Declination	24	20
Inclination	75	40
Alt. Merid.	83	36
Diff. Merid.	25	17
Diſt. ſubſty.	9	32
Alti. Styl.	20	50

Laſtly, hauing found the height of the pole aboue the plane, and gathered the angles at the pole, the arks of the plane from the ſubſtylar to the houre-lines will be as in this table.

Hor.	Ang.Po. Gr.	M.	Arc.Pla. Gr.	M.
8	85	17	76	56
9	70	17	44	47
10	55	17	27	11
11	40	17	16	43
12	25	17	9	32
1	10	17	3	41
	Merid		*Subſtyl*	
2	4	43	1	40
3	19	43	7	16
4	34	43	13	50
5	49	43	22	46
6	64	43	37	0
7	79	43	62	58

This done, if we conſider how the lines doe fall in the fundamentall diagram, wee may there ſee how the North pole is eleuated aboue the lower face, and the South pole aboue the vpper face of the plane, and accordingly make choice of a center, draw the horizontall, the meridian, the ſubſtylar, and the hourelines, and ſet vp the ſtyle as in the other planes.

A third example of a Plane inclining to the Southward.

If in our latitude a plane were propoſed declining from the verticall 24 *gr.* 20 *m.* as before, but inclining to the horizon 14 *gr.* 20 *m.* Southward, the vpper face being open to the Northeaſt, the lower to the Southweſt, this plane ſhall be here repreſented by the circle *B K D* croſſing the meridian in the point *f* betweene the æquator and the horizon, and the proper meridian of this plane by the perpen-

dicular

dicular arke *P g* let downe from the pole to the plane, neere the houre of 11, at the North part of the horizon, as may partly appeare by the neerest extent of the compasses, if the circle *BKD* were drawne round; and the two letters *f* and *g* supplied.

Then in the triangle *B Sf*, knowing the side *B* S the complement of the declination, with the angle of inclination to the horizon at *B*, and the right angle at *S*, we may find *S f* the arke of the meridian betweene the horizon and the plane to be 13 *gr.* 6 *m.* And therefore *P f* the arke of the meridian betweene the pole and the plane to the Southward 115 *gr.* 24 *m.* but 64 *gr.* 36 *m.* to the Northward, the angle *B f S* or *D f N* of the inclination of the plane to the meridian, will bee found 84 *gr.* 9 m; and *B f* or *D f* the arke of the plane between the horizon and the meridian 66 *gr.* 20 *m.*

Latitude	51	30
Declination	24	20
Inclination	14	20
diff. merid.	13	27
dist. substy.	12	8
Alt. Styl.	64	0
Alt. merid.	66	20

Ho.	Aug. Po. Gr.	M.	Arc. Pla. Gr.	M.
6	76	33	75	6
7	61	33	58	56
8	46	33	43	30
9	31	33	28	55
10	16	33	14	58
11	1	33	1	25
	Merid		*Substyl*	
12	13	27	12	8
1	28	27	25	57
2	43	27	40	23
3	58	27	55	38
4	73	27	71	41
5	88	27	88	15

Againe, in the triangle *P g f* knowing the side *P f* the arke of the meridian betweene the pole and the plane, with the angle of inclination to the meridian at *f*, and the right angle at *g*, the angle *f P g* of the inclination of the two meridians will be found to be 13 *gr.* 72 *m.* and *P g* the height of the pole aboue the plane, about 64 *gr.* and *f g* the distance of the substylar from the meridian 12 *gr.* 8 *m.*

Hauing found the height of the pole aboue the plane, and gathered the angles at the pole, the arkes of the plane from the substylar to the houre-lines will be found as in this table.

This done, if we consider how the lines doe fall in the fundamentall diagram, we may there see how the North pole is eleuated aboue the vpper face, and the South pole aboue the lower face of this plane, and accordingly make choice of the center, draw the horizontall, the meridian, the substylar, and the houre-lines, and set vp the style as in the former planes.

CHAP. XI.

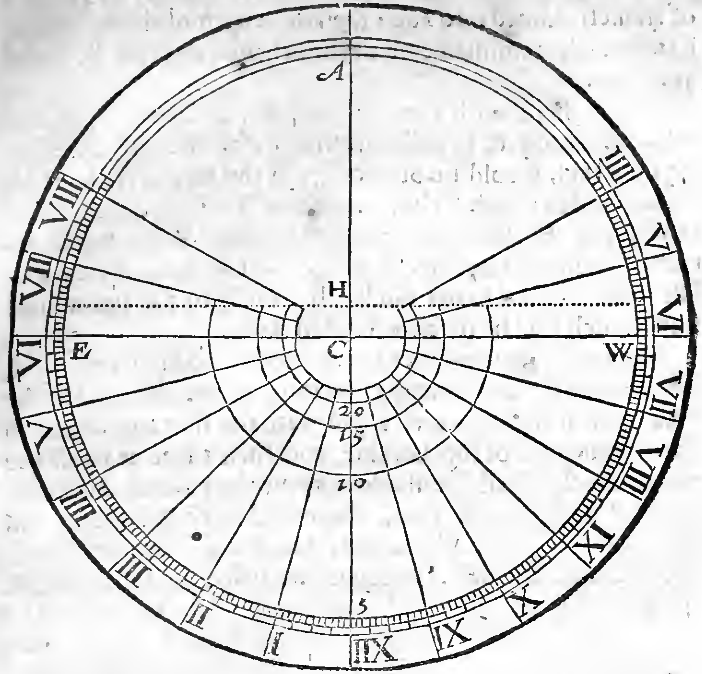
To describe the Tropiques and other circles of declination in an æquinoctiall Plane.

SVch circles as are parallell to the æquinoctiall, and yet fall within the tropiques, may be described on any plane by help of these lines of proportion, but after a different maner, according as the style shall be either perpendicular, or parallell to the plane, or cut the plane with oblique angles.

In an æquinoctiall plane where the style is perpendicular to the plane, the tropiques and other circles of declination will bee perfect circles: wherefore consider the length of the style in inches and parts, and the declination of the circle which you intend to describe in degrees and minutes, the proportion will hold.

As the tangent of 45 *gr.*
to the length of the style:
So the co-tangent of the parallell,
to the semidiameter of his circle.

Suppose the length of the style aboue the plane to bee 10 inches, and that it were required to finde the semidiameter of the tropique, whose declination is knowne to be 23 *gr.* 30 *m*: extend the compasses from the tangent of 45 *gr.* vnto the tangent of 66 *gr.* 30 *m.* the same extent will reach in the line of numbers from 10 vnto 23, which shewes the semidiameter of the tropique to be 23 inches. So if the declination bee 29 *gr.* the semidiameter will bee 27 inches 47. *cent*; if 15 *gr.*



gr. then 37. 32; if 10 gr. then 56. 71; if 5. gr. then 114. 305. and so in the rest.

Or if it were required to proportion the style to the plane,

> As the tangent of 45 gr.
> to the tangent of the declination:
> So the semidiameter of the plane,
> to the length of the style.

As if the semidiameter of the greatest parallell vpon the plane were but six inches, and that parallell should be the fift degree of declination: extend the compasses from the taugent of 45. gr. vnto the tangent of 5 gr. the same extent will reach in the line of numbers from 6. 00 vnto about 0. 53,

 which

which shewes that the length of the style must be 53 parts of an inch diuided into 100; then the length of the style being knowne, the semidiameter of the other circles will be found as before.

I begin here with the fift parallell, and thence proceed vnto the tropique, because the shadow of the rest neere the æquinoctiall, would be ouerlong, and the æquinoctiall it selfe cannot be described. The parallels of North declination are to be set on the North face, and the parallels of South declination on the South face of the plane. Neither need these parallels to be drawne in full circles, but onely to the horizontall line, which shall be described in *Cap.* xviij.

Hauing by these meanes set vp the style to his true height, and drawne the circles of declination, if we shall place the plane so as it shall make an angle with the horizon equall to the complement of the latitude, and then turne it vntill the top of the style cast the shadow vpon the parallel of declination belonging to the time, the meridian of the plane will shew the meridian of the place, and the shadow of the style the houre of the day, without the helpe of a magneticall needle.

CHAP. XII.

To describe the Tropiques and other circles of declination in a polar Plane.

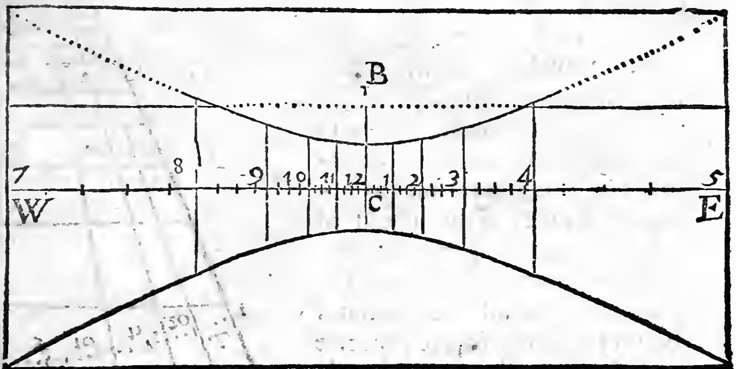
IN all polar planes, whether they be parallel to the meridian or to the circles of the houre of 6, or otherwise declining, the æquinoctiall will be a right line, but the tropiques and other circles of declination will be sections hyperbolicall, and be thus described.

Consi-

Consider the length of the style, the declination of the parallel, and the angle at the pole betweene the substylar and the houre-line, whereon you meane to describe the parallel.

If you would find where the parallels doe crosse the substylar;

As the tangent of 45 *gr.*
to the tangent of declination:
So is the length of the style,
to the distance of the parallel from the æquinoctiall.

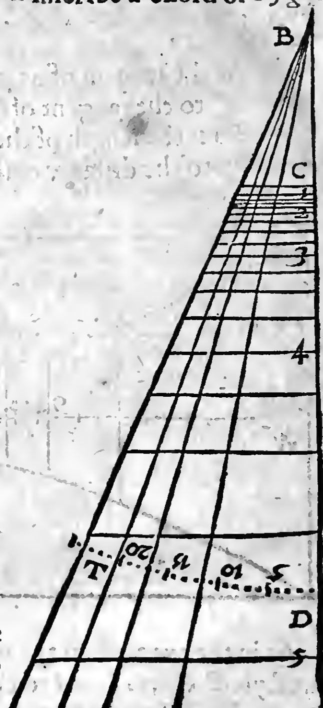


As in the example of the polar plane, where the length of the style *B C* was found to be 1 inch, 61 *cent.* if you desire to know the distance betweene the æquinoctiall and the tropique vpon the substylar line: extend the compasses from the tangent of 45 *gr.* vnto the tangent of 23 *gr.* 30 *m.* the same extent will reach in the line of numbers from 1.61 vnto 0.70; and therefore the distance required is 70 parts of an inch divided into 100. The like reason holdeth for all other parallels of declination crossing the substylar.

But if you would finde where the parallels doe crosse any other of the houre-lines, first find the distance betweene the

the axis of the style and the houre-line, then the distance betweene the æquinoctiall and the parallel, both these may be represented in this maner.

On the center *B* and any semidiameter BD describe an occult arke of a circle, and therein inscribe a chord of 23 *gr.* 30 *m.* form *D* vnto *T*, with such other intermediat declinations as you intend to describe on the plane, so the line BD shall be the æquator, and BT the tropique, and the other intermediate lines the lines of declination.

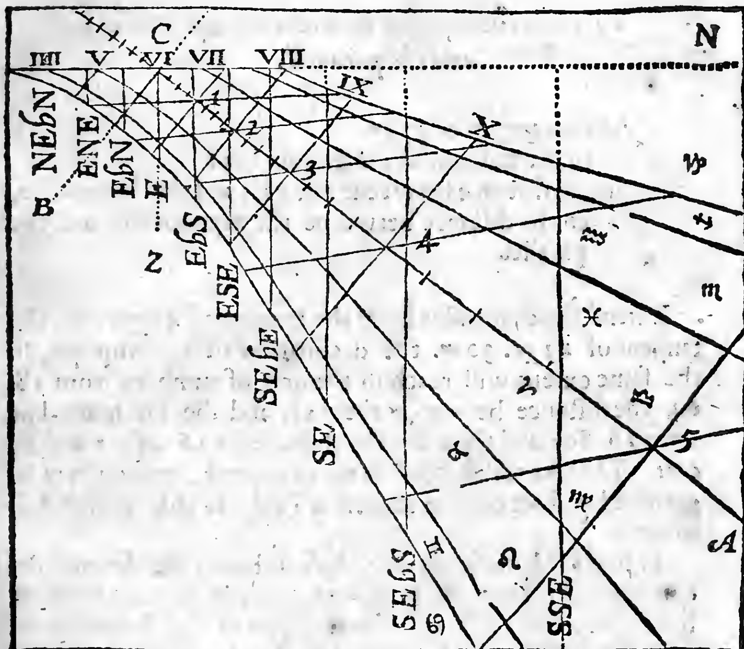


That done, consider your plane, which for example may be either the meridian or the declining polar plane, wherein hauing drawne both the æquator, and the houre-lines as before, first take out the height of the style, and prick that downe in this æquator from *B* vnto *C*; then take out all the distances betweene *B* the top of the style and the seuerall points wherein the houre-lines doe crosse the æquator, transferre them into this æquator B D from the center B, and at the termes of these distances erect lines perpendicular to the æquator, crossing the lines of declination, and note them with the number of the houre from whence they were taken: so these perpendiculars shall represent those houre-lines, and the seuerall distances betweene the æquator and the lines of declination, shall giue the like distances betweene the æquator and the parallels of declination vpon your plane. Vpon this ground it followeth.

To

To finde the distance betweene the axis and the houre-lines.

As the cosine of the houre from the substylar,
is to the sine of 90 *gr.*
So the length of the style,
to the distance between the axis and the houre-line.



As if in the former example of the meridian plane, where B C the height of the style is supposed to be 10 inches, it were required to find the distance between B to the top of the style and the point wherein the houre of 11 in the morning doth crosse the æquator, which is here represented by B 5, because it is the fift houre from the substylar, whose angle at the pole is 75 *gr.* Extend the compasses from the sine of 15 *gr.* the complement of the fift houre from the substylar, vnto the sine of 90 *gr.* the same extent will reach from 10.00 in the line of numbers vnto 38. 64; and therefore the distance B 5 betweene the axis and the houre-line, is 38 inches and 64 *cent.* and may be called the secant of the houre. Then in the rectangle B 5 T, hauing the side B 5, and the angle of declination at B.

To finde the distance betweene the equinoctiall and the paraliell.

As the tangent of 45 *gr.*
to the tangent of the declination:
So the distance betweene the axis and the houre-line,
to the distance betweene the æquinoctiall and the parallel.

Extend the compasses from the tangent of 45 *gr.* vnto the tangent of 23 *gr.* 30 *m.* the declination of the tropique, so the same extent will reach in the line of numbers from 38. 64. the distance betweene the axis and the fift houre-line vnto 16. 80; and therefore the distance is 16 inches and 80 *cent.* The like reason holdeth for all the rest, which may be gathered and set downe in such a Table as this which followeth.

Wherein I haue set downe these distances for seuerall declinations, for 11 *gr.* 30 *m.* for 16 *gr.* 55 *m.* for 20 *gr.* 12. *m.* for 21 *gr.* 41 *m.* and for the declination of the Tropique 23 *gr.* 30 *m.* which may be applied to the like declinations in all meridian and direct polar planes.

As in the former example of the polar plane, where B C the height of the style is found to be 1 inch 61 *cent.* if it were required to find the distance betweene B the top of the style and

and the points wherein the houre-lines of 7 in the morning or 5 after noone, doe crosse the æquator (which distances, I called the secants of those houres,) either you may extend the compasses from the sine of 15 *gr.* the complement of the houre from the substylar vnto the sine of 90 *gr.* so the same

Hor.	An.Po.	Tange.	Secan.	11 30	16 55	20 12	21 41	23 30
	Gr. M.	In. Pa.	In. Pa.	In. Pa.	In. Pa.	In. Pa.	In. Pa.	In. Pa.
0	0 0	0 0	10 0	2 3	3 4	3 68	3 98	4 35
	3 45	0 65	10 02	2 04	3 05	3 69	3 99	4 36
	7 30	1 32	10 09	2 05	3 07	3 71	4 01	4 39
	11 15	1 99	10 20	2 07	3 10	3 75	4 05	4 43
1	15 0	2 68	10 35	2 10	3 15	3 81	4 12	4 50
	18 45	3 39	10 56	2 15	3 21	3 89	4 20	4 59
	22 30	4 14	10 82	2 20	3 29	3 99	4 30	4 70
	26 15	4 93	11 15	2 26	3 39	4 10	4 45	4 85
2	30 0	5 77	11 55	2 34	3 51	4 24	4 60	5 02
	33 45	6 68	12 03	2 44	3 66	4 42	4 78	5 23
	37 30	7 67	12 60	2 56	3 83	4 64	5 02	5 48
	41 15	8 77	13 30	2 70	4 05	4 89	5 29	5 78
3	45 0	10 00	14 14	2 87	4 30	5 20	5 63	6 15
	48 45	11 40	15 17	3 08	4 62	5 58	6 03	6 00
	52 30	13 03	16 43	3 34	5 00	6 04	6 54	7 14
	56 15	14 97	18 00	3 66	5 48	6 62	7 00	7 83
4	60 0	17 32	20 00	4 07	6 08	7 36	7 95	8 70
	63 45	20 28	22 61	4 60	6 88	8 32	9 00	9 83
	67 30	24 14	26 13	5 31	7 95	9 61	10 39	11 36
	71 15	29 46	31 11	6 33	9 47	11 45	12 37	13 53
5	75 0	37 32	38 64	7 86	11 74	14 20	15 36	16 80
	78 45	50 27	51 26	10 43	15 60	18 89	20 38	22 28
	82 30	75 96	76 61	15 58	23 32	28 10	30 47	33 31
	86 15	152 57	152 90	31 10	46 54	56 26	60 81	65 48
6	90 0	Infin.	Infin.	Infin.	Infin.	Infin.	Infin.	Infin.

extent will reach in the line of numbers from 1. 61 the length of the style, vnto 6. 21, according to the former Canon. Or else you may make vse of the former Table, extending the compasses in the line of numbers from 10. 00 the length of the style in the Table, vnto 1. 61 the length of the style belonging to your plane, so the same extent shall reach from 38. 64 the secant in the Table, vnto 6. 21, and such is your secant required, the distance betweene the top of the style and the point of interfection, wherein the fift houre line from the substylar doth crosse the æquator.

Againe, the same extent will reach from 16. 80 the distance in the Table belonging to the fift houre-line betweene the æquatour and the parallel of 23 *gr.* 30 *m.* declination, vnto 2. 70 for the the like distance vpon your plane; and so for the rest, which may be gathered and set downe in a Table.

That done, and the æquator drawne as before, if you would draw the tropiques in the polar plane, looke into the Table, and take 70 *cent.* out of the line of inches, and pricke them downe in the subhylar on either side of the æquatour, and so 72 *cent.* on the first houre, and 80 on the second houre, and 2 inches 70 *cent.* on the fift houre from the substylar, and the rest of these distances on their seuerall houre-lines, and then draw a crooked line through all these points, so as it makes no angles, the line so drawne shall bee the Tropike requirred. In like maner you may draw any other parallell of declination.

Hor.		An. Po Gr.	M.	Tang. In	P.	Secant In.	P.	Trop. In	P.
12		0	0	0	0	1	61	0	70
11	1	15	0	0	43	1	63	0	72
10	2	30	0	0	93	1	85	0	80
9	3	45	0	1	61	2	27	0	99
8	4	60	0	2	79	3	22	1	40
7	5	75	0	6	00	6	21	2	70

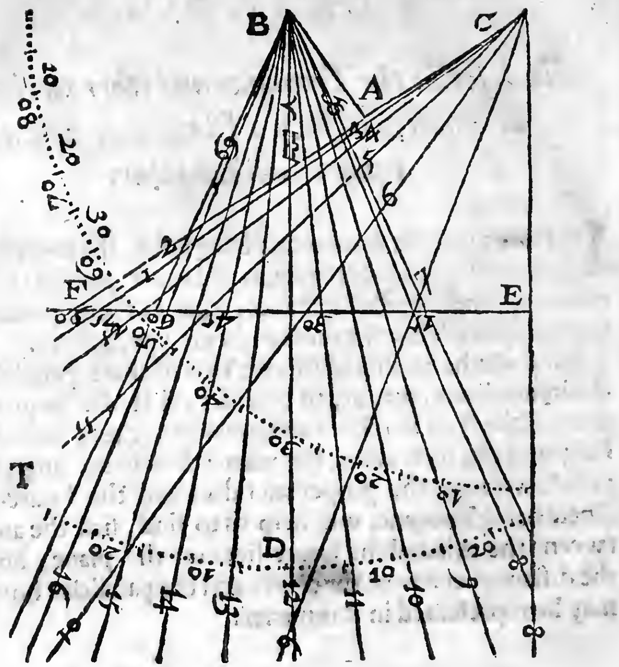
CHAP.

CHAP. XIII.

To describe the Tropiques and other circles of declination in such a Plane as is neither equinoctiall nor polar.

IN Planes neither æquinoctiall nor polar, the æquatour will be a right line, the tropiques and other parallels of declination will bee conicall sections, some of them parabolicall, some ellipticall, but the most of them hyperbolicall.

To finde the points of intersection of these parallels with the houre-lines, wee are to consider, first the length of the axis of the style in inches and parts of inches; secondly the height of the style aboue the plane; thirdly the angles at the pole betweene the proper meridian and the houre-circles. These being knowne, will help vs to find, first the angle betweene the axis and the houre-lines on the plane; and then the distance betweene the center and the parallels; both these may be represented in this maner.



Let the triangle *A B C* be made equall to the ſtyle belonging to your plane, *A C* the ſubſtylar, *B C* the axis of the ſtyle, A B the length of the ſtyle perpendicular to the plane. Then hauing drawne the line B D perpendicular to the axis on the center B, and any ſemidiameter B D deſcribe an occult arke of a circle, and therein inſcribe a chord of 23 *gr.* 30 *m.* from *D.* vnto *T*, on either ſide of the line, with ſuch other intermediate declinations as you intend to deſcribe on the plane, ſo the perpendicular B D ſhall be the æquator, and B T the tropiques, and the other intermediate lines the parallels of declination. Wherefore you may take out the diſtance *C* ♈ from the center to the æquator, and pricke it downe on the ſubſtylar of your plane from the center at *C* vnto ♈, ſo the line drawne through

through ♈ perpendicular to your ſubſtylar, ſhall be the æquator of your plane.

That done, take the diſtance of each houre-line betweene the center aud the equator of your plane, and pricke them downe in the æquator of this figure, from the center at *C*, noting the place, where they croſſe the æquator, with the n umber belonging to the houre, and drawing the houre-lines from *C* through the lines of declination.

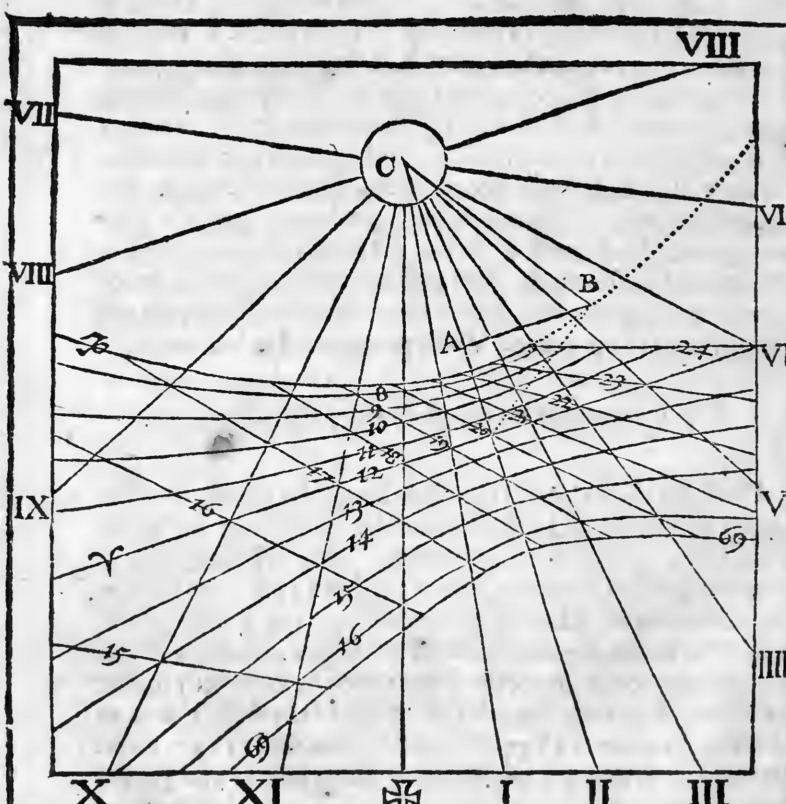
Or hauing the *Sector* you may draw an occult line *C E* perpendicular to the axis B C, and therein pricke downe the tangent of the height of the ſtyle aboue the plane, from *C* vnto *E*. Then draw the line *E F* parallell to the axis, croſſing the ſubſtylar produced in the point *F*, this line *E F* will bee the line of ſines vpon the *Sector*, and therein you may pricke downe the ſines of the complement of the angles at the pole from *E* toward *F*, and draw the houre-lines by thoſe points through the lines of declination, ſo the angles at *C* betweene the axis B C and thoſe houre-lines, ſhall be the angles betweene the axis of your ſtyle and the houre-lines on your plane, and the ſeuerall diſtances betweene the point *C* and the lines of declination, ſhall giue you the like diſtances betweene the center, and the parallels of declination vpon the houre-lines in your plane. Vpon this ground it followeth,

1 *To proportion the ſtyle vnto the plane.*

Conſider the height of the ſtyle aboue the plane, and the length of the ſubſtylar betweene the center and the place which you intend for the trodique. If it bee the tropique which is fartheſt from the center, adde 113 *gr.* 30 *m*: if the neerer tropique, adde 66 *gr.* 30. *m.* vnto the height of the ſtyle, the remainder vnto 180 *gr.* ſhall giue you the altitude of the Sunne aboue aboue the plane when he commeth to that tropique. As in our latitude the height of the ſtyle aboue an horizontall plane is 51 *gr.* 30 *m.* adde vnto this 113 *gr.* 30 *m.* the ſumme is 165 *gr.* which being taken out of 180 *gr.* the remainder

remainder will be 15 *gr.* and such is the altitude of the Sunne aboue this plane when he commeth to be in the Winter tropique: but if you adde 66 *gr.* 30 *m.* vnto 51 *gr.* 30 *m.* the remainder to 180 *gr.* will be 62 *gr.* And such is the altitude of the Sun in the Summer Tropique. Then.

As the sine of 66 *gr.* 30 *m.*
to the sine of the Suns altitude;
So the length of the substylar line,
to the length of the axis of the style.



As in the first examples of the declining verticall, where the height of the style was found to be 34 *gr.* 33 *m.* and is here represented before *pag.* 150. by the angle *B C* ♋ ; adde to this height 113 *gr.* 30 *m.* for the angle C B ♋, the sum will be 148 *gr.* 3 *m.* and the remainder to 180 *gr.* will be 31 *gr.* 57 *m.* and such is the angle *B* ♋ C of the altitude of the Sun aboue the plane, when he cometh to be in the tropique of ♋, which is here the farthest tropique from the center.

Then supposing the length of the substylar line betweene the center and the place which is fit for the farthest tropique to be about 21 inches, extend the compasses from the sine of *66 gr. 30 m.* vnto the sine of 31 *gr.* 57 *m.* the same extent will reach in the line of numbers from 21 vnto 12. 11, and so the length of the axis of the style should be 12 inch. 11 *cent.* Or it may suffice to make it iust 12 inches, as a more easie ground for the rest of the worke.

But if it were required to proportion the style vnto the plane, so as it may cast the shadow to the full length of the substylar line at all times of the yeare, you may then consider the Sun in the tropique, which is to be set nearest vnto the center, and adde *66 gr.* 30 *m.* vnto 34 *gr.* 33 *m.* so the remainder vnto 180 *gr.* will be 78 *gr.* 57 *m.* And if you extend the compasses from the sine of *66 gr.* 30 *m.* vnto the sine of 78 *gr.* 57 *m.* the same extent will reach in the line of numbers from 21 vnto 22.47 for the length of the axis of the style.

2 *Hauing the length of the axis, and the height of the style aboue the plane, to find the length of the sides of the style.*

The style of a plane neither æquinoctiall nor polar, may be either a small rod of iron set parallell to the axis of the world, or perpendicular to the plane, or else a thin plate of iron or brasse made in forme of a rectangle triangle *B A C*, with the base *B C* parallell to the axis of the world, the side *A B* perpendicular to the plane, and the side *A C* the same with the substylar line, wherein knowing *B C*, and the angle B A C,

As the sine of 90 *gr.*
to the length of the axis :
So the sine of the height of the style,
to the length of the perpendicular side :
And so the cosine of the height of the style,
to the length of the substylar side.

Thus in the former example, the length of the axis being supposed to be 12 inches, and the height of the style 34 *gr.* 33 *m.* Extend the compasses from the sine of 90 *gr.* (or else from the sine of 5 *gr.* 45 *m.*) vnto 12 in the line of numbers, the same extent will reach from the sine of 34 *gr.* 33 *m.* vnto 6. 80 in the line of numbers for the length of the perpendicular side, and from the sine of 55 *gr.* 27 *m.* vnto 9. 88 for the length of the substylar side.

3 *To find the distance betweene the center and the æquator vpon the substylar line.*

This is here represented by C, ♈, and may be found by resoluing the rectangle triangle C B ♈.

As the cosine of the height of the style,
is to the sine of 90 *gr.*
So the length of the axis,
to the distance of the æquator from the center.

Extend the compasses from the sine of 55 *gr.* 27 *m.* vnto the sine of 90 *gr.* the same extent will reach in the line of numbers from 12 vnto 14. 57. Wherefore if you take 14 *inch.* 57 *cent.* and pricking them downe on your substylar line from C vnto ♈, draw a line through ♈, crossing the substylar at right angles, the line so drawne shall be the æquator.

4 *To*

4 *To find the angles contained betweene the æquatour and the houre-lines vpon your plane.*

Theſe angles made by B ♈ and the houre-lines, are complements of thoſe which are at C, betweene B C the axis and thoſe ſeuerall houre-lines, and depend vpon the angles at the pole, betweene the proper meridian and the houre-circles.

As the ſine of 90 *gr.*
to the coſine of the angle at the pole:
So the cotangent of the height of the ſtyle,
to the tangent of the angle betweene the æquator and the houre-line.

In our example the height of the ſtyle is 34 *gr.* 33 *m.* and the proper meridian falleth to be the ſame with the circle of the ſecond houre after noone, whereupon the angle at the pole, betweene this proper meridian, and the circles of the houre of 1 on the one ſide, and 3 on the other ſide, will bee 15 *gr*; ſo betweene this meridian and the houre-circles of 12. and 4, the angle will be 30 *gr.* &c. as in the Table.

Ho.		An. Po		Arc. Pla		An. Equ		C ♈		C ♋		C ♑	
		Gr.	M	Gr	M.	Gr.	M.	In.	P.	In.	P.	In.	P.
ſubſty		0	0	0	0	55	27	14	57	20	80	11	21
1	3	15	0	8	38	54	30	14	74	21	36	11	25
12	4	30	0	18	8	51	30	15	33	23	44	11	40
11	5	45	0	29	33	45	45	16	75	29	05	11	76
10	6	60	0	44	30	36	0	20	00	50	84	12	77
9	7	75	0	64	42	20	36	34	10	Infin.		15	82
8	8	90	0	90	0	0	0	Infini-				27	60

If then it be required to find the Angle, which the houre-line of 4 after noone doth make with the plane of the æquator, that is the angle C 4 B contained betweene the houre-line C 4 and the line B 4, drawne from the top of the ſtyle vnto the interſection of the houre-line of 4 with the æquator.

Extend the compaſſes from the ſine of 90 *gr.* vnto the ſine of 60 *gr.* the complement of the angle at the pole, the ſame extent will reach from the tangent of 55 *gr.* 27 *m.* the complement of the height of the pole, vnto the tangent of 51 *gr.* 30 *m.* and ſuch is the angle C 4 B in the diagram *Pag.* 150.

Or in croſſe-worke, if it were required to finde the angle C 9 B, looke into the Table for the houre of 9, and there you ſhall find the angle at the pole to be 75 *gr*; and if you extend the compaſſes from the ſine of 90 *gr.* vnto the tangent of 55 *gr.* 27 *m.* the ſame extent will reach from the ſine of 15 *gr.* the complement of 75 *gr.* vnto the tangent of 20 *gr.* 36 *m.* and ſuch is the angle C 9 B, made at the æquator betweene the line B 9 drawne from the top of the ſtyle, and the houre-line C 9 drawne from the center. The like reaſon holdeth for the reſt, which may be found and ſet downe in a table: then may you either draw theſe angles at C in the former figure more perfectly, and thence finiſh your worke, or elſe proceed

5 *To finde the diſtance betweene the center and the parallels of declination.*

The diſtances betweene the center and the parallels of declination, may be found by reſoluing the triangles made by the axis *B C*, the lines of declination, and the houre-lines. For hauing the angles at the æquator, and knowing the declination of the parallell, if the parallell ſhall fall betweene the æquator and the center, adde the declination vnto the angle at the æquator; or if it ſhall fall without the æquator, take the declination out of the angle at the æquator, ſo ſhall you haue the angle at the parallell Then

As the ſine of the angle at the parallell,
to the coſine of the declination:
So the length of the axis of the ſtyle,
to the diſtance betweene the center and the parallell.

Thus in our example, the angle at the æquator belonging to the houre of 4 after noone, was found before to be 51 *gr.* 30 *m*: if you would find the diſtance betweene the center and the æquator, extend the cōpaſſes from the ſine of 51 *gr.* 30 *m.* vnto the ſine of 90 *gr.* the complement of the declination, the ſame extent will reach in the line of numbers, from 12 vnto 15. 33, and ſuch is the diſtance vpon the houre-line of 4 betweene the center and the æquator.

If you would finde the diſtance vpon this houre-line, betweene the center and the inner tropique, whoſe declination is knowne to be 23 *gr.* 30 *m.* adde the declination to the angle at the æquator, ſo the angle at the parallel will be 75 *gr.* wherefore extend the compaſſes from the ſine of 75 *gr.* vnto the ſine of 66 *gr.* 30 *m.* the complement of the declination, the ſame extent will reach in the line of numbers, from 12 vnto 11. 40, and ſuch is the length of the houre-line of 4 betweene the center and the tropique of ♑.

If you would finde the diſtance vpon this houre-line betweene this center and the tropique of ♋, which is here the fartheſt from the center, take the declination out of the angle at the æquator, ſo the angle at the parallell will be 28 *gr.* wherefore extend the compaſſes from the ſine of 28 *gr.* vnto the ſine of 66 *gr.* 30 *m.* the ſame extent will reach in the line of numbers, from 12 vnto 23.44, and ſuch is the diſtance betweene the center and the tropique of ♋ vpon this houre-line of 4. The like reaſon holdeth for all the reſt, which may be gathered and ſet downe in a table.

That done and the æquator drawne as before, if you would draw the tropique of ♋, looke into the table, and there finding vnder the title C ♋ the diſtance of the ſubſtylar between the center and the parallel of ♋ to be 20 inch. 80 *cent.* take

20 inch 80 *cent.* out of the line of inches, and prick them downe in the substylar of your plane from C vnto ♋.

Or if either the center fall without your plane, or the extent be too large for your compasses, you may prick downe the difference betweene C ♈ and C ♋. As here the distance C ♈ betweene the center and the æquator is 14. 57, the distance C ♋ 20. 80, the difference 6. 23, therefore taking 6 inches 23 *cent.* prick them downe on the substylar from ♈ vnto ♋, and you shall haue the same interlection of the tropique and the substylar, as before; and the like reason holdeth for pricking downe of the rest of these distances on their seuerall houre-lines.

Then hauing the points of interfection betweeu the houre-lines and the parallel, you may ioyne them all in a crooked line without making of any angles, the line so drawne shall be the tropique required. And after this maner may you draw any other parallel of declination, whereof you haue examples in the most of the former Diagrams.

CHAP. XIIII.

To describe the parallels of the Signes in any of the former Planes.

THe æquator and the tropiques before described, doe shew the Suns entrance into 4 of the Signes, the æquator into ♈ and ♎, the one tropique into ♋, and the other into ♑, the rest of the intermediate Signes will be described in the same manner as the tropiques, if first we know their declination.

The manner of finding the declination not onely of the beginning of the Signes, but of all other points of the ecliptique, is is before ſet downe in 2 *Prop. Aſtronomicall, pag.* 52. by which you may find the declination of the beginning of ♉, ♍, and ♏, ♓ to be 11 *gr.* 30 *m.* and of ♊, ♌, ♐ and ♒ to bee 20 *gr.* 12 *m.* If then you inſcribe the chords of 11 *gr.* 30 *m.* and of 20 *gr.* 12 *m.* into the former figure B D T *Pag.* 145. from *D* toward *T*, the lines drawne from B through the termes of thoſe chords ſhall be the Signes required.

And with theſe declinations, the height of the ſtyle, and the length of the axis, you may finde the angles at the parallel, and then the diſtances betweene the center and the parallell, which being pricked downe vpon their ſeuerall houre-lines ſhall giue you the points of interſection, by which you may draw the parallels of the Signes, as in the figures belonging to the polar planes.

CHAP. XV.

To deſcribe the parallels of the length of the day in any of the former Planes.

THe length of the day will alwayes be 12 houres long when the Sunne commeth to be in the æquator, and this holdeth in all latitudes; but at other times of the yeare the ſame place of the Sunne, will not giue the ſame length of the day in another latitude; wherefore the latitude being known, we are firſt

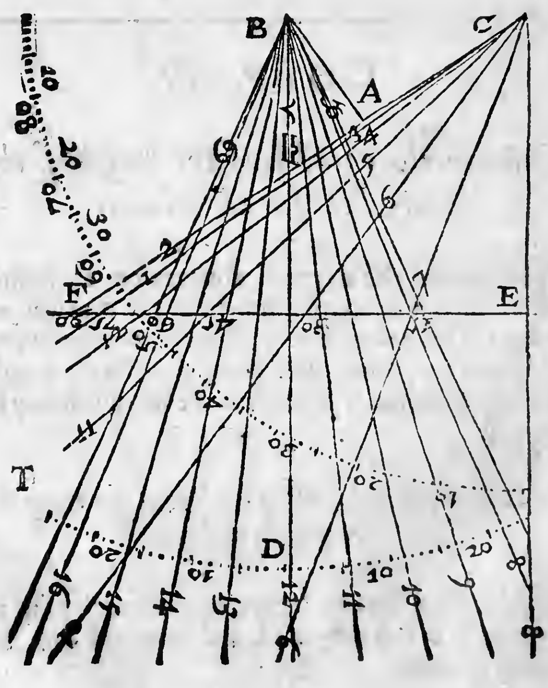
To finde the declination of the Sunne agreeing to the length of the day.

Conſider the difference betweene the length of an æquinoctiall day and the day propoſed, and turne the time into degrees and minutes.

As

As the ſine of 90 *gr.*
is to the ſine of halfe the difference :
So the cotangent of the latitude.
to the tangent of the declination.

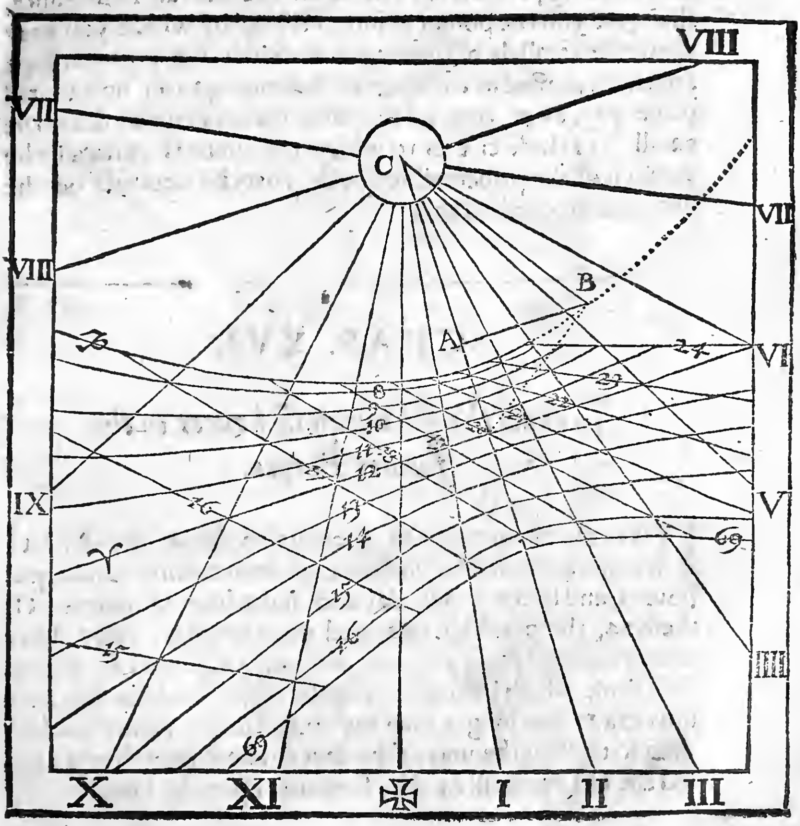
As if the length of the day propoſed were 15 houres, the difference betweene this and an æquinoctiall day (whoſe length is alwaies 12 houres) would be three houres, which make 45 *gr.* and the halfe difference is 22 *gr.* 30 *m.* wherefore extend the compaſſes from the ſine of 90 *gr.* vnto the tangent of 38 *gr.* 30 *m.* the complement of the latitude, the ſame extent will reach from the ſine of 22 *gr.* 30 *m.* vnto the tangent of 16 *gr.* 55 *m.* for the declination of the Sunne at



ſuch

ſuch time as the length of the day is either 9 or 15 houres; and from the ſine of 30 *gr.* vnto the tangent of 21 *gr.* 40 *m.* for the declination belonging to 8 or 16 houres, and from the ſine of 15 *gr.* vnto the tangent of 11 *gr.* 38 *m.* for the declination belonging to 10 or 14 houres, and from the ſine of 7 *gr.* 30 *m.* vnto the tangent of 5 *gr.* 56 *m.* for the declination of the Sun when the length of the day is either 11 or 13 houres.

If then you inſcribe the chords of theſe arkes into the for-



mer figure *B D T*, the lines drawne from *B* through the termes of these arks, shall be the lines belonging to the diurnall arkes, and the seuerall distances betweene them and the point *C* giue the like distances betweene the center and the parallels of the length of the day vpon the houre-lines in your plane.

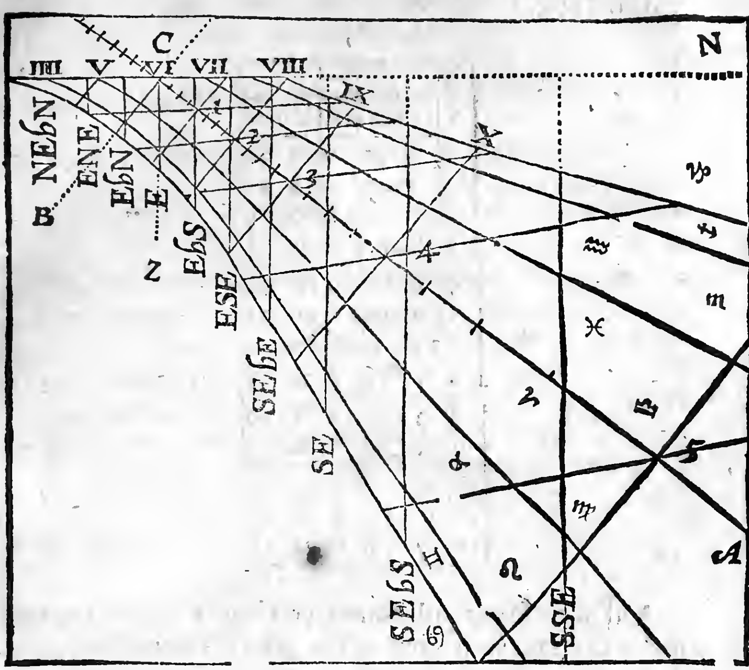
Or comparing these angles of declination with the angles at the æquator, you may haue the angles at the parallel, and then find the distances betweene the center and the parallel, which being pricked downe vpon the seuerall houre-lines, shall giue you the points of intersection, by which you may draw the parallels of the length of the day, whereof you haue another example in the diagram belonging to an horizontall plane *pag.* 105: And by the same reason you may draw the parallels of those circles to which the Sunne is verticall, the parallels of the principall feasts, or what else depends on the declination of the Sunne.

CHAP. XVI.

To draw the old vnequall houres in the former Planes.

IT was the manner of the Ancients to diuide the day into twelue equall houres, and the night into twelue other equal houres, and so the whole day and night into 24 houres. Of these 24, those which belonged vnto the day, were either longer or shorter (excepting the two æquinoctiall dayes) then those which belonged vnto the night; and the Summer houres alwayes longer then the houres in the Winter, according to the lengthening of the dayes, whereupon they are called the old vnequall (and by some the Planetary) houres.

To



To expresse these in the former Planes: first draw the common houre-lines, the æquaror, and the tropiques, as before: then describe two occult parallels of the length of the day, one for 9 houres, the other for 15 houres; for so you may draw a straight line for the first vnequall houre through 5 *ho.* 45 *m* in the parallel of 15, and through 8 *ho.* 15 *m.* in the parallel of 9. This straight line shall passe directly through 7 *ho.* 0 *m.* in the æquator, and so cut off a twelfth part of the arkes aboue the horizon, both from these two parallels and the æquator: and being continued vnto the tropiques, it shall also cut off about a twelfth part from them, and all the rest of the parallels of declination, without any sensible error.

In like manner may you draw the second vnequall houre through 7 *ho.* in the parallel of 15, through 8 *ho.* in the æquator, and through 9 *ho.* in the parallel of 9, and so in the rest, as in this Table.

Hora.	15		Eq.	9	
	Ho.	M.	Ho	Ho.	M
0	4	30	6	7	30
1	5	45	7	8	15
2	7	0	8	9	0
3	8	15	9	9	45
4	9	30	10	10	30
5	10	45	11	11	15
6	12	0	12	12	0
7	1	15	1	0	45
8	2	30	2	1	30
9	3	45	3	2	15
10	5	0	4	3	0
11	6	15	5	3	45
12	7	30	6	4	30

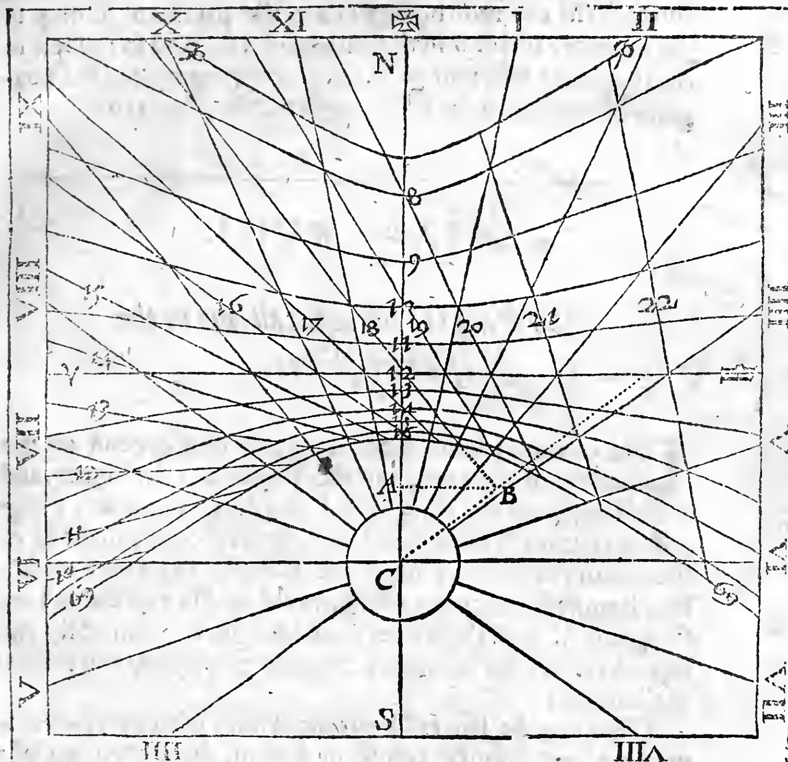
And of these vnequall houres you haue a farther example in the diagram belonging to the polar declining plane, *Pag.* 130.

CHAP. XVII.

To draw the houres from Sunne rising and Sunne setting in the former Planes.

TO know how many houres are past since the Sun rising, or how many remaine to the Sun setting; first draw the common

common houre-lines, the æquator, and the tropiques, as before: then describe two occult parallels of the length of the day, one for 8 houres, and the other for 16 houres. For so



you may draw the first houre from the Sun rising through the common houres of 5 in the parallell of 16, of 7 in the æquator, and of 9 in the parallel of 8. In like manner the second houre from Sun rising through the common houres of 6 in the parallel of 16, of 8 in the æquator, and of 10 in the parall l of 8. And so the rest in their order.

The first houre before Sun setting, or the 23 houre from

the

the laſt Sun ſetting, may be drawne in like ſort through the common houres of 3 after noone in the parallel of 8 of 5 in the æquator, and of 7 in the parallel of 16. The ſecond houre before Sun ſetting, or the 22 houre after the laſt Sun ſetting through the common houres of 2 in the parallel of 8, of 4 in the æquator, and of 6 in the parallel of 16. And ſo the reſt in the like order, whereof you haue another example in the Diagram belonging to the declining verticall, *Pag.* 116.

CHAP. XVIII.

To draw the horizontall line in the former planes.

THe common houre-lines doe common depend on the ſhadow of the axis, but the parallels of the Signes, and of the length of the day, the houre-lines from Sun riſing and Sun ſetting, with many others, depend on the ſhadow of the top of the ſtyle, or ſome one point in the axis, which here ſignifieth the center of the world, and is repreſented by the point B. And theſe lines ſo depending, are then onely vſefull when they fall betweene the two tropiques, and within the horizon.

There may be ſeuerall horizontall lines drawne vpon euery plane, as I ſhewed before in finding the inclination of a plane; but the proper horizontall line which is here meant, muſt alwaies be in the ſame plane with B the top of the ſtyle; ſo that in an horizontall plane there can be no ſuch horizontall line, but in all other planes it may be found by applying the horizontall legge of the *Sector* vnto the top of the ſtyle, and then working as before; and the interſection of this line with the meridian or ſubſtylar line, may be found by propor-

1 *To finde the intersection of the horizon with the meridian, in an æquinoctiall plane.*

As the tangent of 45 *gr.*
to the tangent of the latitude:
So is the height of the style,
to the distance between the style and the horizontall line.

As in the example of the former æquinoctiall plane, *Pag.* 142. extend the compasses from the tangent of 45 *gr.* vnto 51 *gr.* 30 *m.* the tangent of the latitude, the same extent will reach in the line of numbers, from 52 the length of the style vnto *66*, and such is the distance betweene the style and the horizontall line; wherefore I take *66* parts out of a line of inches, and prick them downe in the meridian line from C vnto H aboue the style in the upper face, but below the style in the lower face of the plane, so a right line drawne through *H*, parallel to the houre of *6*, shall be the horizontall line.

2 *To find the intersection of the horizon with the meridian, in a direct polar plane.*

As the tangent of 45 *gr.*
to the cotangent of the latitude:
So the length of the style,
to the distance betweene the style and the horizontall line.

As in the example of the former polar plane, *Pag.* 144. extend the compasses from the tangent of 45 *gr.* vnto tangent of 38 *gr.* 30 *m.* the complement of the latitude, the same extent will reach in the line of numbers, from 1. 61 the length of the style, vnto 1. 28, and such is the distance vpon the meridian

ridian betweene the style and the horizontall line.

In all vpright planes, whether they be direct verticall, or declining, or meridian planes, the horizontall line must alwayes be drawne through *A* the foot of the style, as may appeare in the examples before, *Pag*, 102. 107. 116.

And generally in all planes whatsoeuer, the horizontall line must be drawne through the intersection of the æquatour with the houre of 6. Or if that intersection fall without the plane, yet if any arks of the length of the day be drawne on the plane, the horizontall line may be drawne through their intersections, with the houres of the Suns rising or setting.

CHAP. XIX.

To describe the verticall circles in the former Planes.

THe verticall circles commonly called Azimuths, are great circles drawne through the zenith, by which we may know in what part of the heauen the Sun is, how far from the East or West, and how neere vnto the meridian.

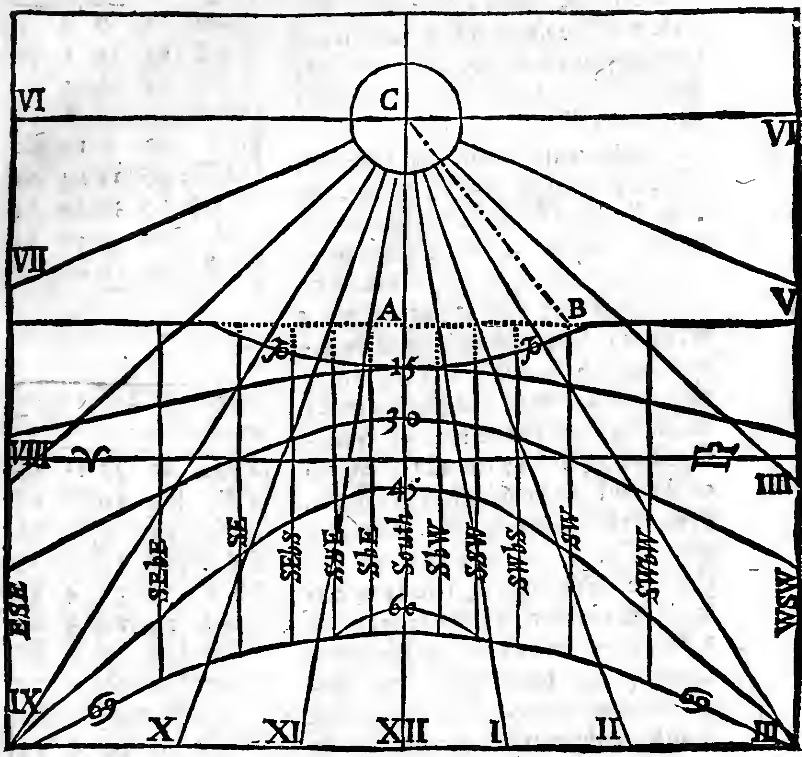
In all vpright planes, whether they be direct verticals, or declining, or meridian planes; the semidiameter of the horizon will be the same with *A B* the perpendicular side of the style, and these Azimuths will be parallels one to the other, and the distance of each Azimuth, from the foote of the style vpon the horizontall line, may be found in this manner.

Consider the length of the style in inches and parts of inches, and the distance of each Azimuth from the style, according to the angle at the zenith in degrees and minutes.

As the tangent of 45 *gr.*
to the tangent of azimuth:

So

So the length of the ſtyle, [illegible]
to the length of the horizontall line betweene the ſtyle and the azimuth.



As if it were required to draw the common azimuths on the South face of the verticall plane before deſcribed, where *A B* the length of the ſtyle may be ſuppoſed to be 10 inches.

Here the plane hauing no declinatiō, the ſtyle is in the plane of the meridian, and ſo pointeth directly into the South. The point of S *b E* is 11 *gr.* 15 *m.* diſtant from the ſtyle, and

S S E 22 *gr.* 30 *m.* and the rest in their order: wherefore extend the compasses from the tangent of 45 *gr.* vnto 10 in the line of numbers, the same extend will reach from the tangent of 11 *gr.* 15 *m.* vnto 1, 99 in the line of numbers for the length of the tangent line, betweene the style and the point S *b E*, and from the tangent of 22 *gr.* 30 *m* vnto 4. 14 for S S *E*, and so for the rest, as in this Table.

Azimuths.	An Zen Gr.	M.	Tangen In.	Pa
South	0	0	0	0
S b E	11	15	1	99
S S E	22	30	4	14
SEbS	33	45	6	68
S E	45	0	10	00
SEBE	56	15	14	97
E S E	67	30	24	14
E b S	78	45	50	27
East	90	0	Infin.	

In like maner in the first example of the declining plane, where the style standeth according to the declination 24 *gr.* 20 *m.* distant from the South toward the West. The next point of S *b W* is but 13 *gr.* 5 *m.* distant from the style; and the second of S S W onely 1 *gr.* 50 *m.* and the third of *SWbS* is againe 9 *gr.* 25 *m.* and the rest in their order. Wherfore hauing before found the length of the style to be 6 inches 80 parts, extend the compasses from the tangent of 45 *gr.* vnto 6. 80 parts in the line of numbers, the same extent will reach from the tangent of 24 *gr.* 20 *m.* vnto 3, 07 in the line of numbers for the length of the tangent line betweene the style and the South, and from the tangent of 13 *gr.* 5 *m.* vnto 1. 58 for the point of *S b W*; and so for the rest, as in this Table.

Azimuths.	An Zen. Gr.	M.	Tangen In.	Pa.
SEbE	80	35	41	00
S E	69	20	18	03
SEbS	58	5	10	91
S S E	46	50	7	25
S b E	35	35	4	86
South	24	20	3	07
S b W	13	5	1	58
S S *W*	1	50	0	22
The	*foote of*		*the styl*	
SWbS	9	25	1	13
S W	20	40	2	57
SWbW	31	55	4	24
W S W	43	10	6	37
W b S	54	25	9	50
West	65	40	15	02
W b N	76	55	29	26
W N W	88	10	212	45

That done, if you take these parts out of a line of inches, and pricke them downe in the horizontall line on either side of the style, drawing right

right lines perpendicular to the horizon through these intersections, but so as they may be contained betweene the horizontall and the tropiques, the lines so drawne shall be the azimuths required.

In an horizontall plane these azimuths are drawne more easily. For here the perpendicular side of the style is the same with the axis of the horizon, and the foote of the style is the verticall point, in which all the azimuth lines doe meete as their circles doe in the zenith: wherefore let any circle described on the center *A*, at the foote of the style, be diuided first into foure parts, beginning at the meridian, and then each quarter subdiuided either into eight equall parts, according to the points of the Mariners compasse, or into 90 *gr.* according to the Astronomicall diuision; if you draw right lines through the center and these diuisions, the lines so drawne shall be the azimuths required.

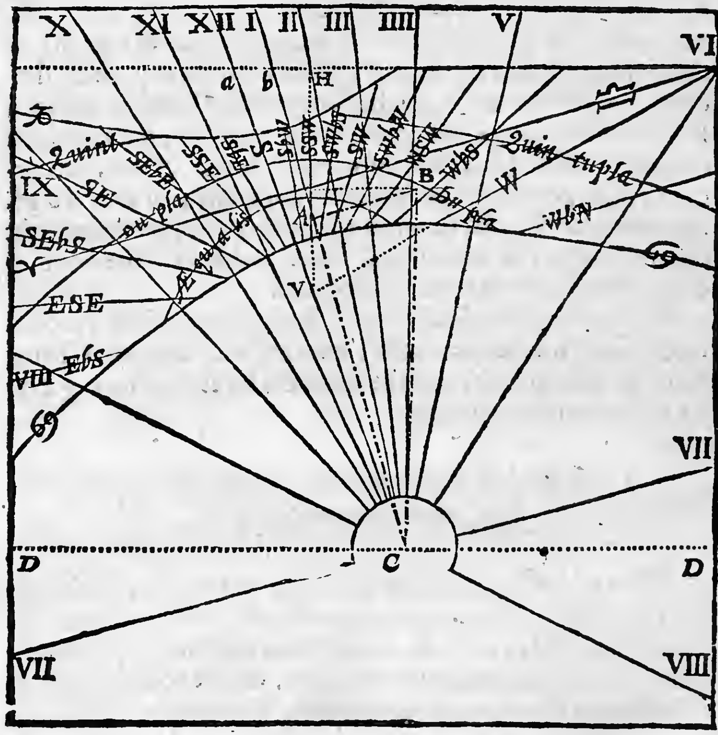
In all other planes inclining to the horizon, these verticall circles will meete in a point, but that verticall point being more or lesse distant from the foote of the style, the angles at this point will be vnequall.

1 *To find the distance betweene the foote of the style, and the verticall point.*

The verticall point wherein all the verticall lines do meet, will be alwayes in the meridian, directly vnder or ouer the top of the style; and the angle betweene the perpendicular side of the style and the verticall line, will be equall to the inclination of the plane to the horizon. Wherefore

As the tangent of 45 *gr.*
 to the tangent of the inclination of the plane:
So is the length of the style,
 to the distance betweene the foote of the style and the verticall point.

Thus in the first example of the declining inclining planes, where the vpper face of the plane looking Southwest, the declination was 24 *gr.* 20 *m.* the inclination 36 *gr*; and you may suppose *A B* the length of the style to be 6 inches: if you extend the compasses from the tangent of 45 *gr.* vnto the tan-



gent of 36 *gr.* the same extent will reach in the line of numbers from 6.00 vnto 4.36, for the distance *A V* betweene *A* the foote of the style and *V* the verticall point.

2 To

2 *To find the distance betweene the foote of the style and the horizontall line.*

As the tangent of the inclination of the plane,
is to the tangent of 45 *gr.*
So the length of the style,
to the distance betweene the foote of the style and the horizontall line.

So the same extent of the compasses as before, will reach in the line of numbers from 6.00 vnto 8 26 for the distance *AH* betweene the foote of the style and the horizontall line.

Then may you take 4 inches 36 *cent.* and pricking them downe from *A* the foot of the style vnto *V* the verticall point in the meridian, draw the line *V A*, which being produced shall cut the horizon in the point *H* with right angles, and be that particular azimuth which is perpendicular to the plane.

Or you may take 8 inches 26 *cent.* and pricke them downe in the former line *V A* produced from *A* vnto *H*, and so draw the horizontall line through *H* perpendicular vnto *V H*, which horizontall line being produced will crosse the æquator in the same point wherein the æquator crosseth the houre-line of 6, vnlesse there be some former error.

3 *To find the angles made by the azimuth lines at the verticall point.*

The angles at the zenith depend on the declination of the plane, as in our example; where the style standeth according to the declination 24 *gr.* 20 *m.* distant from the South toward the West, the azimuth of 10 *gr.* from the meridian Eastward will be 34 *gr.* 20 *m.* the azimuth of 10 *gr.* Westward will be onely 14 *gr.* 20 *m.* diſtant from the ſtyle, and ſo the reſt in their order.

Or if you would rather deſcribe the common azimuths, the point of S *b E* will be 35 *gr.* 35 *m.* the point of S *b W* 13 *gr.* 5 *m.* diſtant from the ſtyle, and ſo the reſt in their order. Then

As the ſine of 90 *gr.*
to the coſine of the inclination of the plane:
So the tangent of the angle at the zenith,
to the tangent of the angle at the verticall point betweene the line drawne through the foot of the ſtyle and the azimuth required.

Wherefore the inclination of the plane in our example being 36 *gr.* extend the compaſſes from the ſine of 90 *gr.* vnto the ſine of 54 *gr.* the ſame extent ſhall reach in the line of tangents, from 24 *gr.* 20 *m.* vnto 20 *gr.* 5 *m.* for the angle *HVa* at the verticall point, betweene the line *VH* drawn through *A* the foote of the ſtyle and the South. Againe, the ſame extent will reach from the tangent of 13 *gr.* 5 *m.* vnto 10 *gr.* 38 *m.* for the angle belonging to S *b W*; and ſo for the reſt, as in this table.

Azimuths.	Ang. Ze. Gr.	Ang. Ze. M	Ang. Ve. Gr.	Ang. Ve. M.
SEbE	80	55	78	25
S *E*	69	20	65	0
S *E b* S	58	5	52	25
S S E	46	50	40	46
S b E	35	35	30	3
South	24	20	20	5
S b W	13	5	10	39
S S W	1	50	1	29
	Style.		0	0
SW b S	9	25	7	38
S W	20	40	16	58
S*WbW*	31	55	26	45
W S *W*	43	10	37	11
W b S	54	25	48	30
Weſt	65	40	60	48
WbN	76	55	73	58
WNW	88	10	87	44

Theſe angles being knowne, if on the center *V*, at the verticall point, you deſcribe an occult circle, and therein inſcribe the chords of theſe angles from the line *VH*, and then draw right lines through the verticall point, and the terms of thoſe chords, the lines ſo drawne ſhall be the azimuths required.

The

The like reason holdeth for the drawing of the azimuths vpon all other inclining planes, whereof you haue another example in the Diagram belonging to the meridian incliner, *Pag.* 126.

Or for further satisfaction you may finde where each azimuth line shall crosse the equator.

As the sine of 90 *gr.*
to the sine of the latitude :
So the tangent of the azimuth from the meridian,
to the tangent of the æquator from the meridian.

Extend the compasses from the sine of 90 *gr.* vnto the sine of our latitude 51 *gr.* 30 *m.* the same extent will reach in the line of tangents from 10 *gr.* vnto 7 *gr.* 50 *m.* for the intersection of the æquator with the azimuth of 10 *gr.* from the meridian. Againe, the same extent will reach from 20 *gr.* vnto 15 *gr.* 54 *m.* for the azimuth of 20 *gr.* And so the rest, as in these tables.

Azim.		*Equat.*	
Gr.	*M*	*Gr.*	*M.*
10	0	7	50
20	0	15	54
30	0	24	20
40	0	33	18
50	0	43	0
60	0	53	35
70	0	65	3
80	0	77	18
90	0	90	0

Azim.		*Equa.*	
Gr.	*M.*	*Gr.*	*M.*
11	15	8	51
22	30	17	58
33	45	27	36
45	0	38	2
56	15	49	30
67	30	62	6
78	45	75	44
90	0	90	0

By which you may see that the azimuth 90 *gr.* distant from the meridian, which is the line of East and West, will crosse the æquator at 90 *gr.* from the meridian in the same point, with the horizontall line and the houre of 6. And that the azimuth

zimuth of 45 *gr.* will crosse the æquator at 38 *gr.* 2 *m.* from the meridian, that is, the line of S E will crosse the æquator at the houre of 9 and 28 *m.* in the morning, and the line of SW at 2 *ho.* 32 *min.* in the afternoone; and so for the rest, whereby you may examine your former worke.

CHAP. XX.

To describe the parallels of the horizon in the former planes.

THe parallels of the horizon, commonly called Almicanters, or parallels of altitude (whereby we may know the altitude of the Sun aboue the horizon) haue such respect vnto the horizon, as the parallels of declination vnto the æquator, and so may be described in like maner.

In an horizontall plane, these parallels will be perfect circles; wherefore knowing the length of the style in inches and parts, and the distance of the parallell from the horizon in degrees and minutes.

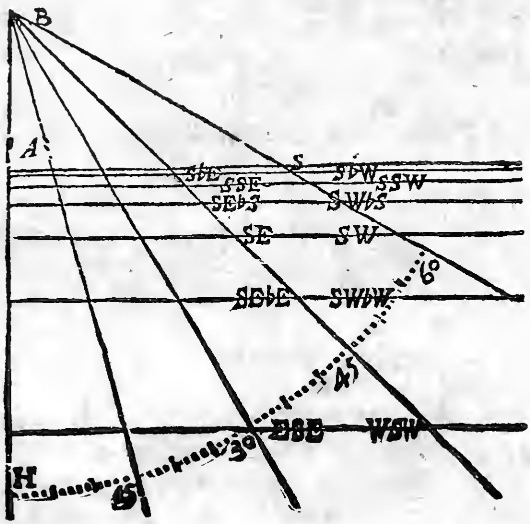
As the tangent of 45 *gr.*
is the length of the style:
So the cotangent of the parallell
to the semidiameter of his circle.

Thus in the example of the horizontall plane, *Pag.* 164. if *A B* the length of the style shall be 5 inches, and that it were required to finde the semidiameter of the parallell of 62 *gr.* extend the compasses from the tangent of 45 *gr.* vnto 5. 00 in the line of numbers, the same extent will reach from the tangent of 28 *gr.* the complement of the parallell vnto 2. 65, and if you describe a circle on the center *A* to the semidiameter of 2 inches 65 *cent.* it shall be the parallell required.

In

In all vpright planes, whether they be direct verticals, or declining, or meridian planes, these parallels will be conicall sections, and may be drawne through their points of intersection, with the azimuth lines, in the same maner as the parallels of declination, through their points of intersection with the houre-lines. To this end you may first finde the distance between the top of the style and the azimuth; and then the distance betweene the horizon and the parallell, both which may be represented in this maner.

On the center *B* and any semidiameter *B H*, describe an occult arke of a circle, and therein inscribe the chords of such parallels of altitude as you intend to draw on the plane, (I haue here put them for 15. 30. 45 and 60 *gr.*) then draw right lines through the center and the termes of those chords, so the line *B H* shall be the horizon, and the rest the lines of altitude, according to their distance from the horizon.



That done, consider your plane (which here for example is

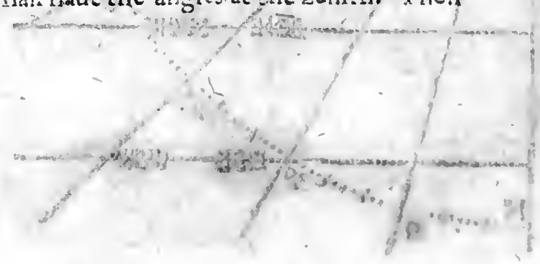
 the

the South face of our verticall plane, *page* 168) wherein hauing drawne both the horizontall and verticall lines, as I shewed before, first take out *A B* the length of the style, and pricke that downe in this horizontall line from *B* vnto *A*; then take out all the distances betweene *B* the top of the style and the seuerall points wherein the verticall lines doe crosse the horizontall, transferre them into this horizontall line B H, from the center B, and at the termes of these distances erect lines perpendicular to the horizon, noting them with the number or letter of the azimuth from whence they were taken, so these perpendiculars shall represent those azimuths, and the seuerall distances betweene the horizon and the lines of altitude shall giue the like distances, betweene the horizontall and the parallels of altitude vpon the azimuths in your plane. Vpon this ground it followeth,

---

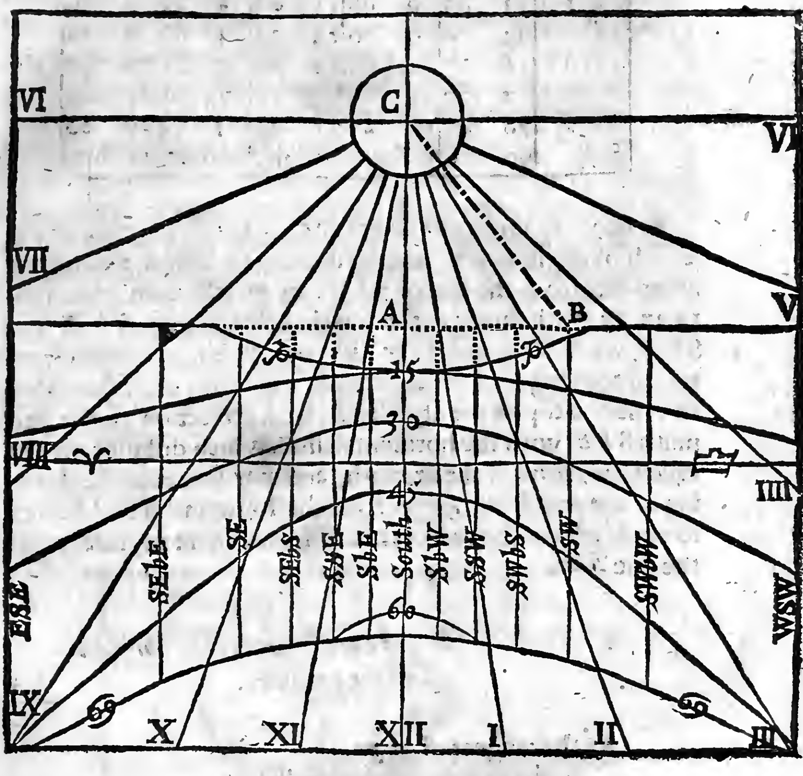
1 *To find the distance betweene the top of the style, and the seuerall points wherein the azimuths doe crosse the horizontall line.*

Hauing drawne the horizontall and azimuth lines as before, looke into the table by which you drew them, and there you shall haue the angles at the zenith. Then



As

As the cosine of the angle at the zenith,
is to the sine of 90 *gr.*
So the length of the style,
to the distance required.



 As

Azimuths.	Ang. Ze. Gr.	M.	Tangent Inch.	P.	Secant. Inch.	P.	Par. 15. Inch.	P.	Par. 30. Inch.	P.
South	0	0	0	0	10	00	2	68	5	77
S b E	11	15	1	99	10	20	2	73	5	90
S S E	22	30	4	14	10	82	2	90	6	24
S E b S	33	45	6	68	12	03	3	23	6	94
S E	45	0	10	00	14	14	3	80	8	16
S E b E	56	15	14	97	18	00	4	82	10	40
E S E	67	30	24	14	26	13	7	02	15	08
E b S	78	45	50	27	51	26	13	73	29	60
East.	90	0	Infinit.		Infinit.		Infinit.		Infinit.	

As in our example of the verticall plane, where *A B* the length of the style was supposed to be 10 inches, extend the compasses from the sine of 78 *gr.* 45 *m.* (the complement of 11 *gr.* 15 *m.* the angle at the zenith, belonging to S *b E* and S *b W*) vnto the sine of 90 *gr.* the same extent will reach from 10. 00 the length of the style, vnto 10. 20 for the distance betweene the top of the style and the intersection of the azimuth S *b E* with the horizontall line, which distance may be called the *secant* of the azimuth, and may serve for the drawing of the parallel of 45 *gr.* from the horizon. The like reason holdeth for the rest of these distances here represented in the line *B H*.

2 *To finde the distance betweene the horizon and the parallels.*

As the tangent of 45 *gr.*
to the tangent of the parallel:
So the secant of the azimuth,
to the distance required.

As if it were required to draw the parallell of 15 *gr.* from the

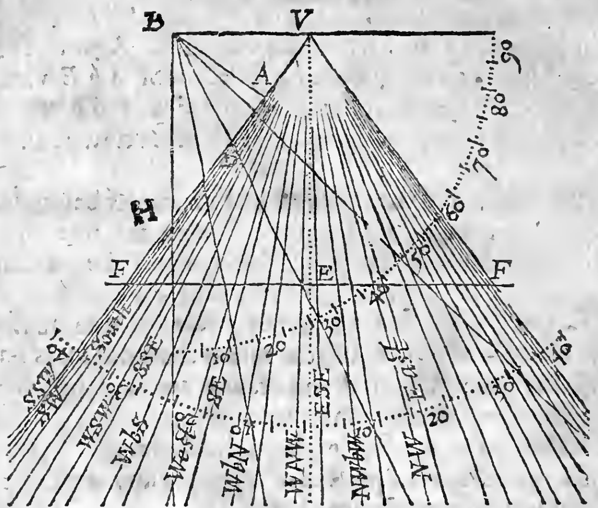
the horizon, vpon this verticall plane; extend the compaſſes from the tangent of 45 *gr.* vnto the tangent of 15 *gr.* the ſame extent will reach in the line of numbers from 10. 00 the ſecant of the South azimuth vnto 2. 68, and therefore the diſtance betweene the horizon and the parallell of 15 *gr.* is 2 inches 68 *cent.* vpon the South azimuth. Againe, the ſame extent will reach from 10. 20 the ſecant of *S b E* vnto 2. 73 for the like diſtance belonging to *S b E* and *S b W*; and ſo for the reſt, which may be gathered and ſet downe in the table.

That done, and the horizon and azimuths being drawne, pricke downe 10 inches from the horizontall line vpon the South azimuth, and 10 inches 20 *cent.* on the azimuths of *S b E* and *S b W*, and 10 inches 82 *cent.* on the azimuths of *S S E* and *S S W*, and 12 inches 3 *cent.* on the azimuth of *S E b S* and *S W b S*, and ſo the reſt of theſe diſtances on their ſeuerall azimuths: then if you draw a crooked line through theſe points, that may make no angles, the line ſo drawne ſhall be the parallell of 45 *gr.* from the horizon. In like manner may you draw the parallel of 15 *gr.* or any other parallell of altitude vpon any verticall plane.

If the plane incline to the horizon, after we haue found the verticall point, and drawne the horizontall line, we are farther to finde the length of the axis of the horizon, then the angles betwixt this axis and the azimuth lines, and ſo the ſeuerall diſtances betweene the parallels and the verticall point, all which may be repreſented in this manner.

On the center B, and any ſemidiameter, deſcribe an occult quadrant of a circle, and therein inſcribe the chords of ſuch parallels of altitude as you intend to draw on the plane, drawing right lines through the center and the termes of theſe chords, ſo the line B H ſhall be the horizon, and his perpendicular B V the axis of the horizon, and the reſt the lines of altitude, according to their diſtance from the horizon.

That done, conſider your plane, which here for example is the first of our three declining inclining planes, wherein hauing drawne both the horizontall and verticall lines as I shewed before, first take out the axis of the horizon, which



is the line between *B* the top of the style and V the verticall point, and pricke that downe in this figure from *B* vnto *V*; then take out both the line *V H* and all the rest of the distances betweene V the verticall point and the seuerall points wherein the verticall lines doe crosse the horizontall line of this figure, from the point *V*, noting the place where they crosse the horizontall line with the number or letter of the azimuth from whence they were taken, and drawing the azimuth lines from V through the lines of altitude.

Or hauing the *Sector* you may draw an occult line *V E* perpendicular to the axis *V* B, and therein prick downe the tangent of the complement of the inclination of the plane from *V* vnto *E*: then draw the line *E* F parallel to the axis, crossing the line V H produced in the point F, so this line E F will be as the line of sines vpon the *Sector*, and therein you

you may prick downe the sines of the complement of the angles at the zenith from E towards F, and draw the vertical lines by those points through the lines of altitude, so the angles at V, betweene the axis V B and those azimuth lines, shall be the angles betweene the axis of the horizon and the azimuth lines on your plane, and the seuerall distances betweene the point V and the lines of altitude, shall giue the like distances betweene the verticall point and the parallels of altitude vpon the azimuths in your plane. Vpon this ground it followeth,

1 *To finde the length of the axis of the Horizon.*

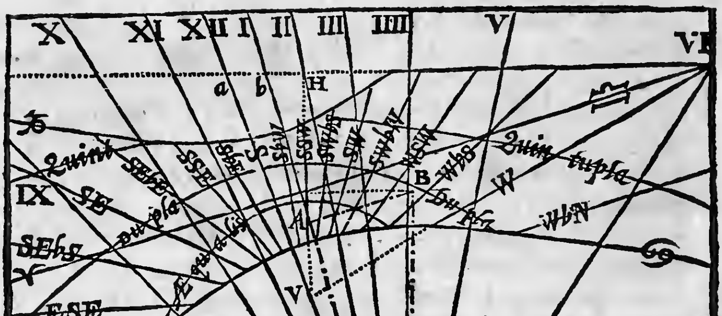
The verticall point is alwayes either directly ouer or vnder the top of the style, and the distance betweene them is that which I call the axis of the horizon, which may thus be found,

As the cosine of the inclination,
to the sine of 90 *gr.*
So the length of the style,
to the length of the axis of the horizon.

For example in the first of the three declining inclining planes, the inclinatiou to the horizon is 36 *gr.* the length of the style A B sixe inches, extend the compasses from the sine of 54 *gr.* the complement of the inclination vnto the sine of 90 *gr.* the same extent will reach in the line of numbers from 6.00 vnto 7.42, and such is V B the length of the axis required.

2 *To finde the angles contained betweene the horizon and the verticall lines vpon your plane.*

The angles at the verticall point betweene the axis of the horizon and the azimuth lines vpon your plane are represented in this figure by those at V, betweene V B and the azimuths. The angles betweene the horizon and the azimuth lines being complements to the former, are represen-



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ted either by those which are made by V E or by B H, and the azimuth lines which are drawne from V.

That you may finde them, looke into the Table, by which you drew the azimuth lines, there shall you finde the angles at the zenith. Then

As the sine of 90 *gr.*
to the cosine of the angle at the zenith:
So the tangent of the inclination to the horizon,
to the tangent of the angle betweene the horizon and the verticall line.

In our example where the inclination to the horizon is 36 *gr.* and the angle at the zenith betweene the azimuth at the style and the meridian, is according to the declination 24 *gr.* 20 *m.* extend the compasses from the sine of 90 *gr.* vnto the tangent of 36 *gr.* the same extent will reach from the sine of 65 *gr.* 40 *m.* the complement of the angle at the zenith, vnto the tangent of 33 *gr.* 30 *m.* for the angle contained betweene the horizon and the South part of the meridian line. Againe, the same extent will reach from the cosine of 35 *gr.* 35 *m.* the angle at the zenith belonging to *S b E* vnto the tangent of 30 *gr.* 3 *m.* for the angle betweene the horizon and the azimuth line of *S b E.* The like reason holdeth for the rest, which may be found and set downe in the Table.

Azi-	Ang Ze		Ang. V.		Ang Ho		Horizon		11	18	26	34	45	0
muths.	Gr.	M.	Gr.	M.	Gr.	M.	Inch.	P.	Inch.	P.	Inch.	P.	Inch.	P.
East.	114	25	119	12	16	40	In-				38	60	11	05
E b S	103	5	106	2	9	20	fi-		210	24	22	40	9	00
E S E	91	50	92	16	1	20	nite.		41	98	15	57	7	60
SE b E	80	35	78	25	6	47	62	82	23	44	12	07	6	68
S E	69	20	65	0	14	23	29	87	16	79	10	12	6	00
SE b S	58	5	52	25	21	0	20	70	13	61	8	99	5	79
S S E	46	50	40	46	26	25	16	68	11	90	8	31	5	53
S b E	35	35	30	3	30	35	14	58	10	90	7	90	5	42
South	24	20	20	5	33	30	13	44	10	32	7	66	5	35
S b W	13	5	10	39	35	17	12	84	10	02	7	55	5	33
S S W	1	50	1	29	35	59	12	62	9	90	7	47	5	31
	Style.		0	0	36	0	12	62	9	90	7	47	5	31
SW b S	9	25	7	38	35	37	12	74	9	96	7	50	5	32
S W	20	40	16	58	34	12	13	20	10	20	7	59	5	34
SW b W	31	55	26	45	31	40	14	13	10	67	7	81	5	39
W S W	43	10	37	11	27	55	15	85	11	50	8	15	5	49
W b S	54	25	48	30	22	55	19	05	12	94	8	73	5	66
West.	65	40	60	48	16	40	25	87	15	51	9	60	5	96
W b N	76	55	73	58	9	20	45	75	20	64	11	32	6	46
W N W	88	10	87	44	1	20	318	88	33	27	14	18	7	25
NW b W	99	25	101	35	6	47	Infi-		92	40	19	60	8	48
N W	111	40	110	0	14	23	nite.				31	44	10	30

Then may you either draw these angles at V in the former figure more perfectly, and thence finish your worke, or else proceed.

3 *To finde the distance betweene the verticall point, and the parallels of the horizon.*

These distances may be found by resolving the triangles in the last figure made by the axis, the lines of altitude, and the azimuth

azimuth lines. For hauing the length of the axis and the angles at the horizon, if you adde the diſtance of the parallell from the horizon vnto the angle at the horizon, you ſhall haue the angle at the parallel. Then

As the ſine of the angle at the parallel,
to the coſine of the altitude;
So the length of the axis,
to the diſtance betweene the verticall point and the parallell.

Thus is our example if it were required to finde the diſtance vpon the ſtylar azimuth V H, betweene the verticall point and the horizon, you haue the rectangle triangle V B H wherein the angle at the horizon here repreſented by B H V is (equall to the inclination of the plane) 36 *gr.* and B V the axis of the horizon betweene the plane and the top of the ſtyle, is 7 inches 42 *cent.* Wherefore extend the compaſſes from the ſine of 36 *gr.* vnto the ſine of 90 *gr.* the complement of the altitude, the ſame extent will reach in the line of numbers from 7. 42 vnto 12, 62, and ſuch is the diſtance of the perpendicular azimuth line V H betweene the verticall point and the horizon.

In like manner if you would finde the diſtance vpon the meridian between the verticall point and the horizon, extend the compaſſes from the ſine of 33 *gr.* 30 *m.* the angle at the horizon, to the ſine of 90 *gr.* the ſame extent will reach in the line of numbers from 7. 42 vnto 13. 44, and ſuch is *V a* the diſtance betweene the verticall point and the horizon vpon the line of the South azimuth, that is, upon the meridian line.

But if you would finde the diſtance vpon the meridian betweene the verticall point and any other parallell of the horizon, as vpon the parallel of 26 *gr.* 34 *m.* then adde theſe 26 *gr.* 34 *m.* vnto 33 *gr.* 30 *m.* the angle at the horizon, ſo ſhall you haue 60 *gr.* 4 *m.* for B D V the angle at the parallel. And if you extend the compaſſes from the ſine of 60 *gr.* 4 *m.* vnto the sine of 63 *gr.* 26 *m.* the complement of the parallell from the horizon, the same extent will reach in the line of numbers from 7.42 the length of the axis, vnto 7.66, and such is the distance *V D* betweene the verticall point and the paparallell of 26 *gr.* 34 *m.* vpon the meridian line. The like reason holdeth for all the rest, which may be gathered and set downe in the table.

That done, and the horizon drawne as before, if you would draw the parallel of 26 *gr.* 34 *m.* from the horizon, looke into the table, and there finding vnder the title of the parallel of 26 34, the distance on the Sonth azimuth line to be 7.66, take 7 inches 66 *cent.* out of a line of inches, and prick them down on the meridian of your plane, from the verticall point at *V*.

Or if either the verticall point fall without your plane, or the extent at any time be too large for your compasses, you may pricke downe the distance betweene the horizon and the parallel. As here the distance betweene the verticall point and the parallel is 7 66, betweene the verticall point and the horizon 13.44, the difference betweene them 5.78 is the distance from the horizon to the parallel, which being pricked downe upon the meridian, shall giue the same intersection as before. And the like reason holdeth for the pricking downe the rest of these distances on their seuerall azimuths.

Hauing the points of intersection betweene the azimuths and the parallel, you may ioyne them all in a crooked line without making of angles, the line so drawne shall be the parallell required. And vpon this ground it followeth,

To describe such parallels on the former planes, as may shew the proportion of the shadow vnto the gnomon.

The proportion of a mans shadow vnto his height, or other shadow to his gnomon set perpendicular to the horizon, may be shewed by parallels to the horizon, if they be drawne to a due altitude, which may thus be found:

As the length of the ſhadow,
to the length of the gnomon:
So the tangent of 45 *gr.*
to the tangent of the altitude.

As if it were required to finde the altitude of the Sunne when the ſhadow of a man ſhall be decuple to his height, extend the compaſſes from 10 vnto 1 in the line of numbers, the ſame extent will reach in the tangent of 45 *gr.* vnto the tangent of 5 *gr.* 42 *m*; which ſhewes that when the Sun commeth to the altitude of 5 *gr.* 42 *m*, your ſhadow, vpon a levell ground, will be ten times as much as your height. In the ſame maner you may ſinde that at 7 *gr.* 7 *m.* of altitude your ſhadow will be octuple, at 9 *gr.* 27 *m.* ſextuple, at 11 *gr.* 18 *m.* quintuple, at 14 *gr.* 2 *m.* quadruple, at 18 *gr.* 26 *m.* triple, at 26 *gr.* 34 *m.* double to your height, at 33 *gr.* 41 *m.* as 3 vnto 2, at 36 *gr.* 52 *m.* as 4 vnto 3, at 38 *gr.* 40 *m.* as 5 vnto 4, at 45 *gr.* equall, at 51 *gr.* 20 *m.* as 4 vnto 5, at 53 *gr.* 7 *m.* as 3 vnto 4, at 56 *gr.* 19 *m* as 2 vnto 3, at 59 *gr.* 2 *m.* as 3 vnto 5, at 63 *gr.* 26 *m.* as 1 vnto 2, &c.

If then you draw a parallell to the horizon at 5 *gr.* 42 *m.* another at 7 *gr.* 7 *m.* and ſo the reſt, when the ſhadow of the ſtyle falleth on the parallell, you haue the proportion, and thereby may you know the ſhadow by the height, and the height by the ſhadow, whereof you haue examples *Pag.* 126. and 137.

I might here proceed to ſhew the deſcription of the circles of poſition, the Signes of the Zodiack in the meridian, the Signes aſcending and deſcending, with ſuch other gnomonicall concluſions; but theſe would proue ſuperfluous to ſuch as vnderſtand the doctrine of the Sphere; and for others, that which is deliuered may ſuffice for ordinary vſe, it being my intention not ſo much to explane the full vſe of ſhadowes (whereof I haue lately giuen a large example in an other place) as the vſe of theſe lines of proportion, that were not extant heretofore.

An Appendix concerning
The description and vse of a small portable Quadrant, for the more easie finding of the houre and Azimuth.

CHAP. I.
Of the description of the Quadrant.

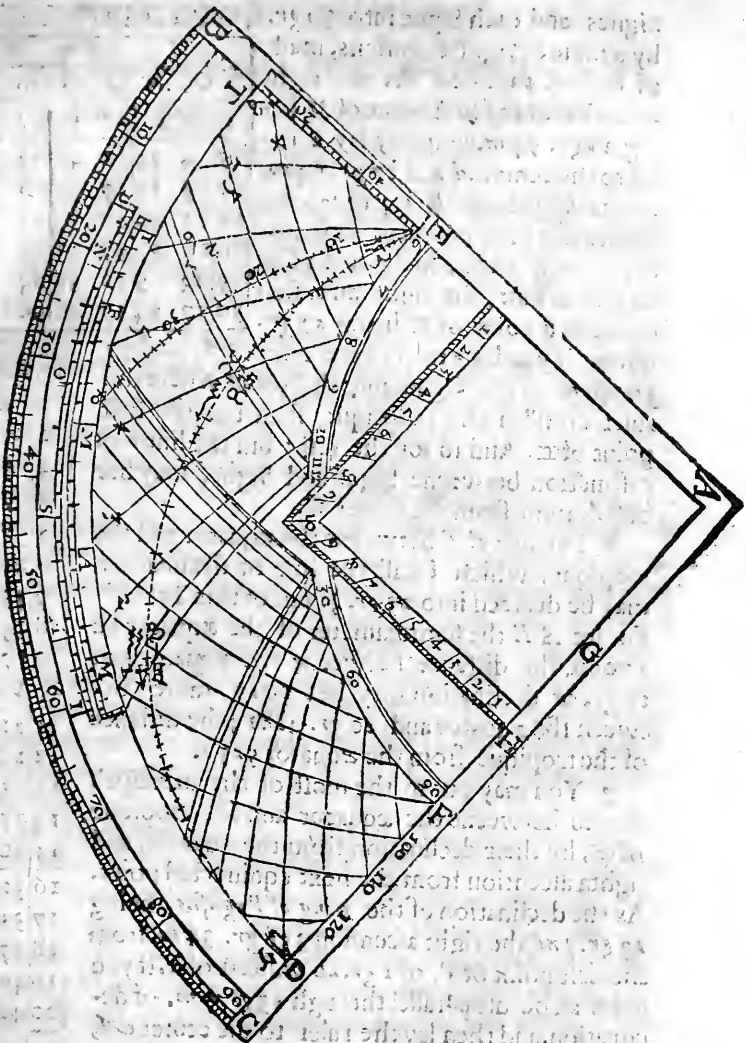
HAuing described these standing planes, I will now show the most of these conclusions by a small Quadrant: This might be done generally for all latitudes, by a quarter of the generall Astrolabe, described before in the vse of the *Sector*, *pag.* 58 . and particularly for any one latitude, by a quarter of the particular Astrolabe, there also described, *pag.* 63. which if it be a foote semidiameter, may shew the azimuth vnto a degree, and the time of the day vnto a minute; but for ordinary vse this smaller Quadrant may suffice, which may bee made portable in this manner.

1 Vpon the center *A*, and semidiameter *A B*, describe the arke *B C*: the same samidiameter will set of 60 *gr.* and the halfe of that will be 30 *gr.* which being added to the former 60 *gr.* will make the arke *B C* to be 90 *gr.* the fourth part of the whole circle, and thence comes the name of a Quadrant.

2 Leauing some little space for the inscription of the moneths and dayes, on the same center *A*, and semidiameter *A T*, describe the arke *T D*, which shall serue for either tropique.

3 Diuide the line *A T* in the point *E*, in such proportion, as that *A T* being 10000, *A E* may be 6556, and there draw another arke *E* F, which shall serue for the Equator, or *A E* being 10000 let *E T* be 5253.

4 Diuide *A* F the semidiameter of the æquator in the point *G*, so as *A* F being 10000, the line *A G* may bee 4343, and



nd on the center *G* and semidiameter *G D* describe the arke *E D*, which shall serue for a fourth part of the ecliptiq u.

5 This part of the ecliptique may be divided into three Signes,

Signes, and each Signe into 30 *gr.* by a table of right ascensions, made as before, *pag.* 60. As the right ascension of the first point of ♉ being 27 *gr.* 54 *m.* you may lay a ruler to the center *A* and 27 *gr.* 54 *m.* in the Quadrant *B C*, the point where the ruler crosseth the Ecliptique, shall be the first point of ♉. In like manner the right ascension of the first point of ♊ being 57 *gr.* 48. *m.* if you lay a ruler to the center *A*, and 57 *gr.* 48 *m.* in the Quadrant, the point where the ruler crosseth the ecliptique, shall bee the first point of ♊. And so for the rest: but the lines of distinction betweene Signe and Signe, may bee best drawne from the center *G*.

A Table of right Ascensions.

Gr.	♈ Gr.	♈ M.	♉ Gr.	♉ M.	♊ Gr.	♊ M.
0	0	0	27	54	57	48
5	4	35	32	42	63	3
10	9	11	37	35	68	21
15	13	48	42	31	73	43
20	18	27	47	33	79	7
25	23	9	52	38	84	32
30	27	54	57	48	90	0

6 The line *E* T betweene the æquator and the tropique, which I call the line of declination, may be divided into 23 *gr.* ½. out of this Table. For let *A E* the semidiameter of the æquator be 10000, the distance betweene the æquator and 10 *gr.* of declination may bee 1917. more; betweene the æquator and 20 *gr.* 4281; the distance of the tropique from the æquator 5252.

Gr.	Parts.
1	176
2	355
3	537
4	723
5	913
6	1106
7	1302
8	1503
9	1708
10	1917
11	2130
12	2348
13	2571
14	2799
15	3032
16	3270
17	3514
18	3763
19	4019
20	4281
21	4550
22	4825
23	5108
Tro	5252

7 You may put in the most of the principall starres betweene the æquator and the tropique of ♋, by their declination from the æquator, and righta ascention from the next equinoctiall point. As the declination of the *wing of Pegasus*, being 13 *gr.* 7 *m.* the right ascension 358 *gr.* 34 *m.* from the first point of ♈, or 1 *gr.* 26 *m.* short of it. If you draw an occult parallel through 13 *gr.* 7 *m.* of declination, and then lay the ruler to the center *A*, and 1 *gr.* 26 *m.* in the quadrant *B C*, the point where the ruler crosseth the parallell shall be the place for the *wing of Pegasus*, to which you may set

et the name and the time when he cometh to the South, at midnight in this maner, *W. Peg.* * 23 *Ho.* 54 *M.* and so for the rest of these fiue, or any other starres.

			Ho.	M.	R.	Ascen	Decl.	M
Pegasus wing	*	*March* 8	23	54	1	26	13	7
Arcturus	*	*October* 14	13	58	29	37	21	10
Lions heart	*	*August* 7	9	48	32	58	13	45
Buls eye	*	*May* 16	4	15	63	33	15	42
Vultures heart	*	*Ianua.* 1	19	33	66	56	7	58

8 There being space sufficient betweene the æquator and the center, you may there describe the quadrat, and diuide each of the two sides farthest from the center *A* into 100 parts, so shall the Quadrant be prepared generally for any latitude.

But before you draw the particular lines, you are to fit foure tables vnto your latitude.

First a table of meridian altitudes for diuision of the circle of dayes and moneths, which may be thus made: Consider the latitude of the place and the declination of the Sun for each day of the yeare. If the latitude and declination be alike both North or both South, add the declination to the complement of the latitude, if they bee vnlike, one North, and the other South, substract the declination from the complement of the latitude, the remainder will be the meridian altitude belonging vnto the day.

Thus in our latitude of 51 *gr.* 30 *m.* Northward, whose complement is 38 *gr.* 30 *m.* the declination vpon the tenth day of Iune will be 23 *gr.* 30 *m.* Northward, wherefore I adde 23 *gr.* 30 *m.* vnto 38 *gr.* 30 *m,* the summe of both is 62 *gr.* for the meridian altitude at the tenth of Iune, The declination vpon of December will be 23 *gr.* 30 *m.* Southward, wherefore I take these 23 *gr.* 30 *m.* out of 38 *gr.* 30 *m.* there will remaine 15 *gr.* for the meridian altitude at the tenth of December; and in this maner you may find the meridian altitude for each day of the yeere, and set them downe in a table.

Dies / Mo	0		5		10		15		20		25		30	
	Gr.	M.	Gr.	M.	Gr.	M.	Gr.	M.	Gr.	M.	Gr.	M.	Gr.	M.
Ianuary	16	31	17	24	18	26	19	37	20	57	22	24	23	58
February	24	17	25	59	27	45	29	35	31	29	33	25		
March	34	35	36	33	38	32	40	30	42	27	44	22	46	15
April	46	37	48	26	50	11	51	50	53	25	54	53	56	15
May	56	15	57	29	58	35	59	33	60	22	61	2	61	31
Iune	61	36	61	54	62	0	61	58	61	45	61	22	60	49
Iuly	60	49	60	6	59	14	58	13	57	4	55	48	54	24
August	54	7	52	36	50	59	49	17	47	31	45	41	43	49
September	43	26	41	30	39	33	37	36	35	38	33	41	31	46
October	31	46	29	53	28	3	26	16	24	35	22	59	21	29
Nouember	21	12	19	51	18	39	17	36	16	43	16	0	15	28
December	15	28	15	5	17	0	15	2	15	17	15	44	16	22

The Table being made, you may inſcribe the moneths, and dayes of each moneth into your quadrant, in the ſpace left below the tropique. For lay the ruler vnto the center *A*, and 16 *gr.* 31 *m.* in the quadrant *B C*, there may you draw a line for the end of December and beginning of Ianuary; then laying your ruler to the center *A*, and 24 *gr.* 17 *m.* in the quadrant, there draw the end of Ianuary and beginning of February, and ſo the reſt, which may be noted with *I*, *F*, *M*, *A*, *M*, *I*, &c. the firſt letters of each moneth, and will here fall betweene 15 *gr.* and 62. *gr.*

The ſecond Table which you are to fit, may ſerue for the drawing and diuiding of the horizon. For drawing of the horizon.

As the cotangent of the latitude,
to the tangent of the greateſt declination:
So the ſine of 90 *gr.*
to the ſine of interſection, where the horizon ſhall croſſe the tropiques.

So in our latitude of 51 *gr.* 30 *m.* we ſhall find the horizon to to cut the tropique in 33 *gr.* 9 *m:* wherefore if you lay the ruler to the center *A*, and 33 *gr.* 9. *m.* in the quadrant, the point where the ruler crosseth the tropique shall be the point where the horizon crosseth the tropique. And if you finde a point at *H*, in the line *AC*, whereon setting the compasses, you may bring the point at *E*, and this point in the tropique both into a circle, the point *H* shall be the center, and the arke so drawne shall be the horizon. Then for the diuision of this horizon.

As the sine of 90 *gr.*
to the sine of the latitude:
So the tangent of the horizon,
to the tangent of the arke in the quadrant, which shall diuide the horizon.

So in our latitude of 51 *gr.* 30 *m.* we shall finde 7 *gr.* 52 *m.* belonging to 10 *gr.* in the horizon, and 15 *gr.* 54 *m.* belonging 20 *gr.* And to the rest, as this Table.

Ho	Gr.	M.	Ho	Gr.	M.	Ho	Gr.	M	Ho	Gr.	M	Ho	Gr	M.	Ho	Gr.	M.
0	0	0	15	11	51	30	24	19	45	38	2	60	53	35	75	71	5
	0	47		12	39		25	11		39	1		54	41		72	19
	1	34		13	27		26	4		40	0		55	48		73	33
	2	21		14	16		26	57		41	0		56	56		74	48
	3	8		15	4		27	50		42	0		58	4		76	3
5	3	55	20	15	54	35	28	43	50	43	0	65	59	13	80	77	18
	4	42		16	43		29	37		44	1		60	22		78	33
	5	29		17	33		30	32		45	3		61	31		79	49
	6	17		18	22		31	27		46	5		62	41		81	5
	7	4		19	12		32	22		47	8		62	52		82	21
10	7	52	25	20	2	40	33	18	55	48	11	70	65	3	85	83	37
	8	39		20	53		34	14		49	14		66	15		84	53
	9	27		21	44		35	10		50	19		67	27		86	10
	10	14		22	36		36	7		51	24		68	39		87	26
	11	2		23	27		37	4		52	29		69	52		88	43
15	11	51	30	24	19	45	38	2	60	53	35	75	71	5	90	90	0

Wherefore you may lay the ruler to the center A, and 7 *gr.* 52 *m.* in the quadrant B C, the point where the ruler crosseth the horizon shall be 10 *gr.* in the horizon; and so for the rest: but the lines of distinction betweene each fift degree, will be best drawne from the center H.

The third table for drawing of the houre-lines, must be a Table of the altitude of the Sunne aboue the horizon at euery houre, especially when he cometh to the æquator, the tropiques, and some other intermediate declinations.

If the Sunne be in the æquator, and so haue no declination.

As the sine of 90 *gr.*
to the cosine of the latitude:
So the cosine of the houre from the meridian,
to the sine of the altitude.

Thus in our latitude of 51 *gr.* 30 *m.* at six houres from the meridian the Sun will haue no altitude, at fiue the altitude will be 9 *gr.* 17 *m*; at foure 18 *gr.* 8 *m*; at three 26 *gr.* 7 *m*; at two 32 *gr.* 37 *m.* at one 36 *gr.* 58 *m*; at noone it will be 38 *gr.* 30 *m.* equall to the complement of the latitude.

If the Sun haue declination, the meridian altitude will be found as before, for the Table of dayes and moneths.

If the houre proposed be six in the morning or six at night.

As the sine of 90 *gr.*
to the sine of the latitude:
So the sine of the declination
to the sine of the altitude.

Thus in our latitude the declination of the Sun being 23 *gr.* 30 *m.* the altitude will be found to be 18 *gr.* 11 *m*: the declination being 11 *gr.* 30 *m.* the altitude will be 9 *gr.*

If the houre proposed be neither twelue nor six.

As the cosine of the houre from the meridian,
to the sine 90 *gr.*

So

So the tangent of the latitude,
to the tangent of a fourth arke.

So in our latitude and one houre from the meridian, this fourth arke will be found to be 52 *gr.* 28 *m.* at two 55 *gr.* 26 *m.* at three 60 *gr.* 39 *m.* at foure 68 *gr.* 22 *m.* and at fiue houres from the meridian 78 *gr.* 22. *m.*

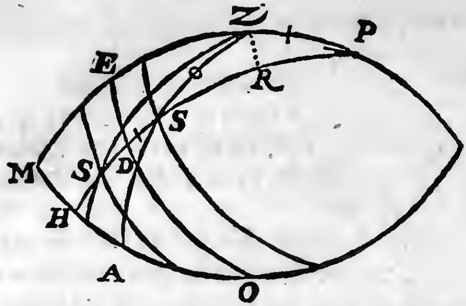
Then consider the declination of the Sun and the houre proposed; if the latitude and declination be both alike, as with vs in North latitude, North declination, and the houre fall betweene noone and six, take the declination out of the fourth arke, the remainer shall be your fift arke.

But if either the houre fall betweene six and midnight, or the latitude and declination shall be vnlike, adde the declination vnto the fourth arke, and the summe of both shall be your fifth arke: or if the summe shall exceed 90 *gr.* you may take the complement vnto 180 *gr.* This fifth arke being knowne:

As the sine of the fourth arke,
to the sine of the latitude:
So the cosine of the fift arke,
to the sine of the altitude.

Thus in our latitude of 51 *gr.* 30 *m.* Northward, the Sun hauing 23. *gr.* 30 *m.* of North declination, if it shall be required to finde the altitude of the Sun for seuen in the morning; here because the latitude and declination are both alike to the Northward, and the houre proposed falleth betweene noone and six, you may take 23 *gr.* 30 *m.* the arke of the declination out of 78 *gr.* 22 *m.* the fourth arke belonging to the fift houre from the meridian, so there will remaine 54 *gr.* 52 *m.* for your fift arke. Then working according to the Canon, you shall find,

As the sine of 78 *gr.* 22 *m.* your fourth arke,
to the sine of 51 *gr.* 30 *m.* for the latitude,



In rectangulo O D H,
vt O E Radius
ad E M Cotan.lat.
ita O D Cosi.horæ.
ad D H Tan.DH.
Cui æqualis est P R cuius compl. D R, novis dr. arcus quartus.

Conferatur arcus D H cum arcu declinationis D S, ita dabitur arcus H S, cuius compl. est S R & prius dr. arcus quintus. Vnde erit

vt Cosi. P R	*hoc est*	*vt Sin. D R,*
ad Cosi. P Z		*ad Sin. E Z,*
ita Cosi. S R		*ita Sin. H S*
ad Cosi. S Z		*ad Sin. A S.*

Hinc forte præstabit vocare H S arcum quintum ita secunda operatio instituetur per solos sinus,

Vel si libet subtractionem sinus quarti arcus evitare, inveniatur angulus O H D quod fieri potest varijs modis. Nam

1 vt Radius	*2 vt Sin. D H*	*3 vt Sin. D H*	*4 vt Sin. D R,*
ad Sin. ang. O	*ad Sin. O*	*ad Tan. D O*	*ad Sin. E Z,*
ita Cosi.lat. O D	*ita Sin. D O*	*ita Radius*	*ita Rad.*
ad Cosi.an. O H D	*ad Sin. H*	*ad tan.ang. H.*	*ad Sin. H.*

Idvento vtcunque angulo ad H, erit in rectangulo H A S,

vt sinus recti anguli H A S,
ad sinum arcus quinti H S,
ita sinus anguli ad horiz. S H A,
ad sin. solaris altitudinis S A.

As

So the sine of 35 *gr.* 8 *m.* the complement of your fift arke,
to the sine of 27 *gr.* 17 *m.* the altitude required.

If in the same latitude and declination, it were required to finde the altitude for fiue in the morning, here the houre falling betweene sixe and midnight; if you adde 23 *gr.* 30 *m.* vnto 78 *gr.* 22 *m.* the summe will be 101 *gr.* 52 *m.* and the complement to 180 *gr.* will be 78 *gr* 8 *m.* for your fifth arke. Wherefore

As the sine of 78 *gr.* 22 *m.*
to the sine of 51 *gr.* 30 *m.*
So the cosine of 78 *gr.* 8 *m.*
to the sine of 9 *gr.* 32 *m.* for the altitude required.

If in the same latitude of 51 *gr.* 30 *m.* Northward, the Sunne hauing 23 *gr.* 30 *m.* of South declination, it were required the altitude for nine in the morning; here because the latitude and declination are vnlike, the one North, and the other South, you may adde 23 *gr.* 30 *m.* the arke of declination, vnto 60 *gr.* 39 *m.* the fourth arke belonging to the third houre from the meridian, so shall you haue 84 *gr.* 9 *m.* for your fift arke. Wherefore

As the sine of 60 *gr.* 39 *m.*
to the sine of 51 *gr.* 30 *m.*
So the cosine of 84 *gr.* 9 *m.*
to the sine of 5 *gr.* 15 *m.* for the altitude required.

And so by one or other of these meanes you may finde the altitude of the Sunne for any point of the ecliptique at all houres of the day, and set them downe in such a Table as this.

A Table

A Table for the altitude of the Sunne in the beginning of each Signe at all houres of the day, calculated for 51 gr. 30 m. of North latitude.

Hor.		♋		♊	♌	♉	♍	♈	♎	♓	♏	♒	♐	♑	
		Gr.	M.	Gr.	M.	Gr.	M.	Gr.	M.	Gr.	M.	Gr.	M	Gr.	M
12		62	0	58	42	50	0	38	30	27	0	18	18	15	0
11	1	59	43	56	34	48	12	36	58	25	40	17	6	13	52
10	2	53	45	50	55	43	12	32	37	21	51	13	38	10	30
9	3	45	42	43	6	36	0	26	7	15	58	8	12	5	15
8	4	36	41	34	13	27	31	18	8	8	33	1	15		
7	5	27	17	24	56	18	18	9	17	0	6				
6	6	18	11	15	40	9	0	0	0						
5	7	9	32	6	50									11	37
4	8	1	32											21	40

Laſtly, you may find what declination the Sun hath when he riſeth or ſetteth at any houre,

> As the ſine of 90 *gr.*
> to the ſine of the houre from ſixe:
> So the cotangent of the latitude,
> to the tangent of the declination.

And ſo in the latitude of 51 *gr.* 30 *m.* you ſhall finde that when the Sun riſeth, either at fiue in the Summer, or ſeuen in the Winter, his declination is 11 *gr.* 37 *m.* when he riſeth at foure in the Summer, or eight in the Winter, his declination is 21 *gr.* 40 *m.* which may be alſo ſet downe in the Table.

That done, you may there ſee that in this latitude the meridian altitude of the Sunne in the beginning of ♋ is 62 *gr.* in ♊ 58 *gr.* 42 *m.* in ♉ 50 *gr.* in ♈ 38 *gr.* 30 *m.* &c. But the beginning of ♋ and ♑ is repreſented by the tropiques *T* D, drawne at 23 *gr.* 30 *m.* of declination, and the beginning of ♈ and ♎, by the æquator *E F.* If you draw an occult parallell betweene the æquator and the tropique, at 11 *gr.* 30 *m.* of declination,

clination, it shall represent the beginning of ♉, ♍, ♏, and ♓; if you draw an other occult parallell through 20 *gr.* 12 *m.* of declination, it shall represent the beginning of ♊, ♌, ♐, and ♒.

Then you may lay a ruler to the center *A*, and 62 *gr.* in the quadrant B C, and note the point where it crosseth the tropique of ♋; then moue the ruler to 58 *gr.* 52 *m.* and note where it crosseth the parallell of ♊; then to 50 *gr.* and note where it crosseth the parallell of ♉, and againe to 38 *gr.* 30 *m.* noting where it crosseth the æquator; so the line drawne through these points shall shew the houre of 12 in the Summer, while the Sunne is in ♈, ♉, ♊, ♋, ♌, or ♍. In like maner if you lay the ruler to the center *A*, and 27 *gr.* in the quadrant, and note the point where it crosseth the parallel of ♓, then moue it to 18 *gr.* 18 *m.* and note where it crosseth the parallell of ♒; and againe to 15 *gr.* noting where it crosseth the tropique of ♑; the line drawne through these points shall shew the houre of 12 in the Winter, while the Sunne is in ♎, ♏, ♐, ♑, ♒ and ♓, and so may you draw the rest of these houre-lines: onely that of 7 from the meridian in the Summer, and 5 in the Winter, will crosse the line of declination at 11 *gr.* 37 *m.* and that of 8 in the Summer, and 4 in the Winter at 21 *gr.* 40 *m.*

The fourth table for drawing of the azimuth lines, must likewise be fitted for the altitude of the Sun aboue the horizon at euery azimuth, especially when he commeth to the æquator, the tropiques, and some other intermediate declination.

If the *Sunne* be in the æquator, and so haue no declination:

As the sine of 90 *gr.*
to the cosine of the azimuth from the meridian:
So the cotangent of the latitude,
to the tangent of the altitude at the æquator.

Thus in our latitude of 51 *gr.* 30 *m.* at 90 *gr.* from the meridian, the *Sunne* will haue no altitude; at 80 *gr.* the altitude will be 7 *gr*. 52 *m*; at 70 *gr*. it will be 15 *gr*. 30 *m*; at 60 *gr*. it will be 21 *gr*. 41 *m*.

If the Sun haue declination, the meridian altitude will be easily found as before, for the table for dayes and moneths. And for all other azimuths.

As the sine of the latitude,
to the sine of the declination:
So the cosine of the altitude at the æquator,
to the sine of a fourth arke.

When the latitude and declination are both alike in all azimuths from the prime verticall vnto the meridian, adde this fourth arke vnto the arke of altitude at the æquator.

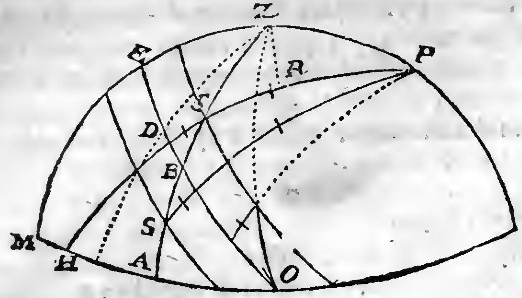
When the latitude and declination are both alike, and the azimuth more then 90 *gr*. distant from the meridian, take the altitude at the æquator out of this fourth arke.

When the latitude and declination are vnlike, take this fourth arke out of the arke of altitude at the æquator, so shall you haue the altitude of the Sun belonging to the azimuth.

Thus in our latitude of 51 *gr*. 30 *m*. Northward, if it were required to finde the altitude of the Sunne in the azimuth of 60 *gr*. from the meridian, when the declination is 23 *gr*. 30 *m*. Northward, you may finde the altittde at the æquator belonging to this azimuth to be 21 *gr*. 41 *m*. by the former Canon, and by this last Canon you may finde the fourth arke to be 28 *gr*. 15 *m*. Then because the latitude and declination are both alike to the Northward, if you adde them both together, you shall haue 49 *gr*. 56 *m*. for the altitude required.

If the declination had been 23 *gr*. 30 *m*. to the Southward, you should then haue taken this fourth arke out of the ark at the æquator, which because it cannot here be done, it is at a signe that the *S*unne is not then aboue the horizon. But if you take the arke at the æquator out of this fourth arke, you shall haue 6 *gr*. 34 *m*. for the altitude of the Sunne when he is

in



O M Radius
Me Cotan. lat.
O A Cosi. azim.
AB Tan. æqua.

EZ Sin. lat.
ZB Cosi. AB.
DS Sin. decli.
SB Sin. arc. 4.

Tables for the altitude of the sun in the beginning of each signe for every tenth azimuth.

	Merid	10	20	30	40	50	60	70	80	90
					Lat. 50 Gr. 0 M					
♋	63 30	63 14	62 23	60 54	58 42	55 32	51 25	46 2	39 17	31 22
♊	60 12	59 54	59 0	57 23	55 1	51 43	47 18	41 40	34 47	26 48
♉	51 30	51 9	50 3	48 10	45 23	41 34	36 38	30 30	23 12	15 5
♈	40 0	39 34	38 15	36 0	32 44	28 20	22 45	16 0	8 17	0 0
♓	28 30	28 0	26 27	23 50	20 5	15 6	8 52	1 30	6 38	
♒	19 48	19 14	17 31	14 37	10 27	4 57	1 48	9 40	18 13	
♑	16 30	14 54	14 7	11 6	6 46	1 8	5 55	14 2	22 43	
					Lat. 51 Gr.					
♋	62 30	62 14	61 22	59 54	57 40	54 35	50 27	45 8	38 33	30 53
♊	59 12	58 54	57 59	56 23	54 0	50 43	46 22	41 51	34 6	26 23
♉	50 30	50 7	49 3	47 11	44 25	40 40	35 47	29 48	22 43	14 52
♈	39 0	38 34	37 16	35 3	31 49	27 30	22 2	15 29	8 0	0 0
♓	27 30	27 1	25 29	22 55	19 13	14 20	8 17	1 10	6 43	
♒	18 48	18 14	16 33	13 43	9 38	4 17	2 18	9 53	18 6	
♑	15 30	14 54	13 10	10 12	5 58	0 25	6 23	14 10	22 33	
					Lat. 52 Gr.					
♋	61 30	61 14	60 22	58 52	56 38	53 33	49 29	44 14	37 58	30 24
♊	58 12	58 54	56 28	56 22	53 0	49 43	44 25	40 0	33 28	26 0
♉	49 30	49 9	48 3	46 11	43 26	39 44	34 58	29 6	22 15	14 40
♈	38 0	37 35	36 17	34 5	30 54	26 40	21 20	14 57	7 44	0 0
♓	26 30	26 1	24 31	22 0	18 22	13 26	7 42	0 48	6 46	
♒	17 48	17 16	15 36	12 48	8 49	3 37	2 45	10 6	18 0	
♑	14 30	14 56	12 12	9 18	5 10	0 13	6 49	14 19	22 30	

in the azimuth of 60 *gr.* from the North, and 120 gr. from the South part of the meridian. The like reason holdeth for the rest of these altitudes, which may be gathered and set downe in a table.

Lastly when the Sun riseth or setteth vpon any azimuth, to find his declination.

> As the sine of 90 *gr.*
> to the cosine of the latitude:
> So the cosine of azimuth from the meridian,
> to the sine of the declination.

And thus in our latitude of 51 *gr.* 30 *m.* when the azimuth is 80 *gr.* from the meridian, the declination will be found to be 6 *gr.* 12 *m*; if the azimuth be 70 *gr.* the declination will be found 12 gr. 18 m; if 60 gr. then 18 gr. 8 m. And so for the rest, which may be also set downe in the Table.

A Table for the altitude of the Sunne in the bginning of each signe for euery tenth azimuth, in 51 gr. 30 m. of North latitude.

Az.	♋		♊ ♌		♉ ♍		♈ ♎		♓ ♏		♒ ♐		♑	
	Gr.	M.	Gr.	M.	Gr.	M.	Gr.	M.	Gr.	M.	Gr.	M	Gr.	M.
0	62	0	58	42	50	0	38	30	27	0	18	18	15	0
10	61	43	58	24	49	38	38	4	26	30	17	45	14	25
20	60	51	57	28	48	33	36	46	25	0	16	5	12	41
30	59	52	55	52	46	40	34	34	22	27	13	15	9	45
40	57	10	53	29	43	55	31	21	18	48	9	14	5	34
50	54	3	50	12	40	11	27	5	13	58	3	57	0	6
60	49	56	45	53	35	23	21	41	8	0				
70	44	40	40	25	29	27	15	13	1	0				
80	38	11	33	46	21	29	7	52						
90	30	38	26	10	14	25	0	0						
100	22	27	18	2	6	45							6	12
110	14	14	9	58									12	18
120	6	34	2	30									18	8

That

That done, if you would draw the line of East or West, which is 90 gr. from the meridian, lay the ruler to the center *A*, and 30 gr. 38. *m.* numbred in the quadrant from *C* toward *B*, and note the point where it crosseth the tropique of ♋; then moue the ruler to 26 gr. 10 m. and note where it crosseth the parallell of ♊; then to 14 gr. 45 m. and note where it crosseth the parallell of ♉; then to 0 gr. 0 m. and you shall find it to crosse the æquatour in the point F; so a line drawne through these points, shall shew the azimuth belonging to East and West. The like reason holdeth for all the rest.

These lines being thus drawne, if you set two sights vpon the line *A C*, and hand a thread and plummet on the center, *A* with a bead vpon the thread, the foreside of the quadrant shall shall be fully finished.

On the backside of the quadrant you may place the Nocturnall described before in the vse of the Sector pag. which consisteth of two parts.

The one is an houre-plane diuided æqually according to the 24 houres of the day and each houre into quarters, or minutes as the plane will beare. The center represents the North pole, the line drawne through the center from XII to XII, stands for the meridian and the lower XII stands for the houre of XII at midnight.

The other part is a rundle for such starres as are neere the north pole together with the twelue moneths, and the dayes of each moneth fitted to the right ascension of the Sunne and starres this in manner.

First consider where the Sun will be at the beginning of the 5, 10, 15, 20, 25, 30, and if you will euery day of each moneth, and finde the right ascension belonging to the place of the sun as I shew before *Pag.*

For example the sun at midnight the last of December or beginning of Ianuary will be *communibus annis* about 20 gr. 40 *m.* of ♑ whose right ascension is 292 gr. 20 m. At midnight the last of Ianuary or beginning of February he will be about 22 gr. 12 m. of ♒ whose right ascension is 324 gr. 35 m. and so the rest which may be set downe in a table.

That done consider the longitude and latitude of the starres and thereby finde their right ascension and declination as I shew before, *Pag.* and set them downe in a Table. These Tables thus made, let the vttermost part of the rundle be made euen with the innermost circle of the houre-plane, and a conuenient space allowed to containe the devisions for the dayes and names of the moneths. Then lay the center of this rundle vpon the center of some other circle divided into 360 *gr.* and by the center and 292 *gr.* 20 *m.* in that circle draw a line for the beginning of January. In like maner by the center and 324 gr. 35 *m.* draw a line for the end of January and beginning of February, and so the rest of the dayes of each moneth.

For the inscription of the starres let one of the lines from the center as that at the beginning of July, or rather let a moueable index be diuided from the center toward the inward circle of the moneths into 40 gr. more or lesse, which may be done for speed equally, but for exactnesse in such maner as the semidiameter of the generall Astrolabe was divided before, *Pag.* So laying the Index to the right ascension in the outward circle you may prick downe the starres by their declination in the Index.

For example, if the right ascension of the pole-starre be 6 gr. 28 *m.* end his declination 87 gr. 20 m. hauing set the center of the Index both to the center of the rundle and of the other circle, turne the Index to 6 gr. 28 m. in that outward circle, and prick downe the starre by 87 gr. 20 m. in the edge of the Index, that is at the distance of 2 gr. 40 m. from the pole. The like reason holdeth for the rest of the starres, which may be distinguished according to their magnitudes, and then be reduced into their formes, as in the example. So the quadrant will bee fitted both for day and night.

Of

CHAP. II.

Of the vse of the Quadrant in taking the altitude of the Sunne, Moone, and Starres.

THe Quadrant is the fourth part of a circle, diuided equally into 90 gr. and here numbred by 10. 20. 30. &c. vnto 90 gr. each degree being subdiuided into 4.

Lift vp the center of the Quadrant, so as the thread with the plummet may play easily by the side of it, and the *Sunne* beames may passe through both the sights; so shall the degrees cut by the thread, shew what is the altitude at the time or obseruation, as may appeare by this example.

Vpon the 14 day of Aprill, about noone, the *Sun*-beames passing through both the sights, the thread fell vpon 51 gr. 20 *m.* and this was the true meridian altitude of the *Sunne* for that day in this our latitude of 51 gr. 30 *m.* for which this Quadrant was made.

Againe, towards three of the clock in the afternoone, the thread fell vpon 38 gr. 40 m. and such was the *Sunnes* altitude at that time.

CHAP.

CHAP. III.

Of the Ecliptique.

1 *The place of the Sunne being giuen to finde his right ascension.*

THe Ecliptique is here represented by the arke, figured with the characters of the twelve *Signes*, ♈, ♉, ♊, &c. each *Signe* being diuided vnequally into 30 gr. and they are to be reckoned from the character of the *Signe*.

Let the thread be laid on the place of the *Sunne* in the Ecliptique, and the degrees which it cutteth in the Quadrant shall be the right ascention required.

As if the place of the *Sunne* giuen be the fourth degree of ♊, the thread laid on this degree shall cut 62 degrees in the Quadrant, which is the right ascension required.

But if the place of the Sunne giuen be more then *90 gr.* from the beginning of ♈, there must be more then *90 gr.* allowed to the right ascension; For this instrument is but a quadrant: and so if the Sunne be in 26 gr. of ♋, you shall find the thread to fall in the same place, and yet the right ascension to the 118 gr.

2 *The right ascention of the Sunne being given, to finde his place in the Ecliptique.*

Let the thread be laid on the right ascension in the Quadrant, and it shall crosse the place of the Sun in the Ecliptique, as may appeare in the former example.

CHAP.

CHAP. IIII.

Of the line of declination.

1 The place of the Sunne being given to finde his declination.

THe line of declination is here drawne from the center to the beginning of the Quadrant, and divided from the beginning of ♈ downward into 23 gr. 30 m.

Let the thread be laid, and the beade set on the place of Sunne in the ecliptique; then moue the thread to the line of declination, and there the bead shall fall upon the degrees of the declination required.

As if the place of the Sunne giuen be the fourth degree of ♊, the bead first set to this place, and then moued to the line of declination, shall there shew the declination of the Sunne at that time to be 21 gr. from the æquator.

2 The declination of the Sunne being given, to finde his place in the Ecliptique.

Let the thread and beade be first laid to the declination, and then moued to the Ecliptique.

As if the declinatiou be 21 gr. the bead first set to this declination, and then moued to the ecliptique, shall there shew the fourth of ♊, the fourth of ♐, the 26 of ♋, and the 26 of ♑; and which of these foure is the place of the Sunne, may appeare by the quarter of the yeere.

CHAP. V.

Of the circle of Moneths and Dayes.

THis circle is here represented by the arke, figured with these letters, *I*, *F*, *M*, *A M*, &c. signifying the moneths January, February, March, Aprill, &c. each moneth being divided vnequally, according to the number of the dayes that are therein.

A Table for the inscription of the moneths in the Nocturnall.

Dies	0		5		10		15		20		25		30	
M	Gr.	M.	Gr.	M.	Gr.	M.	Gr.	M.	Gr.	M.	Gr.	M.	Gr.	M.
Ian.	292	20	297	46	303	7	308	21	313	30	318	36	323	36
Feb.	324	35	329	28	334	16	339	1	343	42	348	21		
Ma.	351	17	355	52	0	26	4	58	9	30	14	2	18	34
Apr.	19	30	24	4	28	42	33	23	38	5	42	52	47	42
May	47	42	52	35	57	32	62	34	67	39	72	45	77	52
Iune	78	55	84	5	89	17	94	28	99	39	104	48	109	55
Iuly	109	55	115	0	120	0	124	58	129	54	134	45	139	30
Aug	140	27	145	9	149	48	154	25	159	0	163	32	168	0
Sept.	168	57	173	26	177	56	182	26	186	56	191	28	196	5
Octo	196	5	200	45	205	25	210	12	215	3	220	0	225	0
No.	226	2	231	10	236	23	241	40	247	2	252	30	258	2
Dec.	258	2	263	35	269	8	274	42	280	16	285	46	291	15

CHAP.

1 *The day of the moneth being given, to finde the altitude of the Sunne at noone.*

Let the thread be laid to the day of the moneth, and the degrees which it cutteth in the Quadrant ſhall be the meridian altitude required.

As if the day giuen be the 15 of May, the thread laid on this day ſhall cut 59 *gr.* 30 *m.* in the quadrant, which is the meridian altitude required.

2 *The meridian altitude being given, to finde the day of the moneth.*

The thread being ſet to the meridian altitude, doth alſo fall on the day of the moneth.

As if the altitude at noone be 59 *gr.* 30 *m.* the thread being ſet to this altitude, doth fall on the 15 of May, and the 9 of July; and which of theſe two is the true day, may be knowne by the quarter of the yeere, or by another dayes obſeruation. For if the altitude proue greater, the thread will fall on the 16 day of May and the 8 of July: or if it proue leſſer, the thread will fall on the 14 of May and the 10 of July; whereby the queſtion is fully anſwered.

CHAP. VI.

Of the Houre-lines.

THat arke which is drawne vpon the center of the quadrant by the beginning of declination, doth here repreſent the æquator: that arke which is drawne by 23 *gr.* 30 *m.*

of declination, and is next aboue the circle of moneths and dayes, representeth the tropiques: those lines which are betweene the æquator and the tropiques, being vndivided and numbred at the æquator by 6,7,8,9,10,11,12. at the tropique by 1,2,3,4,&c. do represent the houre-circles: that which is drawne from 12 in the æquator to the middle of Iune, representeth the houre of 12 at noone in the Summer; and those which are drawn with it to the right hand, are for the houres of the day in the Summer, and the houres of the night in the Winter. That which is drawne from 12 in the æquator to the middle of December, representeth the houre of 12 in the Winter; and those which are drawne with it to the left hand, are for the houres of the day in the Winter, and the houres of the night in the Summer; and of both these, that which is drawne from 11 to 1, serves for 11 in the forenoone, and 1 in the afternoone. That which is drawne from 10 to 2, serues for 10 in the forenoone, and 2 in the afternoone: for the Sunne on the same day is about the same height two houres before noone, as two houres after noone. The like reason holdeth for the rest of the houres.

1. *The day of the moneth, or the height at noone being knowne, to finde the place of the Sunne in the Ecliptique.*

The thread being laid to the day of the moneth, or the height at noone, (for one giues the other by the former proposition) marke where it crosseth the houre of 12, and set the bead to that intersection; then moue the thread till the beade fall on the ecliptique, and it shall fall on the place of the Sunne.

As if the day giuen be the 15 of May, or the meridian altitude 59 *gr.* 30 *m.* lay the thread accordingly, and put the bead to the intersection of the thread with the houre of 12; then moue the thread till the bead fall on the ecliptique, and it shall there shew the fourth of ♊, the fourth of ♐, the 26

 of

of ♋, and the 26 of ♈; and which of these is the place of the Sunne, may appeare by the quarter of the yeare, or another dayes obseruation.

2 *The place of the Sunne in the Ecliptique being knowne, to finde the day of the moneth, &c.*

Let the thread and bead bee first laid on the place of the Sunne in the Ecliptique, and then moued to the line of 12.

As if the place of the Sunne giuen be the fourth of ♊, the bead being laid to this degree, and then moued to the houre of 12, in the Summer, the thread will fall on the 15 day of May, and the 9 of Iuly; or if it be moved to the houre of 12 in the Winter, the thread will fall on the 6 of Ianuary and the 16 of Nouember; which of these is the day of the moneth required, may appeare by the quarter of the yeare.

In this and the former propositions, you haue two wayes to rectifie the bead, by the place of the Sunne, and by the day of the moneth; the better way is by the place of the Sunne, for in the other the Leap-yeare may breed some small difference.

There is yet a third way. For the Sea-men hauing a table for the declination on each day of the yeare, may set the bead thereto in the line of declination.

4 *The houre of the day being giuen to find the altitude of the Sunne aboue the horizon.*

The bead being set for the time by either of the three wayes, let the thread be moved from the houre of 12 toward the line of declination, till the bead fall on the houre giuen; and the degrees which it cuts in the Quadrant, shall shew the altitude of the Sunne at that time.

As if the time giuen be the tenth of Aprill, the Sunne being then in the beginning of ♉, the bead being rectified, you shall finde the height at noone 50 *gr.* 0 *m.* at 11 in the morning 48 *gr.* 12 *m.* at 10 but 43 *gr.* 12 *m.* at 9 but 36 *gr.* at 8 but 27 *gr.* 30 *m.* at 7 but 18 *gr.* 18 *m.* at 6 but 9 *gr.* at 5 it meeteth with the line of declination, and hath no altitude at all, and therefore you may thinke it did rise much about that houre.

Then if you moue the thread againe from the line of declination toward the houre of 12, you shall find that the Sunne is 8 *gr.* 33 *m.* below the horizon at 4 in the morning, and neere 16 *gr.* at 3, and 21 *gr.* 51 *m.* at 2, and 25 *gr.* 40 *m.* at 1, and 27 *gr.* at midnight.

4 *The altitude of the Sunne being giuen, to finde the houre of the day.*

The altitude being obserued as before, let the bead bee set for the time, then bring the thread to the altitude, so the bead shall shew the houre of the day.

As if the 10 of April hauing set the bead for the time, you shall find by the quadrant, the altitude to bee 36 *gr.* the bead at the same time will fall vpon the houre-line of 9 and 3: wherefore the houre is 9 in the forenoone, or 3 in the afternoone. If the altitude be neere 40 *gr.* you shall find the bead at the same time to fall halfe way betweene the houre-line of 9 and 3, and the houre-line of 10 and 2: wherefore it must be either halfe an houre past 9 in the morning, or halfe an houre past 2 in the afternoone; and which of these is the true time of the day, may be soone knowne by a second obseruation: for if the Sunne rise higher, it is the forenoone; if it become lower, it is the afternoone.

5 The

5 *The houre of the night being giuen, to find how much the Sunne is below the horizon.*

The Sunne is alwayes so much below the horizon at any houre of the night, as his opposite point is aboue the horizon at the like houre of the day; and therefore the beade being set, if the question be made of any houre of the night in the Summer, then moue it to the like houre of the day in the Winter; if of any houre of the night in Winter, then moue it to the like houre of the day in Summer; so the degrees which the thread cutteth in the Quadrant, shall shew how much the Sun is below the horizon at that time.

As if it be required to know how much the Sunne is below the horizon the 10 of April at 4 of the clocke in the morning; the bead being set to his place according to the time in the Summer houres, bring it to 4 of the clocke in the afternoone in the Winter houres, and so shall you finde the thread to cut 8 *gr.* and about 30 *m.* in the quadrant; and so much is the Sun below the horizon at that time.

6 *The depression of the Sunne supposed, to giue the houre of the night with vs; or the houre of the day to our Antipodes.*

Here also because the Sunne is so much aboue the horizon at all houres of the day, as his opposite point is below the horizon at the like houre of the night; therfore first set the bead according to the time, then bring the thread to the degree of the Suns depression below the horizon, so shall the bead fall on the contrary houre-lines, and there shew the houre of the night in regard of vs, which is the like houre of the day in regard of vs, which is the like houre of the day to our Antipodes.

As if the 10 of April the Sunne being then in the beginning

ning of ♉, and by supposition 8 *gr.* 30 *m.* below the horizon in the East, it be required to know what time of the night it is; first set the bead according to the day in the Summer houres, then bring the thread to 8 *gr.* 30 *m.* in the quadrant, so shall the bead fall among the Winter houres, on the line of 4 of the clocke in the afternoone: wherefore to our Antipodes it is 4 of the clocke in their afternoone, and to vs it is then 4 of the clocke in the morning.

7 *The time of the yeare or the place of the Sunne being giuen, to find the beginning of day-breake, and end of twi-light.*

This proposition differeth little from the former: for the day is said to begin to breake, when the Sun cometh to be but 18 *gr.* below our horizon in the East, and twi-light to end when it is gotten 18 *gr.* below the horizon in the West: wherefore let the bead be set for the time, and then bring the thread to 18 *gr.* in the quadrant, so shall the bead fall on the contrary houre-lines, and there shew the houre of twi-light as before.

So if it be required to know at what time the day begins to breake on the tenth of April, the Sun being then in the beginning of ♉; first set the bead according to the time in the Summer houres, and then bring the the thread to 18 *gr.* in the quadrant, so shall the bead fall among the Winter houres a little more then a quarter before 3 in the morning; and that is the time when the day begins to breake vpon the tenth of April.

CHAP.

CHAP. VII.

Of the Horizon.

THe Horizon is here represented by the arke drawne, from the beginning of declination towards the end of Febuary, diuided vnequally, and numbred by 10. 20. 30. 40. &c.

1 *The day of the moneth, or the place of the Sunne being knowne, to finde the amplitude of the Sunnes rising and setting.*

Let the bead rectified for the time, be brought to the horizon, and there it shall shew the amplitude required.

As if the day giuen bee the 15 of May, the Sunne being in the fourth degree of ♊, the bead rectified and brought to the horizon, shall there fall on 35 *gr.* 8. *m.* such is the amplitude of the Sunnes rising from the East, and of his setting from the West; which amplitude is alwayes North when the Sunne is in the Northerne signes, and when he is in the Souththerne signes alwayes Southward.

2 *The day of the moneth, or the place of the Sunne being giuen, to finde the ascensionall difference.*

Let the bead rectified for the time, be brought to the horizon, so the degrees cut by the thread in the quadrant, shall shew the difference of ascensions.

As if the day giuen be the 15 of May, the Sunne being in the fourth degree of ♊, let the bead be rectified and brought to the horizon; so shall the thread in the quadrant shew the ascensionall difference to be 28 *gr.* and about 50 *m.*

Vpon the ascensionall difference depends this Corollarie.

To find the houre of the rising and setting of the Sun, and thereby the length of the day and night.

The time of the Sunnes rising may be guessed at by the 3 of the last *Cap.* but here by the ascensionall difference it may be better found, and that to a minute of time. For if the ascensionall difference bee conuerted into time, allowing an houre for 15 *gr.* and 4 minutes of an houre for each degree, it sheweth how long the Sun riseth before six of the clocke in the Summer, and after six the Winter.

As if the day giuen be the 15 of May, the Sun being in the fourth of ♊, and his ascensionall difference found as before 28 *gr.* 50 *m*; this conuerted into time, maketh 1 *ho.* and somewhat more then 55 *m.* of an houre: wherfore the Sun at that time, in regard it was summer, rose 1 *ho.* and full 55 *m.* before 6 of the clocke; and so hauing the quantity of the semidiurnall arke, the length of the day and night need not be vnknowne.

CHAP. VIII.

Of the fiue Starres.

I Might haue put in more starres, but these may suffice for the finding of the houre of the night at all times of the yeare: and first I make choice of *Ala Pegasi*, a starre in the extremity of the *wing of Pegasus*, in regard in wants but 6 minutes of time of the beginning of ♈; but but because it is but of the second magnitude, and not alwayes to be seene, I made choice of foure more, one for each quarter of the Ecliptique.

tique, as *Oculus* ♉ the Buls eye, whose right ascension conuerted into time, is 4 *ho.* 15 *m*; then of *Cor* ♌ the Lions heart, whose right ascension is 9 *ho.* 48 *m*; next of *Arcturus*, whose right ascension is 13 *H.* 58 *m*; and lastly of *Aquila*, or the Vultures heart, whose right ascension is 19 *H.* 33 *m.* These fiue starres haue all of them Northerne declination; and if any others, some of these will be seene at all times of the yeere.

The vse of them is,

The altitude of any of these fiue Starres being knowne to find the houre of the night.

First put the beade to the starre which you intend to obserue, take his altitude, and finde how many houres he is from the meridian by the fourth *Prop.* of the sixt *Chap*; then out of the right ascension of the starre, take the right ascension of the sun conuerted into houres, and marke the difference; for this difference being added to the obserued houre of the starre from the meridian, shall shew how many houres the sunne is gone from the meridian, which is in effect the houre of the night.

As if the 15 of May, the sun being in the fourth of ♊, I should set the beade to *Arcturus*, and obseruing his altitude should find him to be in the West about 52 *gr.* high, and the bead to fall on the houre-line of 2 afternoone, the houre would be 11 *ho.* 50 *m.* past noone, or 10 *m.* short of midnight.

For 62 *gr.* the right ascension of the sunne, conuerted into time, makes 4 *ho.* 8 *m.* which if we take out of 13 *ho.* 58 *m.* the right ascension of *Arcturus*, the difference will be 9 *ho.* 50 *m.* and this being added to 2 *ho.* the obserued distance of *Arcturus* from the meridian, shewes the houre of the night to be 11 *ho.* 50 *m.* Another example will make all more plaine.

If the 9 of Iuly the sunne being then in 26 *gr.* of ♋, I should set the beade of *Oculus* ♉, and obseruing his altitude should find him to be in the East about 12 *gr.* high, and the bead to fall on the houre-line of 6 before noone, which is

18 *ho.* past the meridian, the houre of the night would be better then a quarter past 2 of the clocke in the morning.

For 118 *gr.* the right ascension of the Sun, conuerted into time, makes 7 *ho.* 52 *m*; this taken out of 4 *ho.* 15 *m.* the right ascension of *Oculus* ♉, adding a whole circle, (for otherwise there could be no substraction) the difference will be 20 *ho.* 23 *m.* and this being added to 18 *ho.* which was the obserued distance of *Oculus* ♉ from the meridian, shewes that the Sun (abating 24 *ho.* for the whole circle) is 14 *ho.* 23 *m.* past the meridian, and therefore 23 *m.* past 2 of the clocke in the morning.

If the *Nocturnall* bee placed on the backside of the quadrant you may auoid this equation of right ascensions. For knowing the time of the yeere when the starre will be in the south at midnight you may bring that time to the houre obserued, then will the day of the moneth wherein you made the obseruation point at the houre of the night required.

As in the first example where on the 15 of May the bead set to Arcturus fell on the houre-line of 2 afternoone, because Arcturus will be in the south the 14 of October compleat at midnight you may place the 14 of October at the houre of 2, so the 15 of May will point to 11 *ho.* 50 *min.*

In the second example, where the 9 of Iuly the bead set to the Bulls eye fell on the houre-line of 6 before noone, because the Bulls eye will be in the south the 16 of May compleat at midnight you may tourne the 16 of may to the houre of 6, and so you shall finde the 9 of Iuly to point to 2 *ho.* 23 *min.* as before.

CHAP.

CHAP. IX.

Of the Azimuth-lines.

THose lines which are drawne betweene the æquator and the tropiques, on that side of the quadrant which is nearest vnto the sights, and are numbred by 10. 20. 30. &c. doe represent the azimuths, the vttermost to the left hand representeth the meridian, that which is numbred with 10 the tenth azimuth from the meridian, and that which is numbred with 20 the twentith, and so the rest. Those lines which are drawne from the æquator to the left hand, doe shew the azimuth in the Summer; and those other to the right hand, doe shew the same in the Winter. The vse of them is.

1 *The azimuth whereon the Sunne beareth from vs being knowne, to find the altitude of the Sun aboue the horizon.*

First let the bead be set for the time, as in the former Chapter, then moue the thread vntill the bead fall on the azimuth; so the degrees which the thread cutteth in the quadrant, shall shew the altitude of the Sun at that time. Where you are to obserue, that seeing the azimuths are drawne on the right side of the quadrant, you are also to begin to number the degrees of the Sunnes altitude from the right hand toward the left. As if the sights had been set on the line *AB*, and you had turned your right hand towards the Sun in obseruing of of his altitude, contrary to our practise in the former Chapter.

As if the time giuen were the 2 of August, when the Sun hath about 15 *gr.* of North declination, you may set the bead for the time, so you shall find the height at noone when the

Sun is in the south, to be 53 *gr.* 30 *m.* when he is 10 *gr.* from the south 53 *gr.* 10 *m.* when 20 *gr.* then about 52 *gr.* 8 *m.* when 30 *gr.* then 50 *gr.* 20 *m.* when 40 *gr.* then 47 *gr.* 48 *m.* when 50 *gr.* then 44 *gr.* 12 *m.* when 60 *gr.* then 39 *gr.* 35 *m.* when 70 *gr.* then 33 *gr.* 50 *m.* when 80 *gr.* then 27 *gr* when he is in the East or West 90 *gr.* from the meridian, then is the height neare 19 *gr.* 20 *m*; when he comes to be 100 *gr.* then 11 *gr.* 15 *m.* when 110 *gr.* then 3 *gr.* 20 *m*; and before he commeth to the azimuth of 120 *gr.* he hath no altitude. For the sun hauing 15 *gr.* of North declination, will rise and set at 114 *gr.* 34 *m.* from the meridian.

2 *The altitude of the Sun being giuen, to find on what azimuth he beareth from vs.*

Let the beade be set for the time, and the altitude obserued as before; then bring the thread to the complement of that altitude, so the bead shall shew the azimuth required.

As if the second of August, hauing set the beade for the time, you shall find the altitude of the sun to be 19 *gr.* 20 *m.* remoue the thread vnto 70 *gr.* 40 *m.* the complement of the altitude; or, which is all one, to 19 *gr.* 20 *m.* from the right hand toward the left, and the bead will fall on the line of 90 *gr.* from the meridian. And therefore the point whereon the sunne beareth from vs, is one of these two, either due East or due West. And which of these is the true point of the compasse, may be soone knowne by a second obseruation: for if the sunne rise higher, it is the forenoone; if it be lower, it is the afternoone.

By knowing the azimuth or point of the compasse whereon the sunne beareth from vs, it is easy to find,

A meridian line, and thereby
The coasting of the Countrey.
The site of a building.
The variation of the Compasse.

As

As if the second of August in the afternoone, I should find by the height of the sun that he beares from me 60 *gr.* from the meridian toward the West: then there being 90 *gr.* belonging to each quarter, the West will be 30 *gr.* to the right hand; the East is opposite to the West, the North and South lie equally betweene them.

CHAP. X.

Of the Quadrat.

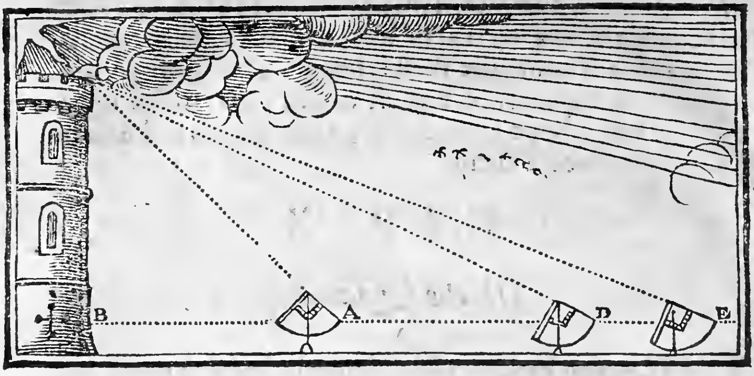
THE Quadrat hath two sides diuided, the other two sides next the Center may be supposed to be diuided, each of them into 100 equall parts: of the sides diuided, that which is next the horizontall line containes the parts of right shadow, the other next the sights, the parts of contrary shadow. The vse of the Quadrat is,

1 *Any point being giuen, to finde whether it be leuell with the eye.*

Lift vp the center of the quadrant, so as the thread with the plummet may play easily by the side of it: then looke through the sights to the place giuen: for now if the thread shall fall on *AB* the horizontall line, then is the place giuen leuell with the eye: but if it shall fall within the said line on any of the diuisions, then it is higher: if without, then it is lower then the leuell of the eye.

2. *To find an height aboue the leuell of the eye, or a distance at one obseruation.*

Looke through the sights to the place, going nearer or farther from it, till the threadfull fall on 100 parts in the quadrat or 45 *gr.* in the quadrant, so shall the height of the place aboue the leuell of the eye, be equall to the distance betweene the place and the eye. If



If the thread fall on 50 parts of a right ſhadow, the height is but halfe the diſtance: if it fall on 25, it is a quarter of the diſtance: if on 75, it is three quarters of the diſtance. For as oft as the thread falleth on the parts of right ſhadow,

As 100 to the parts on which the thread falleth:
So is the diſtance to the height required.

And on the contrary,

As the parts cut by the thread are to 100:
So the height vnto diſtance.

But when the thread ſhall fall on the parts of contrary ſhadowe: if it fall on 50 parts, the height is double vnto the diſtance; if on 25, it is foure times as the diſtance. For as oft as the thread falleth on the parts of contrary ſhadow,

As the parts cut by the thread are vnto 100:
So is the diſtance vnto the height.

And on the contrary,

As 100 are vnto the parts cut by the thread:
So is the height vnto the diſtance.

And what is here ſaid of the height and diſtance, the ſame may be vnderſtood of the height and ſhadow.

3 To

3. To finde a height or a distance at two obseruations.

As if the place which is to bee measured might not otherwise bee approached, and yet it were required to finde the height B C, and the distance: first if I make choice of a station at A, where the thread may fall on 100 parts in the quadrat, and 45 *gr.* in the quadrant, the distance A B will bee equall to the height B C; then if I goe farther in a direct line with the former distance, and make choice of a second station at D, where the thread may fall on 50 parts of right shadow, the distance B D would bee double to the height B C: wherefore I may measure the difference betweene the two stations A and D, and this difference D C will bee equall both to the distance A B and the height A B.

Or if I cannot make choice of such stations, I take such as I may, one at D, where the thread falleth at 50 parts of right shadow; the second at E, where it falleth on 40 parts: and supposing the height B C to bee 100, I find that

As 50 parts are vnto 100, the side of the quadrat:
So 100 the supposed height, vnto 200 the distance B D,
And as 40 parts, at the second station, vnto 100:
So 100 the supposed height, vnto 250 the distance B E.

Wherefore the difference betweene the stations D and E should seeme to bee 50; and then if in the measuring of it, I should finde it to bee either more or lesse, the proportion will hold, as from the supposed difference to the measured difference, so from height to height, and from distance to distance.

As if the difference between the two stations D and E being measured, were found to be 30.

As 50 the suppoſed difference, vnto 30 the true difference:
So 100 the ſuppoſed height, vnto 60 the true height.
And 200 the ſuppoſed diſtance; vnto 120 the true diſtāce:
And 250 at the ſecond ſtation, vnto 150 the diſtance B E.

The like reaſon holdeth in all other examples of this kind: and if an Index with ſights were fitted to turne vpon the Center, it might then ſerue by the ſame reaſon for the finding of all other diſtances.

FINIS.

THE GENERALL VSE OF THE CANON AND TABLE of Logarithmes.

LOgarithmetique is a Logicall kinde of Arithmetique, or artificiall vse of numbers inuented for the ease of the calculation wherein each number is fitted with an Artificiall, and these artificiall numbers so ordered, that what is produced by multiplication of naturall numbers, the same may be effected by the addition of these their artificiall numbers; what they performe by diuision, the same is here done by subtraction: and so the hardest part of calculation auoided by an easy prosthaphæresis.

All this shall be made plane by applying that to these Artificiall numbers, which I haue set downe before for the vse of my Lines of numbers sines and Tangents in the vse of the Sector and Crosstaff. Wherein the Reader is to obserue that, what is to be wrought by round numbers only, is best done by *M. Brigges* his Logarithmes, but the astronomi-

 call

call part concerning arkes and angles, by my Canon of Artificiall ſines and Tangents.

CHAP. I.

Concerning the vſe of the line of Numbers, I ſet downe ten generall Propoſitions in the vſe of the Croſſeſtaff. p. 18. and theſe may bee applied to the table of Logarithmes.

PROP. I.

To multiply one number by another.

THis is the VI. Propoſition of the ten: but I begin with the eaſieſt, adde the Logarithme of the multiplicator to the Logarithme of the multiplied, the ſumme of both ſhall be the Logarithme of the product.

As when we multiply 25 by 30 the product is	750
ſo here adde the Logarithme of 25 viz.	1397. 94001
to the Logarithme of 30	1477. 12125
the ſumme of both will be	2875. 06126

And this is the Logarithme of 750.

In like manner, if we multiply 10 by 10 the prod. is 100. if 100, by 10; the product is 1000. ſo here

The Logarithme of	10 being	1000. 00000
The Logarithme of	1000 ſhall be	2000. 00000
	1000	3000. 00000
	10000	4000. 00000
	100000	5000. 00000

And ſo forward: All intermediate numbers which haue intermediate Logarithmes.

If

If we multiply 101 by 10, the product is 1010 of 102 by 10 the product is 1020: so here

The Logarithme of 10 viz.	1000. 00000
added the Log. of 101	2004. 32137
giues the Log. of 1010	3004. 32137
The same Logarithme of 10	1000. 00000
added to the Logarithme of 102	2008. 60017
giues the Logarithme of 1020	3008. 60017

The difference being only in the first figure, and that is alwayes lesse by one then the number of places, in the number giuen. As when we find the Logarithme to be — 2008 60017 the first figure, 2, is characteristicall, i. the Index shewing that the whole number 102 belonging to this Logarithme, consists of three places. If the Logarithme had beene 1008. 60017 the whole number must haue been 10. 2 consisting of two places, and the rest a fraction of $\frac{2}{10}$.

If the Logarithme were — 0008. 60017 the number belonging to it would be. 1. 02. 1. 1 and $\frac{02}{100}$ And this is one of the reasons why the differences were omitted in the first hundred Logarithmes. All those Logarithmes may be found afterwards vnder a larger Index.

Againe, if we multiply 201. by 5, the product is 1005: so here: if we adde the Logarithme of 5 vnto the Logarithme of 201, the summe of both, shall be the Logarithme of 1005 and the summe of the Logarithmes of 5 and 203 shall be the Logarithme of —1015. Thus the most part of the table may be continued beyond 1000.

PROP. 2.

To diuide one number by another.

Subtract the Logarithme of the Diuisor out of the Logarithme of the Diuidend, the Remainder, shall be the Logarithme of the Quotient.

As when we diuid 750 by 25 the quotient is 30: so here
from the Logarithme of 750 viz 2875.06126
subtract the Logarithme of 25 1397.94001
There remaines the Logarithme of 30. 1477.12125

In like manner when we diuide 11. by 4. the quotient is $2\frac{3}{4}$
so here the Logarithme of 4 viz 0602.05999
taken from the Logarithme of 11 1041.39269
leaues the Logarithme of $2\frac{3}{4}$ 0439.33270
wherefore, if it were required to find the Logarithme of a whole number with a fraction annexed (as one $2\frac{3}{4}$) we might first reduce it into an improper fraction of $\frac{11}{4}$ (or rather of $\frac{275}{100}$) and then subtract as before.

If it were required to find the Logarithme of a single fraction, as of $\frac{4}{11}$, we may subtract as before: But this fraction being lesse then 1, the Logarithme must be lesse then 0. and therefore noted with — a defectiue signe.

So the Logarithme of $\frac{11}{4}$ or $2\frac{3}{4}$ is + 0439.33270
and the Logarithme of $\frac{4}{11}$ — 0439.33270

Prop. 3.

To find the square root of a number.

Halfe the Logarithme of the number giuen is the full Logarithme of the square Root.

So the Logarithme of 144 being 2158.36249
the halfe thereof is 1079.18124
the Logarithme of 12: and such is the square Roo of 144.

Then by conversion having extracted the square Root, we may soone finde the Logarithme.

As, the *Logarithme* of 10,0000 being 1000.00000
the *Logarithme* of the square R. 316227 is 0500.00000
and for the Root of that 177827 0250.00000

Prop. 4.

Prop. 4.

To finde the Cubique Roote of a number.

The third part of the Logarithme of the number giuen is full Logarithme of the *Cubique Roote.*

So the Logarith of	125	is	2096.91001
And $\frac{1}{3}$, the Logarithme of	5		0698.97000

By the same reason we may finde the *Biquidrate Roote,* by diuiding the Logarithme of the number giuen by 4: the solid *Roote,* by dividing by 5; and so forward.

And by conversion, hauing extracted the Roote, we may soone finde the Logarithme.

As the Logarithme of	10.000.&c.	is	1000.00000
The Logar. of the *Cub.R.* 21544.			0333.33333

The Logarithme of	100.000, &c.	2000.00000
the Logarithme of the Cubique R.	4641.	0666.66666

Then multiplying these square and *Cubique Rootes* one by another, we may produce infinite other numbers, and haue all their Logarithmes.

Prop. 5.

Three numbers being giuen, to finde a fourth Proportionall.

This *Golden Rule* the most vsefull of all others, may bee wrought severall wayes as it appeares by this example:

As 12 vnto 24 so 4 to a fourth number,

1. Tactus 2. & 3. divisus per 1. The ordinary way in Arithmetique is by multiplication and diuision. For first they multiply the second into the third, and then diuide the product by the first number giuen. As here multiplying 24 by 4, the Product is *96*, then diuiding *96* by 12 the Quotient will be 8 the fourth number here required.

According to this way we adde the Logarithmes of the second and third, and subtract the Logarithmes of the first, so, that which remaineth, shall be the Logarithme of the fourth number required.

Thus the Logarith. of the first numb.	12	is	1079.18125
the Logarithme of the second	24		1380.21124
the Logarithme of the third	4		0602.05999
the summe of the second and third Logar.			1982.27123
subtract the first and there remaineth			0903.08998

And thus is the Logarithmes of 8. the fourth Proportionall.

2 Quotiens 2. per 1. diuisi multiplicatus in tertium. A second way in Arithmetique is by diuision and multiplication. For where the second number is greater than the first, they may diuide the second by the first, and then multiply the third by the quotient. As here dividing 24 by 12 the quotient is 2: then multiplying 4 by 2, the Product will be 8.

According to this way we take the Logarithme of the first out of the Logarithme of the second, and then adde the difference to the Logarithme of the third. So the summe of this addition shall be the Logarithme of the fourth required.

Thus the Loga. of the first Numb.	12	is	1079.18125
the Logarithme of the second	24		1380.21124
the difference betweene the increasing			300.02999
added to the Logarithme of	4		0602.05999
gives the Logarithme of	8		0902.08998

3 Quotiens 1. per 2. fit divisor 3. A third way in Arithmetique is by diuision and diuision, for where the second number is lesse then the first, they may diuide

diuide the first by the second, and then againe divide the third by the quotient. As here diuiding 12 by 4. the quotient is 3: then diuiding 24 by 3. the quotient is 8.

According to this way we take the Logarithme of the second, out of the Logarithme of the first, and then take the difference out of the Logarithme of the third: So, that which remaineth shall be the Logarithme of the fourth number required.

Thus the Logar. of the first numb.	12	is	1079.18125
the Logarithme of the second	4		0602.05999
The difference decreasing,			477.12126
substracted from the Logarithme of	24		1380.21124
gives the Logarithme of	8		0903.08999

These two latter wayes by difference of Logarithmes, may be considered as the same. Though there be some difference betweene them, yet that may easily be reconciled, if we have regard to the nature of the question. For three numbers being giuen in direct proportion, if the second be greater then the first, the 4. must be greater then the third: If the second be lesse then the first, the 4. must bee lesse then the third, and their Logarithme accordingly. But in reciprocall proportion, considering the first and second numbers to be of one denomination, we are to obserue the contrary.

If we desire to turne subtraction into addition wee may take the Logarithme which is to bee subtracted out of the *Radius*, and adde the complement. So the summe of this addition, the *Radius* being subtracted shall give the required Logarithme as before.

Thus in the last example; where subtracting the difference 477.12126. out of 1380.21124. the Logarithme of 24 we found the remainder to be 0903.08998. the Logarithme of 8.

The *Radius* being	10000.00000
the Logarithme to be subtracted	0477.12126
the complement to the *Radius* is	9522.87874

This

This added to the Logarithme of 24	1380. 21124
giues vs a compound Logarithme	10903.08998

From this, if we subtract the *Radius*,(that is, if we cancell the first figure to the left hand) the rest is 0903.08998 the Logarithme of 8. the fourth Proportionall, as before.

By helpe of this fourth Proportional we may come somewhat neere to finde a Logarithme for a number of 6 places.

As if it were required to finde a logarithme for this number 868624. the table will affoord vs Logarithmes for a lesser and a greater number; and then the intermediate may be found by the part proportionall in this maner.

Here we have the Logarithme of	868	2938.51973
and the Log. of the next following	869	2939.01978
and the tabular difference betweene them		50005

If the Index be fitted to the number of places

the Logarithme of	868000	shall be	5938.51973
and the Logarith. of	869000		5939.01978
the difference being	1000		50005

Then taking 868000, out of 868624, (the number given) the third difference will be 624. And hauing these three differences the proportion will hold,

As	1000	vnto	50005
So	624	vnto	31203

the part proportionall to be added to the lesser Logarithme 5938.51973 so shall we haue 5938.83176. for the logarithme required.

In like maner hauing a logarithme given, we may finde the value of it in a number of sixe places.

As if the Logarithme given were 3938.83182 and it were required to find the number to which it belongeth: This Logarithme is not to be found in the Table; but changing the *Index* and making it 2938.83182

the next lesser logarithme of 868	is	2938.51973
and the tabular difference following		50005
and the proper difference		31209

As

As th Tabular difference	50005	vnto	100000
So the proper difference	31209	vnto	62411

the part proportionall to be ioyned to the end of the former number 868 : so shall we haue 86862411. for the value of this Logarithme. But the Index of the Logarithme being 3. the number required must consist of 4 places: *viz.* 8686 and the rest a fraction of $\frac{24}{100}$.

This I say is somewhat neere the truth. For this number here proposed 868624 is the square of 932,

The true Loga, of the Root	932	is	2969.41591
The true Loga. of the Square	868624.		5938.83182

Prop. VI.

Three numbers being giuen to finde a fourth in a duplicated Proposition.

In questions that hold in a duplicated proportion between *Lines* and *Superficies*, the Logarithmes for lines giuen may be doubled, the Logarithmes for lines required may bee halfed, and then the worke will be the same as in the first part of the former Proposition.

Suppose, the *Diameter* being 14, the content of the circle was 154; the *Diameter* being 28, what may the content bee?

Here the question concerning both lines and superficies, I double the Logarithmes of the 2 lines giuen, and then worke as before in this maner.

The logarithme of	14	is	1146. 12803
the logarithme of	28		1447. 15803
the same againe			1447. 15803
the logarithme of	154		2187. 52072
the summe of these last			5081. 83678
Subtract the double of the first,			2292. 25606
there remaines the logar. of	616		2789.58072

And ſuch is the content of the circle here required.

Suppoſe the content of a Circle being 154, the Diameter of it was 14; the content being 616, what may the diameter be?

Here being one line giuen, and one line required, I double the Logarithme of the line giuen, and then working as before, the halfe of the remainder ſhall be the Logarithme of the line required.

Thus the loga. of	154	is	2187.52072
the logarithme of	616		2789.58072
the logarithme of	14		1146.12803
the ſame againe			1146.12803
the ſumme of theſe laſt			5081.83678
ſubtract the logarithme of the firſt			2187.52072
the remainder will be			2894 31606
the halfe thereof is			1447.15803

The logarithme of 28. the Diameter required.

Or according to the ſecond maner of operation, the difference betweene the logarithmes of lines giuen may be doubled; the difference betweene the logarithmes of the content given may be halfed, and then the worke will be the ſame as in the latter part of the former propoſition.

So, in the firſt queſtion, where the Diameters were giuen and the content required.

The logarithme of	14	is	1146.12803
the logarithme of	28		1447.15803
the difference increaſing			301.03000
the double of this difference			602.06000
added to the logar. of	154		2187.52072
giues the logarith. of	616		2789.58072

In the ſecond queſtion, where the content of both the circles was knowne, and the Diameter of the one required.

The

The logarithme of	154	is	2187.52072
the logarithme of	616		2789.58072
the difference increasing			602.06000
the halfe of this difference			301.03000
added to the logar. of	14		1146.12803
giues the logarith. of	28		1447.15803

PROP. 7.

Three numbers being giuen to finde a fourth in a triplicated proportion.

In questions concerning proportion betweene *Lines* and *Solids* the logarithmes for lines given may bee tripled; the logarithmes for lines required may be diuided into 3. parts, and then the worke will be the same, as in the first way for the rule of Three.

Suppose the *Diameter* of an Iron bullet, being 4 inches, the waight of it was 9 pound, the *Diameter* being 8. inches, what may the waight be?

The logaritbme of	4	is	0602.05999
the logarithme of	8		0903.08999
the Triple of it			2709.26997
the logarithme of	9		0954.24251
the summe of these last			3663.51247
subtract the triple of the first logar.			1806.17997
there remaines the logar. of 72			1857.33251

and such is the waight required.

Suppose the waight of an Iron bullet being 9 pound, the Diameter was foure inches; the waight being 72 pound, what may the Diameter be?

The Logarithme of	9	is	0954.24251
the Logarithme of	72		1857.33250
the Logarithme of	4		0602.05999
the double of this againe			1204.11998
the summe of these last			3663.51247
the first Log. substracted there remaines.			2709.26996
the third part thereof is			0903.08999

the Logarithme of 8. and such is the diameter required.

Or according to the second manner of operation in the rule of three, the difference betweene the Logarithmes of lines giuen may bee tripled; the difference betweene the Logarithmes of the solidity or weight giuen may be diuided into 3 parts.

So in the first question, where the diameters were knowne, and the weight required.

The *Logarithme* of	4	is	0602.05999
the *Logarithme* of	8		0903.08999
the difference encreasing			301.03000
the triple of this difference			903.09000
added to the *Logarithme* of	9		0954.24251
giues the Logarithme of	72		1857.33251

In the second question, where the weight was knowne, and the diameter required.

The Logarithme of	9	is	0954.24251
the Logarithme of	72		1857.33250
the difference increasing			903.08999
the third part of this difference			301.02999
added to the *Logarithme* of	4		0602.05999
giues the *Logarithme* of	8		0903.08998

PROP.

Prop. 8.

Hauing two numbers giuen to find a third in continuall proportion, a fourth, a fifth, a sixt and so forward.

According to the first way in the rule of three, we may subtract the Logarithme of the first number, out of double the Logarithme of the second, the remainder shall be the Logarithme of the third, then subtracting the Logarithme of the first number againe out of the Logarithmes of the second and third, that is, out of triple the Logarithme of the second, the remainder shall be the Logarithme of the fourth, and so forward.

As, when we say: As 1 vnto 2, so 2 vnto 4: and 4 vnto 8; and 8 vnto 16 &c. because the first number is 1, there is no need of diuision, but onely to multiply 2 the second number into it selfe, the product giues the third proportionall number to be 4: then multiplying 2 into 4, the fourth proportionall is 8: and multiplying 2 into 8 the fifth proportionall is 16; and so forward. So here the Logarithme of the first number being 1. there is no need of subtraction.

But, finding the Logarithme of 2 to be.	0301.02999
the double giues the Logarithme of 4	0602.05999
the triple giues the Logarithme of 8	0903.08999
the quadruple giues the Log. of 16	1204.11998

and so forward in *infinitum*.

In all other numbers that begin not with 1, wee may either subtract the Logarithme of the first number, or adde the complement vnto the *Radius*.

As when the numbers giuen are 100 and 108.

The *Logarithme* of the first N. 100. is	2000. 00000
the *Logarithme* of the second 108	2033.42376
the double of this second *Logarithme*.	4066.84752
subtract the first *Log.* there remaines	2066.84752

the *Logarithme* of 116 $\frac{64}{100}$ the third proportionall.

Againe ſubtract the firſt *Logarithme*	2000. 00000
out of the ſumme of the *Logarithmes* of	2033. 42376
the ſecond N. and the third Proportionall	2066. 84752
there remaines the *Logarithme*	2099. 27128

anſwering vnto 125. $\frac{971}{}$ the fourth number in continuall proportion.

According to the ſecond manner of operation we may take the difference between the *Logarithmes* of the two numbers giuen; ſo, this difference applied to the *Logarithme* of the ſecond number ſhall giue the *Logarithme* of the third Proportionall: the ſame difference applied to the *Logarithme* of the third Proportionall, ſhall giue the Logarithme of the fourth Proportionall. Or the double of this difference applyed to the Logarithme of the firſt number ſhall giue the Logarithme of the third Proportionall: the treble of this difference applyed to the Logarithme of the firſt number ſhall giue the Logarithme of the fourth proportionall: and ſo forward.

As in the former example, where the two numbers giuen were 100 and 108: ſuppoſe 100 increaſing to 108, and ſo yearly in continuall proportion after the rate of 8 in 100, and that it were required to find, what this 100 would grow vnto by the end of 20 yeeres?

The Logarithme of the firſt numb.	100 is	2000. 00000
the Logarithme of the ſecond	108	2033. 42376
the yearely difference increaſing		33. 42376
added to the Loga. of the ſecond giues.		2066. 82752

the Logarithme of 116 $\frac{64}{}$ for the third proportionall; And ſuch is the encreaſe at the end of the ſecond yeare.

Againe the ſame yeerely difference added to the Logarithme of the third Proportionall giues 2100. 25128 the Logarithme of 125 $\frac{971}{}$ for the fourth Proportionall and the encreaſe at the end of the third yeare: and ſo the reſt.

But becauſe the queſtion is onely of the 20 yeare without knowing the reſt, we may multiply the former yeerely difference 33. 42376

by 20; ſo the difference of 20 yeare 668. 47520

added

added to the Log. of the first num. 100. vz. 2000. 00000
giues the Logarithme of 466. $^{\underline{095}}$ 2668. 47520
that is 466. l. 1. *s*. 11. *d. ferè*. the summe that 100 would grow vnto by the end of 20 yeares at the rate proposed.

In like manner if the two first numbers giuen were 108 and 100: Suppose 108 decreasing to the 100 and so yeerely in continuall proportion and that it were required to find what 100 would decrease vnto by the end of 20 yeares: Or (which is all one) suppose 100 to be due 20 yeare hence, and that it were required to find the worth thereof in ready money according to the former rate. The Log. of the first N. 108 is 2033. 42376
the Logarithme of the second 100 2000. 00000
the differedce for the yeare decreasing 33. 42376
taken from the Logarithme of 100 leaues 1966. 57624
the Logarithme of 92 $^{\underline{592}}$ for the third proportionall. and such is the present worth of 100 l. due at the yeares end.

The same difference subtracted once more leaues 1933. 15248 the Logarithme of 85 $^{\underline{733}}$ for the fourth propotionall, and the present worth of 100 l. due at the end of two yeares.

The same diff rence multiplyed by 20 makes 668. 47520
and subtracted from the Log. of 100 leaues 1331. 52480
the Logarithme of 21 $^{\underline{4548}}$ that is 21 l. 9 *s*. 1 *d*. and such is the present worth of 100 l. due at the end of 20 yeares: So that this present worth being taken forth of the 100 l. principall debt there remaines 78 l. 10 s. 11 *d*. for the present worth of the continued gaine that may be made either of the loaue of 100 l. or of 8 l. annuity after 20 yeares according to the former rate.

If a lease of 100 l. by the yeare or such other yeerely pension were to continue for 20 yeares, and that it were required to find the worth thereof in ready money. This might bee found vpon the same ground of continuall proportion, and that seuerall wayes.

1 It appeareth before, that 100 l. due at the yeares end is worth but 92 $^{\underline{592}}$ in ready money: If it be due at the end of 2 yeares, the present worth is 85 l. $^{\underline{733}}$: then adding these two together, wee haue 178 l. 326 for the present worth of

100 l.

100. pound Annuity for 2. yeeres and so forward.

2 It appeareth before that the present worth of 8 pound annuity for 20 yeeres is 78 pound 5452: and then it followes by proportion.

As an Annuity of	8.l. 0000	0903.08999
is to the worth thereof	78.5452	1895.11953
		992.02954
So an Annuity of	100.0000	2000.00000
vnto the worth of it	981.8147	2992.02954

3 As the yeerely loane of 100 pound includes an Annuity of 8. pound, So there is a summe equivalent to 100 pound Annuity.

This summe equivalent may be diminished according to the number of yeeres as before: to the complement of the summe diminished to the summe equivalent shall be the present worth of the Annuity.

As the yeerely gaine of	8	0903.08999
to the loane of	100	2000.00000
So an Annuity of	100	2000.00000
to the sum equivalent	1250	3096.91001

Then for diminishing of this sum equivalent wee may multiply the former yeerely difference 33.42376
by 20. so the difference for 20 yeeres 668.47520
taken from the logarithme of 1250 3096.91001
there remaines the logar. of 268.1853 2428.43481
whose complement to 1250. is 981.8147. that is 981. *l*. 16. *s*. 3. *d. ob.* and such is the present worth of 100. pound Annuity for 20. yeeres, at the rate of 8. in 100 *per annum*.

The like reason holdeth for any other rate and time proposed.

PROP.

Prop. 9.

Hauing two extreme numbers giuen, to finde a meane Proportionall betweene them.

Adde the logarithmes of the two extreme numbers the; one halfe of the summe shall be the logarithme of the meane *Proportionall.*

As if the two extreme numbers giuen were 8. and 32

The logarithme of	8	is	0903.08999
The logarithme of	32		1505.14998
The summe of both logarithmes			2408.23997
The halfe of this summe is			1204.11998

the logarithmes of 16: and such is the meane proportionall here required.

Prop. 10.

Hauing two extreme numbers giuen to find two meanes Proportionall's betweene them.

In the ordinary way of Arithmetiqne we commonly multiply the greater extreme by the square of the lesser, so the Cubique root of the Product shall be the lesser meane: then multiplying the lesser meane into the greater extreme, the square root of the Product shall be the greater *Meane Proportionall.* Or hauing found the lesser meane, wee may finde the other meane by continuall proportion.

Accordingly we may adde the logarithme of the greater extreme to double the logarithme of the lesser, so the third part of the summe shall be the logarithme of the lesser meane. Then adding this logarithme of the lesser meane, to the logarithme of the greater extreme, the one halfe of the summe ſhall be the logarithme of the greater *meane Proportionall.*

As if the two extreme numbers given were 8. and 27	
Adde to the logarithme of 8 *viz.*	0903.08999
the ſame againe	0903.08999
and the logarithme of 27	1431.36376
The ſumme of theſe will be	3237.54374
the third part of this ſumme is	1079.18125

the logarithme of 12. the *leſſer meane Proportionall.*

Adde to this logar. of the leſſer meane	1079.18125
the logar. of the greater extreme	1431.36376
The ſumme of both logar. will be	2510.54501
and the halfe of this ſumme is	1255.27250

the logarithme of 18. the greater of the two meane Proportionals here required.

Or according to the ſecond manner of operation in the Rule of Three, (which is the worke that I alwaies follow in the line of numbers) we may take the difference betweene the logarithmes of the two extreme numbers, and diuide this difference into three equall parts, ſo the ſumme of the logarithmes of the leſſer extreme and $\frac{1}{3}$ part ſhall be the logarithme of the leſſer Meane: the ſumme of this logarithme of the leſſer meane and the ſame $\frac{1}{3}$ part ſhall bee the logarithme of the *Greater meane Proportionall.*

So the Logarithme of 8 being	0903.08999
the Logarithme of 27	1431.36376
the difference betweene them	578.27377
The third part of this difference	176.09126
added to the Logarithme of 8. giues	1079.18125

the Logarithme of 12. the *leſſer Meane.*

The ſame added to the Logarithme of 12. giues 1255.27251. the Logarithme of 18. the *Greater Meane Proportionall.*

And

And by the ſame reaſon, if it were required to find three *Meane Proportionals*, we might divide the former difference into 4. equall parts, and ſo forward.

As if it were required to finde the firſt of eleven *Meane Proportionals* betweene 100 and 108. Or (which is all one) ſuppoſe 100 pound increaſing in continuall proportion, ſo as that by the end of 12. moneths it came to 108 pound and that it were required to find what this 100 pound did grow vnto by the end of the firſt moneth.

The Logarithme of the firſt extreme	100	is	2000.00000
the Logarithme of the ſecond	108		2033.42376
the yearely difference betweene them			33.42376
The 12 part or monethly difference			2.78531
added to the Logarithme of	100	giues	2002.78531

the Logarithme of 100. 643403011 the firſt of eleven *meane Proportionals*; and the growth required.

Then having theſe two, 100. and 100. 643403011. together with 108, the laſt of the twelue, the other intermediate may be found by continuall proportion as before.

This Explication of my ten former *Propoſitions* may ſerve for the frugall vſe of the Table of Logarithmes. Thoſe which require more, may haue recourſe to that Treatiſe which is mencioned before in the front of the Table.

CHAP. II.

Concerning the vſe of the *Lines* of *Sines* and *Tangents* I ſhewed in generall, *pag*. 21. how the might ſerue for the reſolution of all Sphæricall triangles. More particularly in the vſe of my *Sector* (pag. 74) I reduced that which is commonly required in a ſphæricall triangle vnto 28 caſes. And for theſe they may be all reſolued by my Tables of Artificiall Sines and Tangents without the help of Secants or verſed Sines.

This manner of the worke will be alwaies ſuch as in the ordinary rule of Three. For, here we haue three numbers giuen whereby to find a fourth Proportionall. And therefore either we may adde the Logarithmes of the ſecond and third, and ſubtract the Logarithme of the firſt:

Or we may take the difference between the Logarithmes of the firſt and ſecond, and apply that difference to the Logarithme of the third.

The firſt of theſe waies is beſt for the reſolution of right angled Triangles where the *Radius*, *viz.* 1000. 0000 is one of the three numbers giuen: But the ſecond way, by differences is more conuenient for the reſt.

The like manner of worke may be obſerved when we are to conſider the Sines or Tangents of Degrees, Minutes, and Seconds. For the Seconds, not expreſſed in the Canon, will be found by the part proportionall: as I will ſhow in the examples following.

1 If it were required to finde the Sine of 51.*gr*. 32′. 15″. I ſhould finde.

The

The Sine of 51 *deg.* 32 *mi.*	is		9893.7452
the Sine of 51 *deg.* 33 *m.*			9893.8455
the Tabular difference betweene them			1003

Then the difference betweene 32 *m.* and 33 *m.* being 60 Seconds, the Proportion will hold,

As 60 Seconds vnto 1003
So 15. vnto 251 the part Proportionall to be added vnto the Sine 51 *deg.* 32 *m.*

So shall we have 9893. 7703. for the sine of 51 *deg.* 32. *m.* 15 *seconds.*

2 If it were required to finde the Degrees, Minutes and Seconds belonging to this Tangent 10099. 9782
I should finde by the Canon that this is somewhat more then

the Tangent of 51 *deg.* 32 *mi.*	10099. 9134
lesse then the Tangent of 51 *deg.* 33. *mi.*	10100.1728
The Tabular difference betweene these is	2594
and the proper difference is	648

betweene the lesser of these Tangents, and the Tangent giuen therefore.

As 2594 vnto 60 Seconds,
So 648 vnto 15 And so, I finde this to be the Tangent of 51 *deg.* 32 *mi.* 15 *seconds.*

3 If it were required to finde the Sine belonging to this Tangent 10099. 9782, I should finde the arke to be somewhat more then 51 *gr.* 32 *m.* and the sine correspondent somewhat more then 9893.7452. then taking out the differences as before, I finde that

As the Tabular difference of Tange.	2594	3413.9700
is to the proper difference	648	2811.5750
		602. 3950
So the Tabular difference of Sines	1003	3001. 3009
to the part Proportionall	251	2398.9059

This part proport. added vnto the former Sine. 9893.7452.

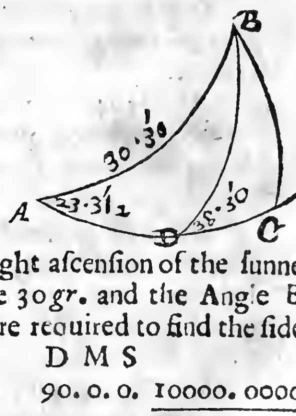
giues 9893 7703 for the ſigne required.

Theſe premiſſes conſidered I come to the 28 Caſes before mentioned wherein I ſet downe a Canon and an Example for each caſe, and theſe for the moſt part the ſame which I vſed before.

Thoſe which haue no further vſe, but of degrees and minutes may take that ſine or Tangent, which they find to be next in the Canon, and neglect the ſeconds.

IN A RECTANGLE TRIANGLE

1 *To finde a ſide by knowing the Baſe and the Angle oppoſite to the inquired ſide.*



As in the Rectangle triangle A C B wherein A ſtands for the æquinoctiall point; A B, an arke of the Ecliptique repreſenting the Longitude of the Sunne in the beginning of ♉; *B C* an arke of the Declination from the Sun to the æquator; and A C an arke of the Æquator repreſenting the right aſcenſion of the ſunne in B: Knowing the Baſe A B to be 30 *gr*. and the Angle B A C 23 *gr*. 31 *m*. 30″. if it were required to find the ſide B C

	D M S	
As the *Radius* the ſine of	90. 0. 0.	10000. 0000
is to the ſine of the Baſe	30. 0. 0	9698. 9700
So the ſine of the oppoſite angle.	23. 31. 30.	9601. 1352
to the ſine of the ſide required	11. 30. 43.	19300. 1052

And ſo writing the ſine 9601 1352 in a paper by it ſelfe and holding it to the ſine of the Baſe in the Canon 1. *gr*. 2, 3, 4, 5, and ſo forward, it would be no long worke to write the

ſumme

summe in a columne by it selfe, and so find the Declination for each degree and Minute of the Ecliptique.

2. *To finde a side by knowing the Base and the other side.*

As in the Rectangle A C B hauing A B 30 *gr.* and B C 11 *gr.* 30 *m.* 43″ S, to finde the side A C.

As the cosine of the side giuen	11. 30. 43.	9991. 1740
is to the *Radius*	90. 0. 0.	10000, 0000
So the cosine of the Base	30. 0. 0.	9937. 5306
to the cosine of the side required.	27. 53. 43.	9946. 3566

3. *To finde a side by knowing the two oblique Angles.*

As in the Rectangle A C B, hauing C A B for the first Angle 23 *gr.* 31 *m.* 30 S. and A B C for the second 69 *gr.* 20 *m.* 35 S: to find the side A C.

As the sine of the next angle	23. 31. 30	9601. 1352
is to the *Radius*	90. 0. 0.	10000. 0000
So the cosine of the opposite angle	69. 20. 35.	9547. 4918
to the cosine of the side required.	27. 53. 43.	9946. 3566

4 *To finde the BASE by knowing both the sides.*

As in the Rectangle A C B hauing A C 27 53 *m.* 43″, and B C, 11 *gr.* 30 *m.* 43 S. to find the Base A B.

As the *Radius*.	90. 0. 0.	10000.0000
to the cosine of the one side	27. 53. 43.	9946. 3566
So the cosine of the other side.	11. 30. 43.	9991. 1640
to the cosine of the Base	30. 0. 0.	9937. 5306

5 *To finde the BASE by knowing one side and the Angle opposite to that side.*

As if in the former triangle A C B we draw B D an Arke of the Horizon for the Latitude of 51 *gr.* 30 *m.* reputing the amplitude of the Sunnes rising from the East, we shall haue two Triangles more, one rectangle B C D, the other obliquadrangled A B D. And so, in the Rectangle D C B, hauing B C 11 *gr.* 30 *m.* 43 *s.* and B D C 38 *gr.* 30 *m.* if it were required, to find the Base D B.

As the sine of the Angle	38 30 0	9794. 1495
to the sine of the side	11 30 43	9300. 1052
So is the *Radius*	90 0 0	10000. 0000
to the sine of the Base	18 41 56	9505. 9556

6 *To finde an Angle by knowing the other oblique angle, and the side opposite to the angle required.*

As in the Rectangle A C B, hauing B A C. 23 *gr.* 31 *m.* 30 *s.* and A C 27 *gr.* 53 *m.* 43 *s.* to find the angle A B C.

As the *Radius*	90 0 0	10000. 0000
to the sine of the angle giuen	23 31 30	9601. 1352
So the cosine of the side	27 53 43	9946. 3566
to the cosine of the angle required	69 20 35	19547. 4918

7 *To*

7 *To finde an angle by knowing the other oblique angle, and the side opposite to the angle giuen.*

As in the Rectangle A C B hauing B A C 23 *gr.* 31 *m* 30 *s.* and B C 11 *d.* 30 *m.* 43 *s.* to finde the angle A B C.

As the cosine of the side	11 30 43	9991. 1740
to the cosine of the angle giuen	23 31 30	9962. 3153
So is the *Radius*	90 0 0	10000. 0000
to the sine of the angle required	69 20 35	9971. 1413

8 *To find an angle by knowing the Base, and the side opposite to the angle required.*

As in the Rectangle B C D hauing B D 18 *gr.* 41 *m.* 56 *s.* and B C 11 *gr.* 30 *m.* 43 *s.* to find the angle B D C.

As the sine of the Base	18 41 56	9505. 0000
is to the *Radius*	90 0 0	10000. 0000
So the sine of the opposite side	11 30 43	9300. 1052
to the sine of the angle	38 30 0	9794. 1495

These eight Propositions haue beene wrought by sines alone; the eight following require ioint help of Tangents.

9 *To find a side, by knowing the other side, and the angle opposite to the side required.*

As in the Rectangle A C B, hauing A C 27 *gr.* 53 *m.* 43 *s.* and B A C 23 *gr.* 31 *m.* 30 *s.* to find the side B C.

As the *Radius*	90	0	0	10000.0000
to the sine of the side giuen	27	53	43	9670.1112
So the Tangent of the opposite angle	23	31	30	9638.8199
to the Tangent of the side required.	11	30	43	19308.9311

10 *To find a side by knowing the other side and the angle next the side required.*

As in the rectangle B C D hauing B C 11 *gr.* 30 *m.* 43 *s.* and B D C 38 *gr.* 30 *m.* to finde D C.

As the Tangent of the angle	38	30	0	9900.6052
to the Tangent of the side giuen	11	30	47	9308.9311
So the *Radius*	90	0	0	10000.0000
to the sine of the side required	14	50	11	9408.3259

11 *To finde a side by knowing the Base and the Angle next the side required.*

As in the rectangle A C B, hauing A B 30 *gr.* 0 *m.* and B A C 23 *gr.* 31 *m.* 30 *s.* to finde the side A C.

As the *Radius*	90 0 0	10000, 0000
to the cosine of the angle	23 31 30	9962, 3153
So the Tangent of the Base	30 0 0	9761, 4393
to the Tang. of the side required	27 53 43	19723, 7546

12 *To find the Base by knowing both the oblique Angles.*

As in the rectangle A C B; hauing B A C 23 *gr.* 31 *m.* 30 *s.* and A B C 69 *gr.* 20 *m.* 35 *s.* to find the Base A B.

As the Tangent of the one angle	23 31 30	9638, 8199
to the cotangent of the other	69 20 35	9576, 3505
So the *Radius*	90 0 0	10000, 0000
to the cosine of the base	30 0 0	9937, 5306

13 *To find the Base, by knowing one of the sides and the Angle next that side.*

As in the rectangle A C B, hauing A C 27 *gr.* 53 *m.* 43 *s.* and B A C, 23 *gr.* 31 *m.* 30 *s.* to find the Base A B.

As the cosine of the angle	23 31 30	9962, 3153
is to the *Radius*	90 0 0	10000, 0000
So the Tangent of the side	27 53 43	9723, 7547
to the tangent of the base	30 0 0	9761, 4394

14 *To finde an Angle by knowing both the sides.*

As in the rectangle A C B, hauing A C 27 *gr.* 53 *m.* 43 *s.* and B C 11 *gr.* 30 *m.* 43 *s.* to finde the angle A B C.

As the sine of the next side	11 30 43	9300,1052
is to the Radius	90 0 0	10000,0000
So the tangent of the opposite side	27 53 43	9723,7547
to the tangent of the angle	69 20 35	10423,6495

15 *To find an angle by knowing the Base, and the side next the angle required.*

As in the rectangle B C D, hauing B D 18 *gr.* 41 *m.* 56 *s.* and B C 11 *gr.* 30 *m.* 43 *s.* to finde the angle B D C.

As the tangent of the Base	18 41 56	9529,5063
to the tangent of the side	11 30 43	9308,9311
So, is the Radius	90 0 0	10000,0000
to the cosine of the angle	53 0 46	9779,4248

16 *To finde an angle by knowing the Base and the other oblique angle.*

As in the rectangle A C B, hauing the Base A B 30 *gr.* and B A C 23 *gr.* 31 *m.* 30 *s.* to find the angle B A C.

As the cosine of the Base	30 0 0	9937,0000
is to the Radius	90 0 0	10000,0000
So the cotangent of the angle giuen	23 31 30	1036,1801
to the tangent of the angle required	69 20 35	10423,6495

These 16 cases are all that can fall out in a Rectangle triangle those which follow doe hold.

In

In any Sphæricall Triangle whatſoeuer.

17. *To finde a ſide oppoſite to an angle giuen by knowing one ſide and two angles, the one, oppoſite to the ſide giuen, the other, to the ſide required.*

As in the triangle A B D, hauing A B 30 *gr.* B D C 38 *gr.* 30 *m.* and B A D 23 *gr.* 31 *m.* 30 *ſ.* to find the ſide B D, which here repreſenteth the amplitude.

As the ſine of the next angle	38 30 0	*9794, 1495*
to the ſine of his oppoſite ſide	30 0 0	*9698, 9700*
		95, 1795
So the ſine of the oppoſite angle	23 31 30	*9601, 1352*
to the ſine of the ſide required	18 41 56	*9505, 9557*

Or changing the ſite of the two middle termes

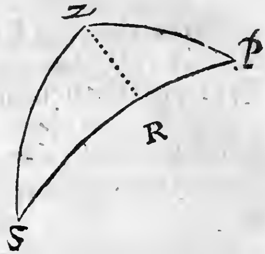
As the ſine of the next Angle	38 30 0	*9794, 1495*
to the ſine of the oppoſite Angle	23 31 30	*9601, 1352*
		193, 0143
So the ſine of the ſide giuen	30 0 0	*9698, 9700*
to the ſine of the ſide required	18 41 56	*9505, 9557*

And ſo writing this difference 193, 0143 in a paper by it ſelfe and holding it to the ſine of the ſide in the Canon. 1, *gr.* 2, 3, 4, 5 and ſo forward, it would bee no long worke to ſubtract and write the remainder in a columne by it ſelfe, and ſo find the amplitude for each degree & minute of the Ecliptique.

Or

Or, in steed of subtracting this difference, we might first take the same out of the *Radius*, and then adde the complement as I shewed before, in the generall explication of the Rule of Three.

18. *To finde an Angle opposite to a side giuen by knowing one angle and two sides, the one opposite to the angle giuen, the other to the angle required.*



As in the triangle Z P S representing the Zenith, Pole, and Sun: where *Z* P is the complement of the Latitude, P S, the complement of the declination, Z S the complement of the Sunnes altitude, P Z S, the Azimuth; Z P S, the houre of the day from the Meridian and P S Z the angle of the Suns Position in regard of the Pole and Zenith; hauing P *Z* S, 130 *gr.* 3 *m.* 11 *s.* PS 70 *gr.* and Z S 40 *gr.* to finde the angle *Z* P S.

As the sine of the next side	70 0 0	9972, 9858
is to the sine of his opposite angle	130 3 11	9883, 9153
		89, 0705
So the sine of the opposite side	40 0 0	9808, 0675
to the sine of the angle required	31 34 26	9718, 9970

19 *To find an Angle by knowing the three siides.*

As in the triangle *Z* P S, hauing *Z P* 38 *gr.* 30 *m.* *P S* 70 *gr.* and *Z S* 40 *gr.* to finde the angle *Z P S*, subtending the Base *Z S*,

As

As the Rectangle contained vnder the ſines of the ſides is to the ſquare of the *Radius* :
So the Rectangle contained vnder the ſines of the halfe-ſumme of the three ſides, and the difference betweene this halfe-ſumme and the Baſe,
to the Square of the coſine of halfe the angle required.

The Baſe ſubtended is	40 *Gr.*	0 *Mi.*
The two ſides including the Angle	38.	30
	70.	0
The ſumme of the 3 ſides	148.	30
The halfe-ſumme of theſe 3.	74.	15
The diff. between this & the Baſe	34.	15

Here for the Square of the *Radius* we take 20000. 0000 to this we adde 9983. 3805 the ſine of 74 *gr.* 15 *m.* and 9750. 3579. the ſine of 34 *gr.* 15 *m.* which make 39733. 7384.

Then for the *Rectangle* of the ſides we adde 9794. 1495 the ſine of 38 *gr.* 30 *m.* and 9972. 9858, the ſine of 70 *gr.* which make 19767.1353. This we take out of 39733.7384 and there remaines for the Logarithme of the ſquare 19966. 6031, the halfe thereof 9983. 3015 we finde to be the coſine of 15. 47′. 13″. And ſo, the whole Angle required is 31. 34′. 26″.

Or for ſuch numbers as are to be ſubtracted, we may take them out of the *Radius*, and write downe their Complements, and then adde them together with the reſt, the manner of the worke in either way will be ſuch as followeth.

40 *gr.*

40 gr. 0			
38. 30	9794.1495		205.8505
70. 0	9972.9858		27.0142
148. 30	19767.1353		
74. 15	9983.3805		9983.3805
34. 15	9750.3579		9750.3579
	20000.0000		
	39733.7384		
	19966.6031		19966.6031
	9983.3015	15.47'.13".	09983.3015
		31.34.26.	

In the like manner we may finde the angle *PZS* to be 130 *gr.* 3 *min.* 11 *seconds*, and the angle *ZSP* 30 *gr.* 28 *min.* 11 *seconds.*

20 *To finde a SIDE by knowing the three Angles.*

If for either of the Angles next the side required, we take the complement to 180 *gr.* these angles will be turned into sides, and the sides into angles. Then may the worke bee the same, as in the former Proposition.

As in the triangle *ZPS*, knowing the angle *ZPS* to be 31. 34'. 26". *PZS* 130. 3'. 11". and *ZSP* 30. 28'. 11". if it were required to finde the side *ZS* opposite to the angle *ZPS*, I would take 130 3' 11" out of 180 *gr.* the remainder will be 49 56 49

Then, as if I had a triangle of 3 knowne sides, one of 31 34' 26", another of 30 28' 11" and the third of 49 56' 49", I would seeke the angle opposite to the first of these sides, by the last Proposition.

So

So the angle which is thus found would be the side which is here required.

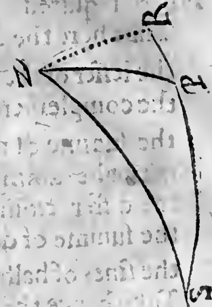
Thus here the Angle oppo. is	31 34′ 26″	
the lesser of the next Angles	30 28 11	9705.0790
the complement of the other	49 56 49	9883.9153
the summe of these three	111 59 26	
the halfe summe	55 59 43	9918,5490
the differ, from the opp.angle	24 25 17	9616.4170
the summe of double the *Radius* and		20000.0000
the sines of halfe summe and difference is		39534.9660
Take hence the sines of the next angles		19588.9943
there remaines for the square		19945 9717
The halfe whereof is the cosine		9972,9858

of 20 *gr.* 0′ and so the side required, 40 *gr*, 0 *m.*

The other sides may be found in the same sort; but when we know either three sides and one angle, or three angles and one side, the rest may be found more readily by the 17 or 18 Proposition.

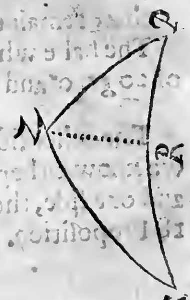
21 *To finde a SIDE by hauing the other two sides and the Angle comprehended.*

This and the Proportion following are best resolved by reducing the oblique-angle triangles given into two Rectangles.



As in the Triangle Z P S, hauing Z P 38 *gr.* 30′. *P S* 70. 0′ and Z *P* S 31. 34′. 26″ to finde the side Z S.

In that we haue *Z* P and Z P S, we may suppose a Perpendicular Z R to be let downe from the angle at Z vpon the greater side P S: So if Z P S the angle giuen be lesse then 90 *gr.* it will fall within the triangle; if more then 90 *gr.* it will fall without the triangle, vpon the side produced, and diuide the triangle giuen into two Rect-angles *Z* R S and Z R P. Wherein



1 We may finde the quantity of this Perpendicular by the first Proposition of Sphæricall Triangles.

2 Wee may finde the side P R either by the second or tenth, or rather by the eleventh Proposition: which side P R will giue the side R S.

3 Hauing *Z* R and R S, wee may find the base Z S by the fourth Proposition, as I shew in the vse of the Sector, *page* 86.

But here for variety, I will shew how the same may bee done of two opperations, both in this and the rest of the cases following, without knowing the quantity of the Perpendicular.

1 As the *Radius* or sine of Z R P	90. 0′ 0″	10000. 0000
to the cosine of the ang. Z P R	31. 34 26	9930. 4223
So the Tangent of the side *Z* P	38. 30 0	9900. 6052
to the tangent of the arke P R	34. 7 30	19831. 0275
2 As the cosine of P R	34. 7 30	9917, 9342

to

to the cosine of ZP	38. 30 0	9893.5443
		24. 3899
So the cosine of RS	35.52 30	9908.6438
to the cosine of ZS	40. 0 0	9884.2539

22 *To finde a* SIDE *by knowing the other two sides and one angle next the side required.*

As in the triangle ZPS hauing ZP, 38. 30′ and ZS 40 *gr.* 0′ and ZPS, 31. 34′ 26″ to find the side PS.

1 Find the arke PR by the 11 Proposition as before.

2	As the cosine of PZ	38. 30′ 0″	9893.5443
	to the cosine of P*R*	34. 7 30	9917.9342
			24.3899
	So the cosine of Z*S*	40. 0 0	9884.2539
	to the cosine of *SR*	35. 52 30	9908.6438

23 *To finde a* SIDE *by knowing one side and the two Angles next the Side required.*

As in the triangle ZP*S* having *ZP* 38 30 *m.* Z*PS*, 31 34 *m.* 26 *se.* and *ZPS* 30. 28 *m.* 11 *se.* to finde the side P*S*.

1 Finde the arke *PR* as before.

2	As the tangent of *ZSP*	30. 28 11	9769.6236
	to the tangent of Z*R S*	31. 34 26	9788.5746
			18.0510
	*S*o the sine of P*R*	34. 7 30	9748.9617
	to the sine of *SR*	35. 52 30	9767.9127

24 *To finde a Side by knowing two angles, and the Side inclosed by them.*

As in the triangle Z P *S* hauing Z P 38 30 *m.* *Z* P *S* 31 34 *m.* 26 *se.* and P *Z S* 130 3 *m.* 11 *sec.* to find the side Z *S*

1 As the cosine of	P *Z*	38	30′	0″	9893.5443
is to the *Radius*		90	0	0	10000.0000
*S*o the cotangent of	Z P S	31	34	26	10211,4253
to the tangent of	P *Z* R	64	18	50	10317.8810
2 As the cosine of	S Z R	65	44	22	9613,7228
to the cosine of	P *Z* R	64	18	50	9636.9311
					23,2083
So the tangent of	P Z	38	30	0	9900,6052
to the tangent of	Z S	40	0	0	9923,8135

25 *To finde an angle by knovving the other tvvo Angles and the side inclosed by them.*

As in the triangle ZPS hauing Z *P* 38 30 *m.* Z *P* S 31 34 *m.* 26 *se.* and *P* Z S 130 3 *m.* 11 *se.* to finde the angle Z S P.

1 Finde the angle P Z R by the 16 Proposition at before.

2 As the sine of	P Z R	64.	18	50	9954.8122
to the sine of	S Z R	65.	44	21	9959.8453
					5.0331
So the cosine of	*Z* P S	31.	34	26	9930.4223
to the cosine of	Z S P	30.	28	11	9935.4554

26 *To finde an angle by knowing the other two Angles and one side next the angle required.*

As in the triangle ZPS, hauing ZP 38. 30 *m.* ZPS, 31 *gr.* 34 *m.* 26 *se.* and ZSP 30. 28 *m.* 11 *se.* to finde the angle PZS.

1 Finde the angle PZR as before.

2 As the cosine of	ZPS	31.	34	26	9930.4223
to the cosine of	ZSP	30.	28	11	9935.4554
					5.0331
So the sine of	PZR	64.	18	50	9954.8122
to the sine of	SZR	65.	44	21	9959.8453

27 *To finde an Angle by knowing two sides and the angle contained by them.*

As in the triangle ZPS, hauing ZP 38. 30 *m.* PS 70 *gr.* and ZPS, 31. 34 *m.* 26 *se.* to finde the angle ZSP.

1 Finde the arke PR as before.

2 As the sine of	SR	35.	52'	30"	9767.9127
to the sine of	PR	34.	7	30	9748.9617
					18.9510
So the tangent of	ZPS	31.	34	26	9788.5746
to the tangent of	ZSP	30.	28	11	9769.6236

28 *To finde an angle by knovving the tvvo next sides, and one of the other angles.*

As in the triangle Z P S hauing Z P 38. 30 *m.* Z S 40 *gr.* and Z P S 31. 34 *m.* 26 *se.* to finde the angle P Z S.

1 Finde the angle P Z R, as before.

2 As the tangent of	Z S	40 0 0	9923.8135
to the tangent of	Z P	38. 30 0	9900.6052
			23.2083
So the cosine of	P Z R	64. 18 50	9636.9311
to the cosine of	S Z R	65. 44 21	9613.7228

These 28 Cases are those which I set downe in the vse of the *Sector*, and all that are commonly required in a sphæricall triangle. I will here adde two more, to shew how that which is found before, by the 22. 23. 26 and 28. Propositions may sometimes be found more easily. *viz.*

29 *To finde a Side by knovving the other tvvo Sides and their opposite angles.*

As in the triangle Z P S, hauing P S 70 *gr.* and P Z S 130 3 *m.* 11 *se.* together with Z S 40 *gr.* and Z P S 31. 34 *m.* 26 *s.* to finde the third side Z P.

As the sine of halfe the difference of the angles giuen,
to the sine of halfe the summe of those angles:
So the tangent of halfe the difference of the sides giuen,
to the tangent of halfe the side required.

30 *To*

30 *To finde an Angle by knowing the other two angles, and their opposite sides.*

As in the triangle Z P S, hauing the former parts P S, P Z S, Z S and Z P S, to finde the third angle Z S P.

As the sine of halfe the difference of the sides giuen,
to the sine of halfe the summe of those sides;
So the tangent of halfe the difference of the angles giuen,
to the cotangent of halfe the angle required.

CHAP. III.

Concerning the ioynt vse of the Lines of Numbers, Sines and tangents, I shewed how they might serue for the resolution of right lined Triangles, whereof I set downe fiue propositions, *page* 24. And these also may be applyed to the Table and Canon of Logarithmes.

The sides of these triangles are measured by absolute numbers, and so represented by Logarithmes.

The angles are measured by degrees and minutes, and so to be found by sines and tangents in the Canon.

PROP.

PROP. I.

Hauine three Angles, and one side to finde the other two SIDES.

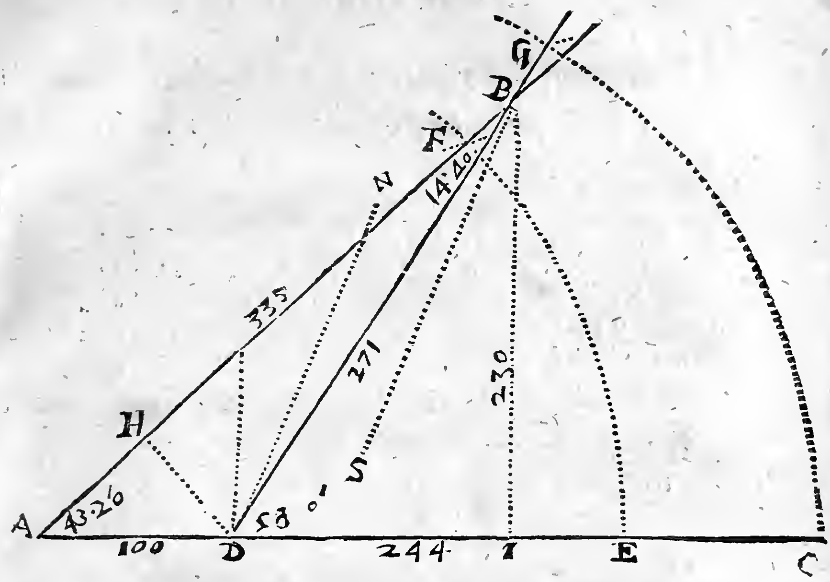
If it be a rectangle triangle, wherein one side about the right angle being knowne, it were required onely to finde the other, this might bee readily done by Sines and Tangents. As in the rectangle A I B, knowing the angle B A I to be 43.20. and the side A I to be 244, if it were required to finde the other side A I.

As the *Radius* (the tangent of)		45 gr. 0 m.	10000.0000
is to the tangent of the angle		43 20	9974.7195
So is the side ginen	A I	244. $\frac{000}{}$	2387.3898
to the side required	B I	230. $\frac{202}{}$	12362.1093

But where both the other sides are required, it is best done by Logarithmes and Sines. As in the same rectangle A I B, hauing the 3 angles and the side A I, to finde both B I and A B.

As the sine of the opposite angle	A B I	46.	40	9861,7575
is to the side giuen	A I	244.	000	2387.3898
				7474.3677
So the sine of the second angle	B A I	43.	20′	9836.4770
to his opposite side	B I	230.	$\frac{202}{}$	2362.1093
As the sine of the third angle	A I B	90.	0	10000.0000
to his opposite side	A B	335.	$\frac{453}{}$	3|2525.6323

The



The like holdeth alſo in obliqu-angled triangles.

As in the Triangle A B D (which I propoſed page 13. as an example for the finding of diſtances) where knowing the diſtance between A and D, to be 100 paces; the angle BAC to be 43. 20 *m.* the angle B D A 122, or the outward angle B D C, 58 *gr.* and conſequently the angle A B D oppoſite to A D the ſide giuen to be 14. 40 *m.* it was required to find the diſtances A B and D B.

		Gr.	*M.*	
As the ſine of the oppoſite angle	A B D	14	40	9403.4554
is to the ſide giuen	A D	100.	000	2000.0000
				7403.4554
So the ſine of the ſecond angle	A D B	58.	0′	9928.4204
to his oppoſite ſide	A B	334.	937	2524.9650
And the ſine of the third angle	D A B	43.	20′	9836.4770
to his oppoſite ſide	D B	271.	032	2433.0216

PROP. II.

Hauing two ſides and one angle oppoſite to either of thoſe ſides to find the other two angles and the third ſide.

As in the triangle A B D, hauing the two ſides A B 335 paces and A D 100 paces, and knowing the angle A D B which oppoſite to the ſide A B, to be 122 *gr.* or the outward angle B D C to be 58 *gr.* if it were required to find the other two angles at A and B, and the third ſide B D. I may firſt find an angle A B D oppoſite to the other knowne ſide AD.

As the oppoſite ſide	A B	$335\frac{000}{}$	2525,0448
to the ſine of the angle giuen	A D B	58. 0′	9928,4204
			7403,3756
So is the next ſide	A D	$100\frac{000}{}$	2000,0000
to the ſine of his opp. angle	A B D	$14.59\frac{5}{6}$	9403,3756

Then knowing theſe two angles at D and B, I take the inward angle ABD 14 59′ 50″ ſ. out of the outward angle BDC 58 0′ and ſo find the thrid angle B A D, to bee 43 20′ 10 ſ. So hauing three angles, and 2 ſides I may well find the third ſide B D by the former Proportion.

As the ſine of the firſt angle	A D B	58 *gr.* 0 *m.*	9928,4204
is to his oppoſite ſide	A B	$335\frac{000}{}$	2525,0448
			7403,3756
So the ſine of the laſt angle	D A B	$43.20\frac{1}{6}$	9836,5033
to his oppoſite ſide	D B	$271\frac{810}{}$	2433,1277

PROP.

PROP. III.

Hauing two sides and the angle betweene them to finde the other two angles and the third side.

If the angle conteined betweene the two sides giuen bee a right angle, the other two angles will be found readily by tangents and Logarithmes. As in the rectangle A I B hauing the side A I 244 and the side I B to find the angles at A and B.

As the greater side	A I	244	2387,3898
is to the lesser side	I B	230	2361,7278
So the *Radius* the tangent of		45 *gr.* 0'	10000,0000
to the tangent of the lesser angle		43 18½	9974,3380

But if it be an oblique angle that is conteined betweene the two sides giuen, the triangle may be reduced into two rectangle triangles, and then resolued as before.

As, in the triangle A D B, hauing the sides A B 335 A D, 100 and the angle B A D 43 20', to finde the angles at B and D, and the third side B D. First, I would suppose a perpendicular D H to bee let downe from D, the end of the lesser side, vpon the greater side A B: so shall I haue two rectangle triangles D H A and D H B. And in the rectangle A H D, the angle at *A* being 43 20' the other angle *A D H* will be 46. 40' by complement and with these angles and the side *A D*, I may find both *A H* and *D H* by the first proportion. Then taking *A H* out of *A B*, there remaines *H B* for the side of the Rectangle *D H B*, and therefore with this side *H B* and the other side *D H*, I may finde the angle at *B*, by the former part of this proportion. And with this angle and the perpendicular *D H*, I may finde the third side *D B*, by the first proposition.

Or hauing two sides and the angle betweene them, wee may finde the other two angles without letting downe any perpendicular, in this manner.

As the ſumme of the two ſides giuen
is to the difference of theſe ſides
So the tangent of halfe the ſum of the two oppoſite angles to the tangent of halfe the difference betweene thoſe angles.

So here hauing the ſide	*AB*	335	
and the other ſide	*AD*	100	
the ſumme of theſe ſides is		435	2638,4892
and the difference of theſe ſides		235	2371,9678
The angle conteined *BAD* is		43 20'	267,4214
the ſumme of the two oppoſite angles		136 40'	
the halfe ſumme of theſe angles		68 20	10400,9092
and by proportion and halfe difference		53 40$\frac{1}{5}$	10133,4878
This halfe ſum & halfe differēce mak		122 0$\frac{1}{5}$	the greater angle
and the difference betweene them		14 19$\frac{4}{5}$	the leſſer angle

PROP. IV.

Hauing three ſides, to finde the three angles.

Let one of the three ſides giuen be the Baſe, (but rather the greater ſide) that the perpendicular may fall within the triangle. Then gather the ſumme and the difference of the two ſides, and the proportion will hold.

As the Baſe of the Triangle
to the ſumme of the ſides
So the difference of the ſides
to the alternate Baſe. This alternate Baſe being taken forth of the true baſe, if wee let downe a perpendicular from the oppoſite angle, it ſhall fall vpon the middle of the remainder. As in the triangle ADB.

The

The lesser side is	*AD*	100	
The other side	*BD*	271	
The Base of the triangle	*AB*	335	2525, 0448
The summe of the sides		371	2569, 3739
			44, 3291
The difference betweene these sides		171	2232, 9961
and so the alternate Base is		189 376/	2277, 3252
This taken out of 335 leaues		145 624/	

the halfe whereof is——— 72 812. And such is the segment *AH*, the distance betweene the angle at *A* and the perpendicular *DH*. So that hauing drawne this perpendicular, wee haue two rectangle triangles *DHA* and *DHB* in which hauing two sides and the right angle, wee may find the other angles by the second proposition.

These foure propositions may suffice for the resolution of the sides and angles in all right lined Triangles.

PRPOP. V.

Hauing the Base and Perpendicular in a right-lined Triangle, to finde the superficiall content.

The perpendicular may bee found, by one or other of the former proposition S, and that being known we may find the superficiall content. As in the Triangle ADB, hauing the Base AB 335, and the perpendicular DH 68, 545.

As the number of	2	0301, 0700
to the perpendicular	68, 545	1835, 9757
		1534, 9457
So the Base	335	2525, 0448
to the content.	11 481 293/	4059, 9905

 Or,

Or, if we would find the content without knowing the perpendicular, we may put two or more operations into one, as in the proportion following.

PROP. VI.

Hauing two sides of a right-lined Triangle, and the angle betweene them, to find the content.

Adde the sine of the Angle, and the Logarithmes of both the sides, from the summe of these subtract —— 10301, 0300 so the Remainder shall be the Logarithme of the content.

As, in the triangle A D B, hauing the sides AB 335, AD 100, and the angle BAD 43 *gr.* 20 *m.*

The sine of the angle 43 *gr.* 20 *m.* is	9836, 4770
the Logarithme of the side A B 335	2525, 0448
the Logarithme of the side A D 100	2000, 0000
The summe of these make	14361, 5218
from which subtract the solemne Logarithme	10301, 0300
the Remainder will be	4060, 4918

the Logarithme of 11494 the content required.

PROP. VII.

Hauing three Angles, and one side of a right-lined Triangle, to finde the content.

Adde the double of the Logarithme of the side giuen, and the sines of the two next angles; from the summe of these subtract the summe of 10301, 0300, and the sine of the opposite angle, so the Remainder shall bee the Logarithme of the content.

As

As in the Triangle ADB ſuppoſing the angles BAC to be 34 D. 20 *m.* BDA 122 D. 0 *m,* ABD 14 *gr.* 40 *m*; and the ſide AD to be 100 parts.

The Logarithme of the ſide AC	100 is	2000,0000
the ſame againe		2000,0000
The ſine of the angle BAC	43 *gr.* 20′	9836,4770
The ſine of the angle BDA	58 0	9928,4204
The ſumme of theſe foure make		23764,8974
Againe if we adde the ſolemne Logarithme		10301,0300
to the ſine of the oppoſite angle	14 *gr.* 40′	9403,4554
The ſumme of both will make		19704,4854
Which ſubtracted from 23764.8974 leaue		4060,4120

the Logarithme of 11492 the content required.

PROP. VIII.

Hauing the third ſides of a right-lined triangle, to finde the content.

Firſt ſet downe the three ſides, the ſumme of them, and the halfe ſumme. Then from this halfe-ſumme ſubtract each ſide ſeuerally, and note the differences. That done, adde the Logarithmes of the halfe-ſumme, and theſe differences, the halfe thereof ſhall be the Logarithme of the content.

Thus in the triangle ADB, the three ſides are	*AB.*	335	
	DB.	271	
	AD.	100	
the ſumme of theſe ſides is		706	
the halfe ſumme		353	2547,7747
the difference from AB		18	1255,2725
the difference from DB		82	1913,8138
the difference from AD		253	2403,1205
The ſumme of their Logarithmes			8119,9815
and the halfe thereof is			4059,9907

the Logarithme of 11481 $\frac{293}{}$ the content required.

PROP.

PROP. IX.

Hauing the three sides of a right-lined triangle, to finde the Perpendicular.

As, in the former triangle ADB, to finde the perpendicular DH. First, find the content of the Triangle by the former proportion, then may the perpendicular bee found by the conuerse of the V. Proposition.

As the Base of the triangle	335	2525,0448
to the superficiall content.	11481 $\frac{29}{}$	4059,9907
		1534,9459
So alwayes the number of	2	0301,0300
to the perpendicular	68 $\frac{545}{}$	1835,9759

PROP. X.

Hauing the Semidiameter of a Circle to finde the Chord for any Arke proposed.

As if in protracting the former triangle ADB it were requnired to find length of a Chord of 43 *gr.* 20 *m.* agreeing to the Semidiameter AE, which wee suppose to be 3 inches. This might be done by the first proportion for, if the chord were drawne from E to F we should haue a triangle EAF of three angles and two sides knowne. But, more generally comparing the sine of 30 *gr.* with the sine of halfe, the arke proposed, the proportion will hold.

As the sine of the *Semiradius*	30 *gr.* 0 *m.*	9698,9700
to the Semidiameter	3 $\frac{000}{}$	0477,1212
		9221,8488
So the sine of halfe the arke	21 *gr.* 40 *m.*	9567,2689
to the Chord required	2 $\frac{215}{}$	0345,4201

So

So that hauing drawne the line A E, and described an occult arke of a Circle vpon the center A, and semidiameter A E at the distance of three inches, if we take out two inches, and 215 parts of 1000, and inscribe them into that arke from E to F, the line A F shall make the angle F A E to be 43 20 *m.* as was required.

Thus hauing applyed that to the Canon and table of Logarithmes which I had set downe before for the generall vse of the lines of numbers, sines, and tangent, it may appeare sufficiently, that, if we obserue the rules of proportion set forth by others, and worke by these Tables, we may vse addition insteed of their multiplication, and subtraction insteed of their diuision, and so apply these generall rules to infinite particulars.

CHAP. IV.

Containing some vse of right-lined triangles, in the practise of Fortification.

IN the late manner of Fortification the ordinary care is.

1 That the angle of the Bulwarke may be either a right angle, or neere vnto it.

2 That this angle may be defended from the flanque and cortin on either side.

3 That the lines of defense may not exceed the reach of a musket, which is said to bee xij. score yards and those make 720 foot.

4 That the depth of the flanques and bredth of the rampart be sufficient to resist a battery; and that may be about 100 foot at the ground.

Vpon these considerations depend the rest of lines and angles: whereof I will set downe some Propositions, beginning with that which may resolue the works of others.

PROP. I.

Hauing the side of a Regular Fort, with the length of the Gorge, the Flanque and the Face of the Bulwarke, to find the rest of the lines and angles.

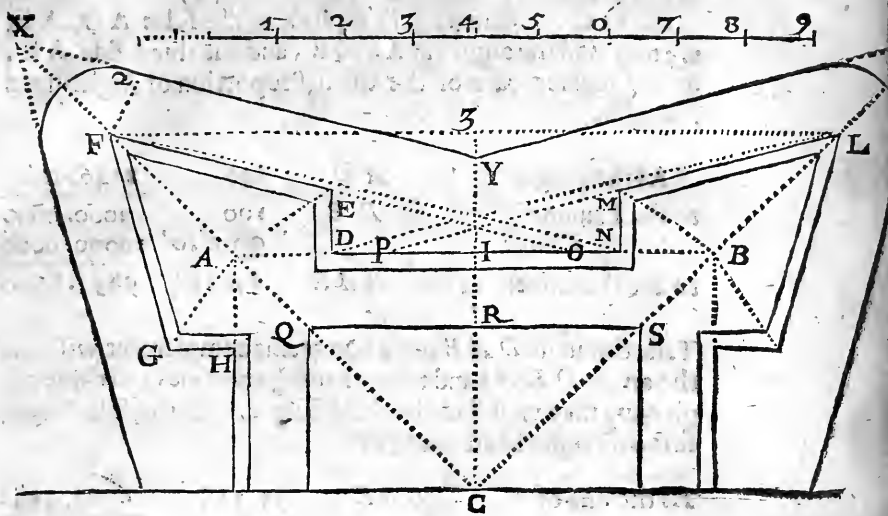
A regular Fort is that, which is made with equall sides and angles, each Bulwarke like vnto other.

Suppose that, by obseruation or otherwise we haue found, that in a square fort, the side was 700 foot, the Gorge 140, the Flanque 100, and the Face 335: In a Pentagonall, hexagonall, heptagonall, as in this table.

		Quadr	Pentag	Hexag	Heptag	Octag.
The side	*AB*	700	800	900	950	1000
The gorge	*AD*	140	180	190	200	230
The flanque	*DE*	100	120	140	150	140
The face	*EF*	335	352	370	360	420

And that it were required to find the rest of the lines, and the quantity of the angles belonging to each Fort, beginning with the quadrate.

First,



First we may protract this Fort, by making a square whose side A B shall bee 700 foot by the scale: then take but 140 for the gorge, and set them of from A vnto D, and from A vnto H. At D and H raise 2, flanques perpendicular to the sides of the fort and there pricke downe 100 from D vnto E, and and from H vnto G. That done, take 335 out of the same scale, and setting one foot of the compasses in the point E, make an occult arke of a circle. Againe, setting one foote of the compasses in the point G, make another occult arke, crossing the former in the point F; So the lines, E F, F G shall represent the face of the Bulwarke.

In like manner, for the Bulwarke at B, wee may set of the gorge from B vnto N, &c. So haue wee diuerse triangles, which may be resolued by the first 3. Propositions of righlined triangles. And the manner of it shall be so set downe, as that the Precept may be easily distinguished from the example, and applied to any other, not onely by this canon and table of Logarithmes, but by the old Canon of sines and tangents, and by the lines of sines and tangents both vpon the Sector and the crosse-staffe.

1 In the Rectangle A D E, hauing the sides AD, A E, we may find the angles at A and E, and the third side A E, by the former part of the third Proportion of Right-lined triangles.

As the gorge	*A D*	140	2146. 1280
to the Flanque	*D E*	100	2000. 0000
So the *Radius*		90. 0'. 0"	10000. 0000
to the the tangent of	*D A E*	35. 32. $\frac{1}{4}$	9853. 8720

Take the angle *D A E* out of 90 *gr*. the complement will giue the angle *D E A*: and then, hauing two sides and three angles, we may well find the third side *A E* by the first Proposition of right-lined triangles

As the sine of	*D A E*	35. 32 $\frac{1}{4}$	9764. 3542
to the side	*D E*	100.	2000. 0000
So the sine of	*A D E*	90. 0'. 0"	10000. 0000
to the side	*A E*	172 $\frac{047}{}$	2235. 6458

2 Because the fort is supposed to bee square, the angle *H A D*, must be 98 *gr*. and the halfe angle *C A D* 45 *gr*. if wee adde this angle C A D vnto the angle *D A E* and take the summe out of 180 *gr*. the remainder 99. 2$\frac{1}{74}$ shall be the angle *E A F*. Then in the triangle *E A* F, hauing the angle at *A*, and the two sides F E, A E, wee may finde the other angles at E and F, by the III. Proposition of right-lined triangles.

As the face	E F	335	2525. 0448
to the sine of	E A F	99. 27$\frac{3}{4}$	9994. 0502
			7469. 0054
So the line	A E	172 $\frac{047}{}$	2235. 6459
to the sine of	A F E	30. 26$\frac{1}{7}$	9704. 6513

Adde

Adde this angle A F E to the angle E A F, and take the summe out of 180 *gr.* the Remainder 50. 6.4″ shall be the angle A E F. And then we haue two sides and three angles; to finde the head-line A F.

As the sine of	E A F	99. 27 $\frac{3}{4}$	9964.0502
to the face	E F	335.	2525.0448
			7469.0054
So the sine of	A E F	50. 6. $\frac{1}{15}$	9884.8958
to the headline	A F	260 $\frac{55}{}$	2415.8904

3 If we produce the face F E vntill it meet the cortin in O; we shall haue the triangle A F O: wherein, knowing the side A F, and the three angles (for, knowing two angles, the third is alwayes knowne by complement vnto 180 *gr.*) wee may finde the other two sides F O, A O.

As the sine of	A O F	14. 33′. 48″	9400.4548
to the head-line	A F	260 $\frac{55}{}$	2415.8904
			6984.5644
So the sine of	F A O	45. 0′. 0″	9849.4850
to the line	F O	732 $\frac{69}{}$	2864.9206
and the sine of	A F O	30. 26. 12″	9704.6513
to the line	A O	524 $\frac{912}{}$	2720.0869

Take the gorge N B 140, out of the side A B 700. there remaines 560 for the line A N. Take this line A O out of A N, and there remaines 35 $\frac{088}{}$ for O N that part of the cortin from whence the face of the Bulwarke may be defended.

4 In the triangle A F N hauing two sides A F, A N, and the angle betweene them F A N, we may finde the other two angles at F and N, by the later part of the third Proposition of right-lined triangles.

As the summe of the sides *A* F, *A* N.		$820\frac{55}{}$	2914.1050
is to the difference of those sides		$299\frac{45}{}$	2476.3245
So the tangent of the halfe summe of the			437.7805
opposite angles at F and N.		22.30'	9617.6153
to the tangent of halfe the difference between those angles.		$8.36\frac{1}{5}$	9179.8348

This halfe difference added to the halfe sum, giues the greater angle. *A* F N $31.6\frac{1}{5}$

and subtracted, the lesser A N F $13.53\frac{4}{5}$

As the sine of *A* N F		13.53.48'	9380.5157
to the headline *A* F		$260\frac{550}{}$	2415.8904
			6964.6253
So the Sine of F *A* N		45.0.0	9849.4850
to the line of defence F N		$767\frac{113}{}$	2884.8597

5 In the triangle A B C we haue the side A B, and the 3. angles, to finde the side C A or C B, from the center to the angles of the Fort.

As the sine of	*A* C B	90.0.0	10000.0000
to the side	*A* B	700.	2845.0980
So the sine of	A B C	45.0.0	9849.4850
to the line	*A* C	$494\frac{975}{}$	2694.5830

This line *A* C added to the headline *A* F, giues the whole C F, from the center of the Fort to the vttermost point of the Bulwark to be $755\frac{525}{}$

6 In the triangle C F L (the side F L being parallel to A B the side of the Fort) we haue the three angles and the side C F; by which we may finde F L the distance between the points of the two next Bulwarks.

As the sine of	C L F	45. 0 0	9849.4850
to the line	C F	755. $\frac{525}{}$	2878.2498
So the sine of	F C L	90. 0 0	10000.0000
to the sine	F L	1068. 464	3028.7648

Thus

Thus by resoluing of six triangles we haue found

The angle at the gorge	D A E	35.	32′	15″	
the angle of the Bulwark	G F E	60.	52	24	
the angle	F E D	104.	33	48	
the angle	A N F	13.	53	48	

		Foote
The length of the line	A E	172. 047
the Headline	A F	260. 550
the Line on the Cortin	O N	35. 088
the Line of defence	F N	767. 113
the semidiameter	C A	494. 975
the line frō the center to the Bulw.	C F	755. 525
the distance betweene the Bulw.	F L	1068. 464

the principall Lines and Angles belonging to the Bulwark at A.

The rest of the lines are either parallell vnto these, or else they may be found in the same manner.

And all these may be vnderstood to be the same in the rest of the *Bulwarkes* belonging to this Fort.

Againe, what is said of a square Fort, the same may be applyed to all regular Forts.

And so, resoluing the workes of other men, it may appeare how neere they haue come to the former grounds.

But that wee may not altogether insist vpon examples, I will set downe some profitable suppositions, and from them proceed to finde the rest of the lines and angles belonging to auy Regular Fort.

1 The angle at the center A C B, betweene the lines C A, C B, drawne from the Center to each Bulwarke, is found by dividing 360 *gr.* by the number of the sides. *So* in a square Fort, this angle will be 90 *gr.* In a Pentagonall Fort, where there are fiue sides, it will be 72 *gr.* &c.

2 Take this angle at the center, out of 180 *gr.* there remaines the angle of the Fort H A D.

3 The

3 The angle *A* D E between the Flanque and the Cortin, may be alway 9o *gr.*

4 The vttermost angle of the Bulwarke E F G, must be lesse then the angle of the Fort, yet not lesse then 6o *gr.* nor doth it need to be much more then 9o *gr.* If we allow it to be $\frac{2}{3}$. of the angle of the Fort, it may be defended from the Flanque and Cortin on either side.

5 The angle at the Gorge D A E, which formes the Flanque D E, may be allowed betweene 35 and 4o *gr.* For in small regular Forts, it may be 4o *gr.* but where the angle of the Fort is great, it may be lesse.

These 5. angles being first settled, the most of the other angles will depend vpon them, as in the Table following.

Or howsoeuer there may bee other angles found to bee more convenient, yet these are sufficient to explane the vse of triangles.

In a Regular Fort.		Quadr. Gr.	Quadr. M.	Pentag. Gr.	Pentag. M.	Hexag. Gr.	Hexag. M.	Heptag. Gr.	Heptag. M.	Octag. Gr.	Octag. M.	Cortin. Gr.	Cortin. M.
Angle at the Center	*A C B*	90	0	72	0	60	0	51	25	45	0	0	0
Angle of the Fort	*H A D*	90	0	108	0	120	0	128	34	135	0	180	0
Angle of the Flanque	*A D E*	90	0	90	0	90	0	90	0	90	0	90	0
Angle of the Bulwarke	*G* F *E*	60	0	72	0	80	0	85	42	90	0	90	0
Angle of the Gorge	*D A E*	40	0	39	0	38	0	37	0	36	0	35	0
The halfe of H A D is C A D		45	0	54	0	60	0	64	17	67	30	90	0
Halfe of G F *E is A* F *E*		30	0	36	0	40	0	42	51	45	0	45	0
Complement of C A D is D A F		135	0	126	0	120	0	115	43	112	30	90	0
*AFE out of C AD leaues AO*F		15	0	18	0	20	0	21	25	22	30	45	0
*Complement of AO*F *is OED*		75	0	72	0	70	0	68	35	67	30	45	0
Complement of O E D is D E F		105	0	108	0	110	0	111	26	112	30	135	0
Complement of D A E is AED		50	0	51	0	52	0	53	0	54	0	55	0
A E D *out of D E F leaues AEF*		55	0	57	0	58	0	58	26	58	30	80	0
A E F and A F *E giue* F *A E*		95	0	87	0	82	0	78	43	76	30	55	0

II. *Hauing*

PROP. II.

Hauing the ordinary angles, with the Flanque and line of Defense, to finde the rest of the lines and angles, in a regular Fort.

SVppose the angles to be such, as in the former table, the depth of the flanque DE 100. foot, and the line of defense FN 720. foote; and that it were required, to find the rest of the lines and angles belonging to a Pantagonall fort.

1 In the triangle ADE, hauing the three angles and the flanque DE, we may find the length of the gorge AD, and the line AE. The angle ADE is alway 90 *gr.* but, the fort being Pentagonall, mad with fiue Bulwarkes at the fiue angles, the table giues the angle DAE to bee 39 *gr.* and the angle AED 51 *gr.* wherefore.

As the sine of	DAE	39.0'.0"	9798.8718
to the flanque	DE	100.	2000.0000
			7798.8718
So the sine of	AED	51.0'.0"	9890.5026
to the gorge	AD	123 $\frac{42}{}$	2091.6308
And the whole sine	*ADE*	90.0.0.	10000.0000
to the line	*AE*	158 $\frac{90}{}$	2201.1282

2 In the triangle AFE, hauing the three angles and the side AE, we may find the face of the Bulwarke FE, and the head-line AF.

As the ſine of	A F E	36. 0. 0.	9769.2186
to the line	A E	158^{90}	2291.1282
			7568.0904
So the ſine of	F A E	87. 0. 0.	9999.4044
to the face	F E	269^{97}	2431.3140
And the ſine of	A E F	57. 0. 0.	9923.5914
to the head-line	A F	226^{72}	2355.5010

3 In the triangle A F O, hauing the three angles and the ſide A F, we may find the other two ſides F O and A O.

As the ſine of	A O F	18. 0. 0.	9489.9823
to the headline	A F	226^{72}	2355.5010
			7134.4813
So the ſine of	F A O	126. 0. 0.	9907.4576
to the line	F O	593^{57}	2773.4763
And the ſine of	A F O	36. 0. 0.	9769.2186
to the line	A O	431^{36}	2634.7373

4 In the triangle A F N, hauing the headline A F the line of defenſe F N, and the angle F A N, wee may find the other two angles at N and F, and the third ſide A N.

As the line of defenſe	F N	720.	2857.3325
to the ſine of	F A N	126. 0. 0″.	9907.9576
			7050.6251
So the headline	A F	226^{72}	2355.5010
to the ſine of	A N F	14. 45. 33.	9406.1261

This angle A N F added to the angle F A N, and the ſumme of both taken out of 180 *gr.* will giue the third angle A F N.

As the ſine of	F A N	126 *gr.* 0. 0.	9907.9576
to the line of defenſe	F N	720.	2857.3325
			7050.6251
So the ſine of	A F N	39. 14′ 27″	9801.1178
to the line	A N	562^{98}	2750.4927

Hauing

Hauing this line A N if we adde the gorge N B, or A D, the ſumme of both ſhall be the ſide of the fort A B.

If wee take the gorge A D, out of this line A N, the remainder ſhall be the cortin D N.

Againe if we take the line A O, out of this line A N, the remainder ſhall be O N, that part of the cortin from whence the face of the Bulwarke may be defended. And ſo here

The length of this line	A N being	562. 98
the gorge	A D	123. 49
the ſide of the fort	A B ſhall be	686. 47
the cortin	D N	439. 49
Againe taking the line	A O	431. 26
from AN, there remaines	O N	131. 72

5 In the triangle A I C, hauing the three angles, and the ſide A I, the one halfe of A B the ſide of the fort, wee may find both O I, the ſemidiameter of the circle inſcribed, and C A, the ſemidiameter of the circle circumſcribed about the fort.

As the ſine of	A C I	36. 0'. 0''.	9769.2186
to the line	A I	343 $\frac{235}{}$	2535.5915
			7233.6271
So the ſine of	C A I	54. 0. 0.	9907.9576
to the line	C I	472. 4225.	2674.3305
And the whole ſine	C I A	90. 0. 0.	10000.0000
to the line	C A	583. 9466.	2766.3729

This line C A added to the head-line A F, giues the diſtance C F betweene the center of the fort, and the vttermoſt point of the Bulwarke.

6 If this fort ſhall be incompaſſed with a dith, whoſe vttermoſt ſides ſhall bee parallell to the face of the Bulwarke; ſuppoſing this ditch to be of a known bredth (and that may be about 100 foot) we haue the triangle F 2 X; wherein, knowing the three angles, & the ſide F 2, we may find the line FX.

As the ſine of	F X 2	36. 0. 0.	9769. 2186
to the bredth-line	F 2	100.	2000. 0000
So the whole ſine	F 2 X	90. 0. 0.	10000. 0000
to the line	F X	$170^{\underline{13}}$	2230. 7814

This line F X added to the line C F, giues the diſtance C X, betweene the center of the fort, and the vttermoſt corner of the ditch. And ſo here,

The length of the head-line	A F is	226. 72
the ſemidiameter	C A	583. 95
Both theſe make the line	C F	810. 67
Adde vnto this the line	F X	170. 13
So, *C A, A* F, F X make	C X	980. 80

7 In the triangle C Y X, hauing the three angles and the ſide C X, we may finde the two other ſides C Y and X Y.

As the ſine of	C Y X	108. 0'. 0''.	9978. 2063
to the line	C X	980^{80}	2991. 5815
			6986. 6248
So the ſine of	C X Y	36. 0. 0.	9769. 2186
to the line	C Y	$606^{\underline{169}}$	2782. 5938
And the ſine	X C Y	36. 0. 0.	9769. 2186
to the line	X Y	$606^{\underline{169}}$	2782. 5938

Take the line C I, from this line C Y, there remaines I Y, the bredth of the ditch from the middle of the cortin.

8 Then, for the lines F L, X Z, and ſuch other parallels to the ſide of the fort A B,

As the ſemidiameter	C A	583. 95	2766. 3729
to the ſide of the fort	A B	686. 47	2836. 6215
			7074. 2486
So the length of	C F	810. 67	2908. 8444
to the diſtance	F L	953. 00	2979. 0930
And the length of	C X	980. 80	2991. 5815
to the diſtance	X Z	1152. 97	3061. 8201

9 The

9 The Perpendiculars C 3, C 4, and such others, let downe from the center vpon the former parallels may bee found in the same sort,

As the semidiameter	C A	583. 95	2766. 3729
to the Perpendicular	C I	472. 42	2674. 3305
			92. 0424
So the length of	C F	810. 67	2908. 8444
to the Perpedicular	C 3	655. 84	2816. 8020
And the length of	C X	980. 80	2991. 5815
to the Perpendicnlar	C 4	793. 48	2899. 5391

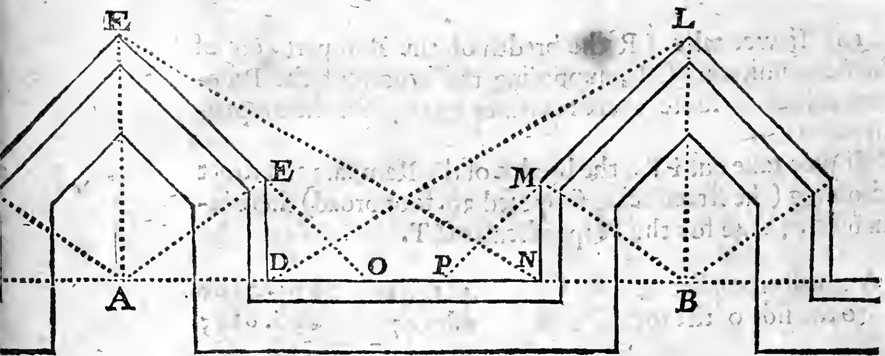
10 If wee take I R the bredth of the Rampart, out of the Perpendicular C I, supposing the bredth of the Rampart to be 100. foote, there remaines 372. 42 for the Perpendicular C R.

If wee take out I T, the bredth of the Rampart and street adioining (the street being supposed 30. foot broad) there remaines 342. 42 for the Perpendicular C T.

As the Perpendicular	C I	472. 42	2674. 3305
to the side of the fort	A B	686. 47	2836. 6215
			162. 2910
So the Perpendicular	C R	372. 42	2571. 0358
to the side of the Rampart	Q S	541. 16	2733. 3268
And the Perpendicu'ar	C T	342. 42	2534. 5622
to the inner side of the street	V W	497. 57	2696. 8532
As the Perpendicular	C I	472. 42	2674. 3305
to the semidiameter	C A	583. 95	2766. 3729
			92. 0424
So the Perpendicular	C R	372. 42	2571. 0378
to the line	C Q	460. 34	2663. 0802
And the Perpendicular	C T	342. 42	2534. 5622
to the line	C V	423. 25	2626. 6046

PROP. III.

Hauing the ordinary angles with the line of defense and face of the Bulwarke, to finde the rest of the lines and angles.



SVppose a long cortin to be fortified with Bulwarkes, the angle of each Bulwarke to bee 90 *gr.* the angle at the gorge forming the flanque 35 *gr.* the rest, as in the former table, the line of defense, 720 foote, and the face of the Bulwarke 300 foote.

1 In the triangle A E F, hauing the three angles and the face F E, wee may finde the headline A F, and the line *A E.*

As

As the fine of	F *A E*	55. 0. 0.	9913. 3645
to the face	F E	300.	2477. 1212
			7436. 2433
So the fine of	*A E* F	80. 0. 0.	9993. 3514
to the head-line	*A* F	360. 668	2557. 1081
And the fine of	*A* F *E*	45. 0. 0.	9849. 4850
to the line	*A E*	258. 965	2413. 2417

2 In the triangle *A D E* hauing the three angles and the line *A E,* we may find both the flanque *D E*, and the gorge *A* D

As the fine of	A D E	90. 0. 0.	10000. 0000
to the line	A E	258. 96.	2413. 2417
			7586. 7583
So the fine of	D A E	35. 0. 0.	9758. 5913
to the flanque	D E	148. 53	2171. 8330
And the fine of	A E D	55. 0. 0	9913. 3645
to the gorge	A D	212. 132	2326. 6062

3 In the triangle F A O, hauing the three angles, and the two equall fides A F, A O, we may finde the length of of F O, the face produced vnto the cortin.

As the fine of	A O F	45. 0. 0″	9849. 4850
to the headline	A F	360. 66	2557. 1081
So the whole fine of	F A O	90. 0. 0	10000. 0000
to the face produced	F O	510.	2707. 6231

4 In the triangle F A N, hauing the headline A F, the line of defenfe F N, and the right angle F A N, wee may finde the other two angles at F and N, and the third fide A N.

As

As the line of defence	F N	720	2857, 3325
to the whole sine of	F A N	90. 0. 0	10000.0000
So the headline	A F	360. 66	2557. 1081
to the sine of	A N F	30. 3 $\frac{2}{3}$	9699. 7756

As the sine of	F A N	90. 0. 0	10000.0000
to the line	F N	720.	2857. 3325
So the sine of	A F N	59. 56 $\frac{1}{3}$	9937. 2735
to the line	A N	623. 1697	2794. 6060

Hauing the line A N, if we adde the Gorge N B, or A D, the summe of both shall be the line A B or F L, the distance betweene both Bulwarks.

If we take the Gorge A D out of this line A N, the remainder shall be the Cortin D N.

Againe, if we take the line A O out of this line A N, the remainder shall be O N, that part of the cortin from whence the face of the Bulwark may be defended.

Thus the length of A N being	623.169
the Gorge N *B*, or A D	212. 132
the distance F L or A *B* shall be	835.301
the Cortin D N	41.037
Againe taking the line A O	360. 668
from A N, there remaine O N	262.501

PROP.

PROP. IIII.

Hauing the Angles of an irregular Fort, with the side betweene them, and the face of the Bulwark, to find the rest of the Lines and Angles.

Suppose the angles of an old walled Towne were to bee fortified with new Bulwarks. The angles of the Bulwarke to be either $\frac{2}{3}$. of the angle at the wall, or (if $\frac{2}{3}$. of the angle be more then *90 gr.*) it may suffice, that they be *90 gr.* The Flanques perpendicular to the Cortin, to be formed by an angle betweene 35 and 40 *gr.* as shall be found more conuenient. And the face of each Bulwarke to be 300 foot.

Let the angle at A be 126 *gr.* then may E F G, the angle of the Bulwark be 84 *gr.* and the angle D A E may be allowed to be 38 *gr.* Let the angle at B be 140 *gr.* then because $\frac{2}{3}$. of this angle are aboue 93 *gr.* the angle of this Bulwarke may well be 90 *gr.* and the angle at the Gorge N B M. 36 *gr.* And let A B, the distance betweene these angles be 750 foot.

In regular Forts the Bulwarkes may be made one like the other, so the head-lines being produced will all meet in the same center. In irregular (such as this) there will bee some difference, yet the worke though somewhat longer will bee still the same.

At the Bulwarke A in the triangle A F E, because the angle of the Fort H A D is 126 *gr.* the halfe angle Q A D 63 *gr.* and the angle at the Gorge D A E supposed to be 38 *gr.* the angle E A F will bee 79 *gr.* Againe the angle A F E (the halfe of G F E the angle of the Bulwarke) being 42 *gr.* the angle A E F will be 59 *gr.* by complement.

As the sine of	F A E	79. 0. 0	9991,9465
to the face	F E	300.	2477.1212
			7514.8253
So the sine of	A E F	59. 0. 0	9933.0656
to the head-line	A F	261. 963.	2418. 2403
And the sine of	A F E	42. 0. 0	9825. 5109
to the line	A E	204. 496.	2310. 6856

In the rectangle A D E the angle, at the Gorge D A E being 38 *gr.* the other angle D E A must bee 52 *gr.* by complement.

As the whole sine of	A D E	90. 0. 0.	10000,0000
to the line	A E	204. 496.	2319.6856
			7689.3144
So the sine of	D A E	38. 0. 0.	9789.3419
to the flanque	D E	125.900.	2100.0275
And the sine of	A E D	52. 0. 0	9896.5321
to the Gorge	A D	161. 145.	2207.2177

In like manner at the Bulwarke *B* in the triangle B L M, because the angle of the fort is 140 *gr.* the halfe thereof S B N 70 *gr.* and the angle at the Gorge N B M supposed to be 36 *gr.* the angle M B L will be 74 *gr.* And then the angle B L M (the halfe of the angle of the Bulwarke) being 45 *gr.* the third angle B M L, must be 61 *gr.* by complement.

As the sine of	*M B L*	74. 0'. 0''.	9982.8416
to the face	*M L*	300.	2477.1212
			7505.7204
So the sine of	*B M L*	61. 0. 0.	9941.8192
to the head-line	*B L*	272.960	2436.0988
And the sine of	B L M	45. 0. 0.	9849.4850
to the line	B M	220.1681.	2343.7646

And

And in the rectangle triangle BNM, allowing NBM, the angle at the Gorge to be 36 *gr.* the other angle BMN must be 54 *gr.* by complement.

As the whole sine	BNM	90. 0. 0.	10000.0000
to the line	BM	220. 681	2343.7646
			7656.2354
So the sine of	NBM	36. 0. 0	9769,2186
to the flanque	NM	129. 713	2112,9832
And the sine of	BMN	54. 0. 0	9907,9576
to the Gorge	BN	178. 534	2251,7222

3 In the triangle AFO, taking the angle AFO 42 *gr.* out of the angle QAO 63 *gr.* there remaines 21 *gr.* for the angle AOF.

As the sine of	AOF	21. 0'. 0"	9554,3291
to the headline	AF	261. 963	2418,2403
			7136,0888
So the sine of	AFO	42. 0. 0	9825,5109
to the line	AO	489. 127	2689,4221
And the sine of	FAO	63. 0. 0	9949,8808
to the face produced	FO	651. 316	2813,7920

And so in the like triangle BLP, taking the angle BLP, 45 *gr.* out of the angle SBP 70 *gr.* there remaines 25 *gr.* for the third angle BPL.

As the sine of	BPL	25. 0. 0	9625,9482
to the headline	*BL*	272. 960	2436,0988
			7189,8494
So the sine of	BLP	45. 0. 0	9849,4850
to the line	*BP*	456. 704	2659,6356
And the sine of	*LBP*	110. 0' 0"	9972,9858
to the face produced	LP	606. 927	2783,1364

Thus the length of the side	*A B*	being	750.
the length of the Gorge	*B N*		178, 534
the length of the line	*A N*		571, 466
Take from this the line	*A O*		489, 127
there remaines for the line	*O N*		82, 339
Againe taking the Gorge	*A D*		161, 145
out of the side *A B* there remaines *B D*			588,855
Take from this the line	*B P*		456, 704
there remaines for the line	*D P*		132, 151
Take *A D* out of A N the cortin D N		is	410,321

4 In the triangle *A F N*, hauing two sides *A* F, *A N*, and F *A N* the angle betweene them, we may finde the other two angles at N and F, and the line of defence F N.

As the summe of the sides A F, A N,	833. 429	2920.8684
is to the difference of those sides	309. 503	2490.6536
So the tangent of halfe the summe of the two		430. 2048
opposite angles at F and N	31.30. 0	9787.3193
to the tangent of	12. 49 ¼	9357.1145

the halfe difference betweene those angles.

This halfe difference added to the halfe summe giues

the greater angle	A F N.	44. 19 ¼.	
and subtracted the lesser	A N F	18. 40 ¾.	
As the sine of	A N F	18. 40′ ¾.	9505.5225
to the headline	A F	261.963	3418.2403
			7087.2822
So the sine of	F A N	63. 0. 0	9949.8808
to the line of defence	F N	728. 783	2862.5986
And the sine of	A F N	44. 19 ¼	9844.2725
to the line	A N	571. 465	2756.9903

And in the like triangle *B D L*, hauing two sides B L, B D, and the angle betweene them L B D, we may finde the other two angles at D and L, and the line of defence L D.

As

As the summe of B L and B D	861. 815	2935. 4138
to the difference of these sides	315.895	2499. 5421
So the tangent of halfe the summe of the two		435.8717
opposite angles at L and D,	35. 0. 0.	9845.2267
to the tangent of	14. 23. $\frac{2}{3}$	9409.3550

This halfe difference added to the halfe summe giues

the greater angle	B L D	49. 23' $\frac{2}{3}$.
and subtracted the lesser	B D L	20. 36. $\frac{1}{3}$.

As the sine of	B L D	20. 36 $\frac{1}{3}$	9546.4542
to the headline	B L	272.960	2436.0988
			7110.3544
So the sine of	L B D	70. 0. 0	9972.9858
to the line of defence	L D	728. 838	2862.6314
And the sine of	B L D	49. 23 $\frac{2}{3}$.	9880.3627
to the line	B D	588. 855	2770.0083

PROP. V.

Hauing the Lines and Angles of a regular Fort, to find the content in feet and acres.

The content of a Fort may be taken feuerall wayes: either from within the Rampert, or from within the outside of the ditch, or else we may take in the Out-workes: And those may be of feuerall sorts, such as are here represented, or the like.

If we consider the content within the Rampart, we haue the triangle Q C S, wherein knowing the Perpendicular C R and the Base Q S, we may finde the content of the triangle. And this content multiplyed by the number of the like triangles belonging to the Fort, shall bee the whole content required.

Thus, in the *Pentagonall* Fort before described, where the

Perpendicular CR was found to be in feet 372. 42. and the Base QS 541. 16.

As the solemne number		2.	0301.0300
is to the Base	QS	541. 16	2733.3268
			2432.2968
So the Perpendicular	CR	372. 42	2571.0358
to the content of the triangle		100773.25	5003.3326
Adde (for 5. triangles) the logarithme of		5.	0698.9700
The content in feet comes to		503866.	5702.3026

Then to reduce this *content* into *acres*, we may either divide the number of feet by 43560, (the number of feet contained in an *acre*) or working by Logarithmes, we may subtract this solemne Logarithme 4639.08787.

Thus, from the Logarithme of	503866.25.	5702.3026
subtract the solemne Logari. of	43560.	4639.0878
there remaines the Logarith. of	11.56.	1063.2148

the content in acres contained within the Rampert.

If it be required to finde the content of this *Pentagonall Fort* within the outward side of the Ditch, we haue 10 such triangles as *XCY*, wherein knowing the two sides *CX*, *CY*, and the angle betweene them *XCY*, we may let down a Perpendiculer from the angle at *Y*, vpon the Base *CX*; and then with the Perpendicular and the Base, we may finde the content of the triangle as before.

Thus the side *CX* being 980. 80, the side *CY* 606. 17, and the angle betweene them *XCY*, 36. 0'. 0''

1	As the whole sine of		90. 0. 0	10000,0000
	to the lesser side	*CY*	607. 17	2782,5938
	So the sine of	*XCY*	36. 0. 0	9769,2186
	to the Perpendicular			2551,8124

2 *As* the solemne number		2	0301,0300
to the Base	C X	980, 80	2991,5819
			2690,5515
So the Perpendicular			2551,8124
to the content of the triangle		174728,60	5242,3639
*A*dde (for ten triangles) the Logari, of		10	1000,0000
the content in feet comes to		1747286	6242,3639
*A*gaine subtract the Logarithme of		43560	4639,0878
the content in acres comes to		40, 11	1603,2761

By the same reason resoluing all into triangles, wee may take in the Counterscarp, and the rest of the Out-workes, *A*nd so finde the content, not onely of a Regular Fort, but of any other piece of ground.

FINIS.

The [illegible] Canon.

This Canon hath the vse as Tables of right Sines and Tangents set forth by others, but the practise somewhat more easie. For except the Sines and [illegible] by these Tables [illegible] multiplication, and [illegible] of their division, and to resolve all sphericall triangles without the helpe of Secants or versed *sines*.

It would serve the like of right-lined Triangles [illegible] with the Logarithmes of my old Colleague & worthy friend M. *Henrie Briggs*. For both proceed from the same ground, and so require the same manner of workes; as I often shew in my publique Lectures at *Gresham* College, when I read.

[illegible] of all the Students
of Mathematicall practise,
E. G.

FINIS.

The use of the Canon.

THis CANON hath like vse as Tables of right *Sines* and *Tangents* set forth by others, but the practise somewhat more easie. For keeping their rules, and working by these Tables, you may vse addition instead of their multiplication, and subtraction in stead of their division, and so resolve all sphæricall triangles without the helpe *Secants* or *versed sines.*

If any desire the like of right-lined Triangles, he may ad joyne the *Logarithmes* of my old Collegue & worthy friend M. *Henrie Briggs.* For both proceed from the same ground, and so require the same maner of workes; as I often shew in my publique Lectures at *Gresham* College: where I rest.

Friend to all that are studious of Mathematicall practise,

E. G.

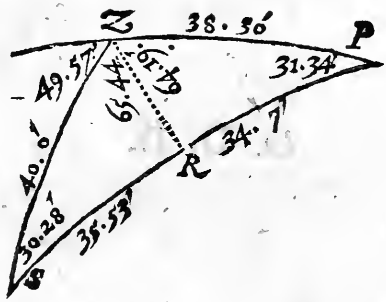
FINIS.

CANON TRIANGVLORVM,

Or Tables of

Artificiall SINES and TANGENTS, to to a *Radius* of 10000,0000 parts, and each minute of the Quadrant.

By EDM. GUNTER Professor of Astonomie in *Gresham Colledge*.



LONDON,
Printed by *William Iones*, for *Iames Bowler*, and are to be sold at the *Marigold* in *Pauls* Church-yard.
1636.

HONORATISSIMO

DOMINO *Dn.* JOHANNI

COMITI de BRIDGEWATER,

VICECOMITI de BRACKLEY,

BARONI de ELLESMERE,

Hunc ſuum Canonem
Triangulorum

D. D. D.

EDM. GUNTER.

The description of the Canon.

THis *Canon* hath six columnes. The first is of degrees and minutes, from the beginning of the Quadrant unto 45 *gr.* the sixt of degrees and minuts, from 45 *gr.* vnto the end of the quadrant; the other foure containe the *Sines* and *Tangents* belonging to each of these degrees and minutes, after the manner of other *Canons*. The difference is in the numbers. For these *Sines* are not such as halfe the chords of the double arke, nor these *Tangents* perpendiculars at the end of the Diameter: but other numbers substituted in their place, for attaining the same end, by a more easie way, such as the *Logarithmes* of the Lord of *Merchiston*, and thereupon I call them *Artificiall Sines* and *Tangents*. So the second and fourth columnes containe the *Sines* and *Tangents* of the degrees and minutes in the first columne; the third and fift containe the *Sines* and *Tangents* of the sixt columne.

As if it were required to finde the artificiall *Sine* belonging to our latitude, which here at *London* is 51 *gr.* 32 *m.* you may find *Sine* 51 in the lower part of the page, and *M.* 32 in the sixt columne, the common angle will giue 9893,7452 for the *Sine* required. And in the same line you haue 9793,8317 for the *Sine* of the complement of this latitude, which in one word may be called the *cosine*. In like maner the *Tangent* of 51 *gr.* 32 *m.* will be found to be 10099,9134, and the *co-tangent* 9900,0865.

The *Secants* (if there were vse of them) may easily be supplied, by taking the *co-sine* out of the double of the *Radius*.

As the double of the *Radius* being	20000,0000
Take hence the *co-sine* of 51 *gr.* 32 *m.*	9793,8317
The *Secant* of 51 *gr.* 32 *m.* will be	10206,1683

The *versed Sines* may also bee supplied by adding 301,0300 vnto the double of the *sine* of halfe the arke, and subtracting the *Radius*. As the halfe of 51 *gr.* 32 *m.* being 25 *gr.* 46 *m.*

Adde to the *Sine* of 25 *gr.* 46 *m.*	9638,1968
The same againe, and the former	9638,1968
number, so the *Radius* being subtracted,	301,0300
the *versed sine* of 51 *gr.* 32 *m.* will be	9577,4236

M	Sin. 0.		Tan. 0.		
0	0	10000,0000	0	*Infinitum.*	60
1	6463,7260	9999,9999	6463,7260	13536,2739	59
2	6764,7560	9999,9999	6764,7561	13235,2438	58
3	6940,8473	9999,9998	6940,8474	13059,1525	57
4	7065,7860	9999,9997	7065,7863	12934,2136	56
5	7162,6959	9999,9995	7162,6964	12837,3035	55
6	7241,8771	9999,9993	7241,8778	12758,1221	54
7	7308,8238	9999,9991	7308,8247	12691,1752	53
8	7366,8157	9999,9988	7366,8169	12633,1831	52
9	7417,9681	9999,9985	7417,9696	12582,0303	51
10	7463,7255	9999,9981	7463,7273	12536,2726	50
11	7505,1180	9999,9977	7505,1202	12494,8797	49
12	7542,9064	9999,9973	7542,9091	12457,0908	48
13	7577,6684	9999,9969	7577,6715	12422,3284	47
14	7609,8529	9999,9964	7609,8565	12390,1434	46
15	7639,8160	9999,9958	7639,8201	12360,1798	45
16	7667,8445	9999,9953	7667,8492	12332,1507	44
17	7694,1732	9999,9947	7694,1785	12305,8214	43
18	7718,9966	9999,9940	7718,0026	12281,9974	42
19	7742,4775	9999,9933	7742,4841	12257,5158	41
20	7764,7536	9999,9926	7764,7610	12235,2389	40
21	7785,9427	9999,9919	7785,9508	12214,0491	39
22	7806,1458	9999,9911	7806,1547	12193,8452	38
23	7825,4507	9999,9902	7825,4604	12174,5395	37
24	7843,9338	9999,9894	7843,9444	12156,0555	36
25	7861,6623	9999,9885	7861,6738	12138,3262	35
26	7878,6953	9999,9875	7878,7077	12121,2922	34
27	7895,0854	9999,9866	7895,0988	12104,9012	33
28	7910,8793	9999,9856	7910,8937	12089,1062	32
29	7926,1189	9999,9845	7926,1344	12073,8656	31
30	7940,8418	9999,9834	7940,8584	12059,1416	30
		Sin. 89.		*Tan.* 89.	M

M	Sin. 0.		Tan. 0.		
30	7940,8418	9999,9834	7940,8584	12059,1416	30
31	7955,0819	9999,9823	7955,0996	12044,9004	29
32	7968,8690	9999,9812	7968,8886	12031,1113	28
33	7982,2333	9999,9800	7982,2534	12017,7466	27
34	7995,1979	9999,9787	7995,2192	12004,7808	26
35	8007,7866	9999,9774	8007,8091	11992,1908	25
36	8020,0206	9999,9761	8020,0445	11979,9555	24
37	8031,9194	9999,9748	8031,9446	11968,0553	23
38	8043,5008	9999,9734	8043,5274	11956,4726	22
39	8054,7814	9999,9720	8054,8193	11945,1806	21
40	8065,7763	9999,9706	8065,8057	11934,1942	20
41	8076,4996	9999,9691	8076,5305	11923,4694	19
42	8086,9646	9999,9675	8086,9970	11913,0029	18
43	8097,1832	9999,9660	8097,2172	11902,7827	17
44	8107,1669	9999,9644	8107,2025	11892,7975	16
45	8116,9262	9999,9628	8116,9634	11883,0365	15
46	8126,4709	9999,9611	8126,5098	11873,4901	14
47	8135,8104	9999,9594	8135,8510	11864,1489	13
48	8144,9532	9999,9576	8144,9955	11855,0044	12
49	8153,9075	9999,9558	8153,9516	11846,0483	11
50	8162,6808	9999,9540	8162,7367	11837,2632	10
51	8171,2803	9999,9522	8171,3281	11828,6718	9
52	8179,7129	9999,9503	8179,7626	11820,2374	8
53	8187,9847	9999,9484	8188,0363	11811,9636	7
54	8196,1020	9999,9464	8196,1555	11803,8444	6
55	8204,0702	9999,9444	8204,1258	11795,8741	5
56	8211,8949	9999,9423	8211,9525	11788,0474	4
57	8219,5810	9999,9403	8219,6407	11780,3592	3
58	8227,1335	9999,9382	8227,1953	11772,8046	2
59	8234,5568	9999,9360	8234,6207	11765,3792	1
60	8241,8553	9999,9338	8241,9214	11758,0785	0
		Sin. 89.		Tan. 89.	M

M	Sin. 1.		Tan. 1.		
0	8241,8553	9999,9338	8241,9214	11758,0785	60
1	8249,0331	9999,9316	8249,1015	11750,8984	59
2	8256,0942	9999,9293	8256,1649	11743,8351	58
3	8263,0423	9999,9270	8263,1152	11736,8847	57
4	8269,8810	9999,9247	8269,9562	11730,0437	56
5	8279,6136	9999,9223	8276,6912	11723,3087	55
6	8283,2433	9999,9199	8283,3234	11716,6765	54
7	8289,7734	9999,9175	8289,8559	11710,1440	53
8	8296,2067	9999,9150	8296,2916	11703,7083	52
9	8302,5460	9999,9125	8302,6335	11697,3664	51
10	8308,7941	9999,9099	8308,8842	11691,1158	50
11	8314,9535	9999,9073	8315,0462	11684,9537	49
12	8321,0268	9999,9047	8321,1221	11678,8778	48
13	8327,0163	9999,9020	8327,1142	11672,8857	47
14	8332,9243	9999,8993	8333,0249	11666,9750	46
15	8338,7529	9999,8966	8338,8563	11661,1437	45
16	8344,5043	9999,8938	8344,6104	11655,3895	44
17	8350,1805	9999,8910	8350,2894	11649,7105	43
18	8355,7834	9999,8882	8355,8952	11644,1047	42
19	8361,3149	9999,8853	8361,4296	11638,5703	41
20	8366,7769	9999,8823	8366,8945	11633,1054	40
21	8372,1709	9999,8794	8372,2915	11627,7084	39
22	8377,4988	9999,8764	8377,6223	11622,3776	38
23	8382,7620	9999,8734	8382,8886	11617,1113	37
24	8387,9621	9999,8703	8388,0918	11611,9081	36
25	8393,1007	9999,8672	8393,2335	11606,7664	35
26	8398,1792	9999,8641	8398,3151	11601,6848	34
27	8403,1990	9999,8609	8403,3381	11596,6619	33
28	8408,1613	9999,8576	8408,3036	11591,6963	32
29	8413,0676	9999,8544	8413,2131	11586,7868	31
30	8417,9190	9999,8511	8418,0678	11581,9321	30
		Sin. 88.		Tan. 88.	M

M	Sin. 1.		Tan. 1.		
30	8417,9190	9999,8511	8418,0678	11581,9321	30
31	8422,7168	9999,8478	8422,8689	11577,1310	29
32	8427,4621	9999,8444	8427,6176	11572,3823	28
33	8432,1561	9999,8410	8432,3150	11567,6849	27
34	8436,7998	9999,8376	8436,9622	11563,0377	26
35	8441,3944	9999,8341	8441,5603	11558,4397	25
36	8445,9409	9999,8306	8446,1103	11553,8897	24
37	8450,4402	9999,8271	8450,6131	11549,3868	23
38	8454,8933	9999,8235	8455,0698	11544,9301	22
39	8459,3012	9999,8199	8459,4814	11540,5186	21
40	8463,6648	9999,8162	8463,8486	11536,1513	20
41	8467,9850	9999,8125	8468,1724	11531,8275	19
42	8472,2625	9999,8088	8472,4537	11527,5462	18
43	8476,4983	9999,8050	8476,6933	11523,3066	17
44	8480,6932	9999,8012	8480,8919	11519,1080	16
45	8484,8478	9999,7974	8485,0505	11514,9495	15
46	8488,9631	9999,7935	8489,1696	11510,8303	14
47	8493,0397	9999,7896	8493,2501	11506,7498	13
48	8497,0784	9999,7856	8497,2927	11502,7072	12
49	8501,0798	9999,7816	8501,2981	11498,7018	11
50	8505,0446	9999,7776	8505,2670	11494,7329	10
51	8508,9736	9999,7735	8509,2000	11490,7999	9
52	8512,8673	9999,7694	8513,0978	11486,9021	8
53	8516,7263	9999,7653	8516,9610	11483,0389	7
54	8520,5513	9999,7611	8520,7902	11479,2098	6
55	8524,3429	9999,7569	8524,5860	11475,4140	5
56	8528,1016	9999,7527	8528,3489	11471,6510	4
57	8531,8281	9999,7484	8532,0797	11467,9203	3
58	8535,5228	9999,7441	8535,7787	11464,2212	2
59	8539,1863	9999,7397	8539,4466	11460,5534	1
60	8542,8191	9999,7353	8543,0838	11456,9162	0
M		Sin. 88.		Tan. 88.	M

M	Sin. 2.		Tan. 2.		
0	8542,8191	9999,7353	8543,0838	11456,9162	60
1	8546,4217	9999,7309	8546,6908	11453,3091	59
2	8549,9947	9999,7264	8550,2683	11449,7317	58
3	8553,5385	9999,7219	8553,8166	11446,1834	57
4	8557,0536	9999,7174	8557,3362	11442,6637	56
5	8560,5404	9999,7128	8560,8276	11439,1724	55
6	8563,9994	9999,7082	8564,2912	11435,7088	54
7	8567,4310	9999,7035	8567,7274	11432,2725	53
8	8570,8357	9999,6988	8571,1368	11428,8631	52
9	8574,2139	9999,6941	8574,5197	11425,4802	51
10	8577,5659	9999,6894	8577,8765	11422,1234	50
11	8580,8923	9999,6846	8581,2076	11418,7923	49
12	8584,1933	9999,6797	8584,5135	11415,4864	48
13	8587,4694	9999,6749	8587,7945	11412,2054	47
14	8590,7209	9999,6700	8591,0509	11408,9490	46
15	8593,9482	9999,6650	8594,2832	11405,7167	45
16	8597,1517	9999,6600	8597,4916	11402,5083	44
17	8600,3317	9999,6550	8600,6766	11399,3233	43
18	8603,4885	9999,6499	8603,8385	11396,1614	42
19	8606,6225	9999,6449	8606,9776	11393,0223	41
20	8609,7341	9999,6397	8610,0943	11389,9056	40
21	8612,8234	9999,6346	8613,1888	11386,8111	39
22	8615,8909	9999,6294	8616,2615	11383,7384	38
23	8618,9369	9999,6241	8619,3127	11380,6872	37
24	8621,9616	9999,6188	8622,3427	11377,6572	36
25	8624,9653	9999,6135	8625,3517	11374,6482	35
26	8627,9484	9999,6082	8628,3402	11371,6598	34
27	8630,9111	9999,6028	8631,3082	11368,6917	33
28	8633,8536	9999,5974	8634,2562	11365,7437	32
29	8636,7764	9999,5919	8637,1844	11362,8155	31
30	8639,6795	9999,5864	8640,0931	11359,9068	30
		Sin. 87.		Tang. 87	M

M	*Sin.* 2.		*Tan.* 2.		
30	8639,6795	9999,5864	8640,0931	11359,9068	30
31	8642,5634	9999,5809	8642,9825	11357,0175	29
32	8645,4282	9999,5753	8645,8528	11354,1471	28
33	8648,2741	9999,5697	8648,7044	11351,2955	27
34	8651,1015	9999,5640	8651,5375	11348,4625	26
35	8653,9106	9999,5584	8654,3522	11345,6477	25
36	8656,7016	9999,5527	8657,1489	11342,8510	24
37	8659,4748	9999,5469	8659,9278	11340,0721	23
38	8662,2303	9999,5411	8662,6891	11337,3108	22
39	8664,9684	9999,5353	8665,4330	11334,5669	21
40	8667,6893	9999,5294	8668,1598	11331,8401	20
41	8670,3932	9999,5235	8670,8696	11329,1303	19
42	8673,0803	9999,5176	8673,5627	11326,4372	18
43	8675,7510	9999,5116	8676,2393	11323,7606	17
44	8678,4052	9999,5056	8678,8996	11321,1003	16
45	8681,0433	9999,4995	8681,5437	11318,4562	15
46	8683,6654	9999,4934	8684,1719	11315,8280	14
47	8686,2717	9999,4873	8686,7844	11313,2155	13
48	8688,8625	9999,4812	8689,3813	11310,6186	12
49	8691,4378	9999,4750	8691,9628	11308,0371	11
50	8693,9980	9999,4687	8694,5292	11305,4707	10
51	8696,5431	9999,4625	8697,0806	11302,9193	9
52	8699,0733	9999,4561	8699,6171	11300,3828	8
53	8701,5889	9999,4498	8702,1390	11297,8609	7
54	8704,0899	9999,4434	8704,6464	11295,3535	6
55	8706,5765	9999,4370	8707,1395	11292,8604	5
56	8709,0490	9999,4306	8709,6184	11290,3815	4
57	8711,5074	9999,4241	8712,0833	11287,9166	3
58	8713,9520	9999,4175	8714,5345	11285,4655	2
59	8716,3829	9999,4110	8716,9719	11283.0281	1
60	8718,8001	9999,4044	8719,3957	11280,6042	0
		Sin. 87.		*Tan.* 87.	M

M	Sin. 3.		Tan. 3.		
0	8718,8001	9999,4044	8719,3957	11280,6042	60
1	8721,2040	9999,3977	8721,8062	11278,1937	59
2	8723,5946	9999,3910	8724,2035	11275,7964	58
3	8725,9720	9999,3843	8726,5877	11273,4123	57
4	8728,3365	9999,3776	8728,9589	11271,0410	56
5	8730,6882	9999,3708	8731,3173	11268,6826	55
6	8733,0271	9999,3640	8733,6631	11266,3368	54
7	8735,3535	9999,3571	8735,9964	11264,0036	53
8	8737,6674	9999,3502	8738,3172	11261,6827	52
9	8739,9691	9999,3433	8740,6258	11259,3742	51
10	8742,2586	9999,3363	8742,9222	11257,0777	50
11	8744,5360	9999,3293	8745,2066	11254,7933	49
12	8746,8015	9999,3223	8747,4792	11252,5207	48
13	8749,0552	9999,3152	8749,7400	11250,2599	47
14	8751,2973	9999,3081	8751,9892	11248,0107	46
15	8753,5278	9999,3009	8754,2268	11245,7731	45
16	8755,7468	9999,2937	8756,4531	11243,5468	44
17	8757,6540	9999,2865	8758,6681	11241,3319	43
18	8760,1511	9999,2792	8760,8719	11239,1280	42
19	8762,3366	9999,2719	8763,0646	11236,9353	41
20	8764,5111	9999,2646	8765,2464	11234,7535	40
21	8766,6747	9999,2572	8767,4174	11232,5825	39
22	8768,8275	9999,2498	8769,5777	11230,4222	38
23	8770,9697	9999,2423	8771,7273	11228,2726	37
24	8773,1013	9999,2349	8773,8664	11226,1335	36
25	8775,2225	9999,2273	8775,9952	11224,0048	35
26	8777,3334	9999,2198	8778,1135	11221,8864	34
27	8779,4340	9999,2122	8780,2217	11219,7782	33
28	8781,5244	9999,2045	8782,3198	11217,6801	32
29	8783,6048	9999,1969	8784,4079	11215,5920	31
30	8785,6752	9999,1892	8786,4860	11213,5139	30
		Sin.86.		Tan.86.	M

M	Sin. 3.		Tan. 3.		M
30	8785,6752	9999,1892	8786,4860	11213,5139	30
31	8787,7358	9999,1814	8788,5544	11211,4455	29
32	8789,7866	9999,1736	8790,6130	11209,3869	28
33	8791,8278	9999,1658	8792,6619	11207,3380	27
34	8793,8593	9999,1580	8794,7013	11205,2986	26
35	8795,8814	9999,1501	8796,7313	11203,2686	25
36	8797,8940	9999,1421	1798,7519	11201,2480	24
37	8799,8974	9999,1342	8800,7632	11199,2368	23
38	8801,8915	9999,1262	8802,7653	11197,2347	22
39	8803,8764	9999,1181	8804,7582	11195,2417	21
40	8805,8523	9999,1100	8806,7422	11193,2577	20
41	8807,8192	9999,1019	8808,7172	11191,2827	19
42	8809,7772	9999,0938	8810,6834	11189,3166	18
43	8811,7263	9999,0856	8812,6407	11187,3592	17
44	8813,6667	9999,8774	8814,5893	11185,4106	16
45	8815,5985	9999,0691	8816,5293	11183,4706	15
46	8817,5216	9999,0608	8818,4608	11181,5391	14
47	8819,4363	9999,0525	8820,3838	11179,6161	13
48	8821,3425	9999,0441	8822,2984	11177,7016	12
49	8823,2403	9999,0357	8824,2046	11175,7953	11
50	8825,1299	9999,0272	8826,1026	11173,8973	10
51	8827 0112	9999,0188	8827,9924	11172,0075	9
52	8828,8843	9999,0102	8829,8741	11170,1258	8
53	8830,7494	9999,0017	8831,7477	11168,2522	7
54	8832,6065	9998,9931	8833,6134	11166,3865	6
55	8834,4557	9998,9844	8835,4712	11164,5287	5
56	8836,2969	9998 9758	8837,3211	11162,6788	4
57	8838,1304	9998 9671	8839,1632	11160,8367	3
58	8839,9560	9998,9583	8840,9977	11159,0022	2
59	8841,7741	9998,9496	8842,8245	11157,1754	1
60	8843,5845	9998,9407	8844,6437	11155,3562	0
		Sin. 86.		Tan. 86.	M

M	Sin. 4.		Tan. 4.		
0	8843,5845	9998,9408	8844,6437	11155,3562	60
1	8845,3873	9998,9319	8846,4554	11153,5445	59
2	8847,1827	9998,9230	8848,2597	11151,7403	58
3	8848,9706	9998,9141	8850,0565	11149,9434	57
4	8850,7512	9998,9051	8851,8460	11148,1539	56
5	8852,5245	9998,8961	8853,6283	11146,3716	55
6	8854,2905	9998,8871	8855,4034	11144,5966	54
7	8856,0493	9998,8780	8857,1713	11142,8286	53
8	8857,8010	9998,8689	8858,9321	11141,0678	52
9	8859,5456	9998,8597	8860,6858	11139,3141	51
10	8861,2832	9998 8506	8862,4326	11137,5673	50
11	8863,0139	9998,8413	8864,1725	11135,8274	49
12	8864,7376	9998,8321	8865,9055	11134,0944	48
13	8866,4545	9998,8228	8867,6317	11132,3682	47
14	8868,1646	9998,8135	8869,3511	11130,6488	46
15	8869,8679	9998,8041	8871,0638	11128 9361	45
16	8871,5646	9998,7947	8872,7699	11127,2300	44
17	8873,2546	9998,7852	8874,4693	11125,5306	43
18	8874,9380	9998,7758	8876,1622	11123,8377	42
19	8876,6149	9998,7662	8877,8487	11122,1513	41
20	8878,2853	9998,7567	8879,5286	11120,4713	40
21	8879,9493	9998,7471	8881,2022	11118,7978	39
22	8881,6069	9998,7375	8882,8694	11117,1305	38
23	8883,2581	9998,7278	8884,5303	11115,4696	37
24	8884,9031	9998,7181	8886,1849	11113,8150	36
25	8886,5418	9998,7083	8887,8334	11112,1665	35
26	8888,1743	9998,6986	8889,4756	11110,5243	34
27	8889,8006	9998,6888	8891,1118	11108,8881	33
28	8891,4209	9998,6789	8892,7420	11107,2580	32
29	8893,0351	9998,6690	8894,3660	11105,6339	31
30	8894,6433	9998,6591	8895,9841	11104,0158	30
		Sin. 85.		Tan. 85.	M

M	Sin. 4.		Tan. 4.		
30	8894,6433	9998,6591	8895,9841	11104,0158	30
31	8896,2455	9998,6492	8897,5963	11102,4036	29
32	8897,8417	9998,6391	8899,2026	11100,7973	28
33	8899,4322	9998,6291	8900,8030	11099,1969	27
34	8901,0167	9998,6190	8902,3977	11097,6022	26
35	8902,5955	9998,6089	8903,9866	11096,013[illegible]	25
36	8904,1685	9998,5988	8905,5697	11094,4302	24
37	8905,7358	9998,5886	8907,1472	11092,8527	23
38	8907,2974	9998,5784	8908,7190	11091,2809	22
39	8908,8534	9998,5681	8910,2853	11089,7146	21
40	8910,4028	9998,5578	8911,8460	11088,1539	20
41	8911,9487	9998,5475	8913,4012	11086,5988	19
42	8913,4880	9998,5371	8914,9508	11085,0491	18
43	8915,0219	9998,5267	8916,4951	11083,5048	17
44	8916,5503	9998,5163	8918,0340	11081,9659	16
45	8918,0733	9998,5058	8919,5675	11080,4324	15
46	8919,5910	9998,4953	8921,0957	11078,9042	14
47	8921,1034	9998,4847	8922,6186	11077,3813	13
48	8922,6104	9998,4742	8924,1362	11075,8637	12
49	8924,1122	9998,4635	8925,6487	11074,3512	11
50	8925,6089	9998,4528	8927,1561	11072,8439	10
51	8927,1003	9998,4422	8928,6581	11071,3418	9
52	8928,5866	9998,4314	8930,1551	11069,8448	8
53	8930,0678	9998,4206	8931,6471	11068,3528	7
54	8931,5439	9998,4098	8933,1340	11066,8659	6
55	8933,0150	9998,3990	8934,6160	11065,3840	5
56	8934,4810	9998,3881	8936,0929	11063,9070	4
57	8935,9421	9998,3772	8937,5649	11062,4350	3
58	8937,3983	9998,3662	8939,0321	11060,9678	2
59	8938,8496	9998,3552	8940,4943	11059,5056	1
60	8940,2960	9998,3442	8941,9517	11058,0482	0
		Sin. 85.		Tan. 85.	M

M	Sin. 5.		Tan. 5.		
0	8940,2960	9998,3442	8941,9517	11058,0482	60
1	8941,7375	9998,3331	8943,4044	11056,5955	59
2	8943,1742	9998,3220	8944,8522	11055,1477	58
3	8944,6063	9998,3109	8946,2954	11053,7046	57
4	8946,0335	9998,2997	8947,7338	11052,2661	56
5	8947,4560	9998,2885	8949,1675	11050,8324	55
6	8948,8739	9998,2772	8950,5966	11049,4033	54
7	8950,2871	9998,2659	8952,0211	11047,9788	53
8	8951,6956	9998,2546	8953,4410	11046,5589	52
9	8953,0996	99[illegible]432	8954,8564	11045,1436	51
10	8954,4990	9998,2318	8956,2672	11043,7327	50
11	8955,8939	9998,2204	8957,6735	11042,3264	49
12	8957,2843	9998,2089	8959,0754	11040,9245	48
13	8958,6702	9998,1974	8960,4728	11039,5271	47
14	8960,0517	9998,1858	8961,8658	11038,1341	46
15	8961,4287	9998,1742	8963,2544	11036,7455	45
16	8962,8013	9998,1626	8964,6387	11035,3612	44
17	8964,1696	9998,1509	8966,0187	11033,9812	43
18	8965,5337	9998,1392	8967,3944	11032,6055	42
19	8966,8934	9998,1275	8968,7658	11031,2341	41
20	8968,2488	9998,1157	8970,1330	11029,8669	40
21	8969,5998	9998,1039	8971,4949	11028,5050	39
22	8970,9467	9998,0921	8972,8546	11027,1453	38
23	8972,2894	9998,0802	8974,2092	11025,7907	37
24	8973,6280	9998,0683	8975,5597	11024,4402	36
25	8974,9624	9998,0563	8976,9060	11023,0939	35
26	8976,2926	9998,0443	8978,2483	11021,7516	34
27	8977,6187	9998,0323	8979,5864	11020,4135	33
28	8978,9408	9998,0202	8980,9206	11019,0793	32
29	8980,2588	9998,0081	8982,2507	11017,7492	31
30	8981,5728	9997,9959	8983,5769	11016,4230	30
		Sin. 84.		Tan. 84.	M

M	*Sin. 5.*		*Tan. 5.*		
30	8981,5728	9997,9959	8983,5769	11016,4230	30
31	8982,8829	9997,9838	8984,8991	11015,1008	29
32	8984,1889	9997,9715	8986,2173	11013,7826	28
33	8985,4909	9997,9593	8987,5316	11012,4683	27
34	8986,7890	9997,9470	8988,8420	11011,1579	26
35	8988,0833	9997,9347	8990,1486	11009,8513	25
36	8989,3737	9997,9223	8991,4513	11008,5486	24
37	8990,6602	9997,9099	8992,7503	11007,2496	23
38	8991,9429	9997,8974	8994,0454	11005,9545	22
39	8993,2217	9997,8850	8995,3367	11004,6632	21
40	8994,4957	9997,8725	8996,6243	11003,3757	20
41	8995,7680	9997,8599	8997,9081	11002,0918	19
42	8997,0356	9997,8473	8999,1883	11000,8117	18
43	8998,2994	9997,8347	9000,4648	10999,5352	17
44	8999,5595	9997,8220	9001,7375	10998,2624	16
45	9000,8159	9997,8093	9003,0066	10996,9933	15
46	9002,0687	9997,7965	9004,2721	10995,7278	14
47	9003,3178	9997,7838	9005,5340	10994,4659	13
48	9004,5633	9997,7710	9006,7923	10993,2076	12
49	9005,8053	9997,7581	6008,0472	10991,9528	11
50	9007,0436	9997,7452	9009,2984	10990,7016	10
51	9008,2784	9997,7323	9010,5461	10989,4539	9
52	9009,5096	9997,7193	9011,7902	10988,2097	8
53	9010,7373	9997,7063	9013,0310	10986,9690	7
54	9011,9615	9997,6933	9014,2682	10985,7317	6
55	9013,1823	9997,6802	9015,5021	10984,4979	5
56	9014,3996	9997,6671	9016,7325	10983,2675	4
57	9015,6134	9997,6540	9017,9594	10982,0405	3
58	9016,8238	9997,6408	9019,1830	10980,8169	2
59	9018,0309	9997,6276	9020,4033	10979,5967	1
60	9019,2345	9997,6143	9021,6202	10978,3797	0
		Sin. 84.		*Tan. 84.*	M

M	Sin. 6.		Tan. 6.		
0	9019,2345	9997,6143	9021,6202	10978,3797	60
1	9020,4348	9997,6010	9022,8338	10977,1662	59
2	9021,6317	9997,5877	9024,0440	10975,9559	58
3	9022,8254	9997,5743	9025,2510	10974,7489	57
4	9024,0157	9997,5609	9026,4548	10973,5452	56
5	9025,2027	9997,5475	9027,6552	10972,3447	55
6	9026,3864	9997,5340	9028,8524	10971,1475	54
7	9027,5669	9997,5204	9030,0464	10969,9535	53
8	9028,7441	9997,5069	9031,2372	10968,7627	52
9	9029,9182	9997,4933	9032,4249	10967,5751	51
10	9031,0890	9997,4797	9033,6094	10966,3906	50
11	9032,2567	9997,4660	9034,7906	10965,2093	49
12	9033,4211	9997,4523	9035,9688	10964,0311	48
13	9034,5824	9997,4386	9037,1439	10962,8561	47
14	9035,7406	9997,4248	9038,3158	10961,6841	46
15	9036,8957	9997,4110	9039,4848	10960,5152	45
16	9038,0477	9997,3971	9040,6506	10959,3493	44
17	9039,1966	9997,3832	9041,8134	10958,1866	43
18	9040,3424	9997,3693	9042,9731	10957,0268	42
19	9041,4852	9997,3553	9044,1298	10955,8701	41
20	9042,6249	9997,3413	9045,2836	10954,7164	40
21	9043,7616	9997,3273	9046,4343	10953,5656	39
22	9044,8954	9997,3132	9047,5821	10952,4178	38
23	9046,0261	9997,2991	9048,7270	10951,2730	37
24	9047,1538	9997,2849	9049,8689	10950,1311	36
25	9048,2786	9997,2707	9051,0078	10948,9921	35
26	9049,4004	9997,2565	9052,1439	10947,8560	34
27	9050,5194	9997,2423	9053,2771	10946,7228	33
28	9051,6354	9997,2279	9054,4075	10945,5925	32
29	9052,7485	9997,2136	9055,5349	10944,4651	31
30	9053,8587	9997,1992	9056,6595	10943,3405	30
		Sin. 83.		Tan. 83.	M

M	Sin. 6.		Tan. 6.		
30	9053,8587	9997,1992	9056,6594	10943,3405	30
31	9054,9661	9997,1848	9057,7812	10942,2187	29
32	9056,0706	9997,1704	9058,9002	10941,0998	28
33	9057,1723	9997,1559	9060,0164	10939,9836	27
34	9058,2711	9997,1414	9061,1297	10938,8702	26
35	9059,3672	9997,1268	9062,2404	10937,7596	25
36	9060,4604	9997,1122	9063,3482	10936,6518	24
37	9061,5508	9997,0976	9064,4532	10935,5467	23
38	9062,6385	9997,0829	9065,5556	10934,4444	22
39	9063,7235	9997,0682	9066,6553	10933,3447	21
40	9064,8057	9997,0534	9067,7522	10932,2477	20
41	9065,8852	9997,0387	9068,8465	10931,1534	19
42	9066,9619	9997,0238	9069,9381	10930,0619	18
43	9068,0359	9997,0090	9071,0269	10928,9730	17
44	9069,1073	9996,9941	9072,1132	10927,8867	16
45	9070,1760	9996,9791	9073,1969	10926,8031	15
46	9071,2421	9996,9642	9074,2779	10925,7220	14
47	9072,3055	9996,9492	9075,3563	10924,6436	13
48	9073,3662	9996,9341	9076,4321	10923,5679	12
49	9074,4243	9996,9191	9077,5053	10922,4947	11
50	9075,4799	9996,9039	9078,5759	10921,4240	10
51	9076,5328	9996,8888	9079,6440	10920,3559	9
52	9077,5832	9996,8736	9080,7096	10919,2903	8
53	9078,6310	9996,8583	9081,7726	10918,2273	7
54	9079,6762	9996,8431	9082,8331	10917,1669	6
55	9080,7188	9996,8278	9083,8910	10916,1089	5
56	9081,7590	9996,8124	9084,9466	10915,0534	4
57	9082,7966	9996,7970	9085,9995	10914,0004	3
58	9083,8317	9996,7816	9087,0500	10912,9499	2
59	9084,8643	9996,7662	9088,0981	10911,9018	1
60	9085,8944	9996,7507	9089,1437	10910,8562	0
		Sin. 85.		Tan. 83.	M

M	Sin. 7.		Tan. 7.		
0	9085,8944	9996,7507	9089,1437	10910,8562	60
1	9086,9221	9996,7351	9090,1869	10909,8130	59
2	9087,9473	9966,7196	9091,2277	10908,7723	58
3	9088,9700	9996,7040	9092,2660	10907,7339	57
4	9089,9903	9996,6883	9093,3020	10906,6980	56
5	9091,0082	9996,6727	9094,3355	10905,6644	55
6	9092,0236	9996,6569	9095,3667	10904,6333	54
7	9093,0367	9996,6412	9096,3955	10903,9045	53
8	6094,0473	9996,6254	9097,4219	10902,5780	52
9	9095,0556	9996,6096	9098,4460	10901,5539	51
10	9096,0615	9996,5937	9099 4678	10900,5322	50
11	9097,0650	9996,5778	9100,4872	10899,5127	49
12	9098,0662	9996,5619	9101,5043	10898,4956	48
13	9099,0651	9996,5459	9102,5192	10897,4808	47
14	9100,0616	9996,5299	9103,5317	10896,4682	46
15	9101,0558	9996,5138	9104,5420	10895,4580	45
16	9102,0477	9996,4977	9105,5500	10894,4500	44
17	9103,0373	9996,4816	9106,5557	10893,4443	43
18	9104,0246	9996,4654	9107,5591	10892,4408	42
19	9105,0096	9996,4492	9108,5604	10891,4395	41
20	9105,9924	9996,4330	9109,5594	10890 4405	40
21	9106,9729	9996 4167	9110,5562	10889,4438	39
22	9107,9511	9996,4004	9111,5507	10888,4492	38
23	9108,9272	9996,3840	9112,5431	10887,4568	37
24	9109,9010	9996,3677	9113,5333	10886,4666	36
25	9110,8726	9996,3512	9114,5214	10885 4786	35
26	9111,8420	9996,3348	9115,5072	10884,4928	34
27	9112,8091	9996,3183	9116,4908	10883,5091	33
28	9113,7741	9995,3017	9117,4724	10882,5275	32
29	9114,7370	9995,2851	9118,4518	10881,5481	31
30	9115,6976	9996,2685	9119,4291	10880,5709	30
		Sin. 82.		Tan. 82.	M

M	Sin. 7.		Tan. 7.		
30	9115,6976	9996,2685	9119,4291	10880,5709	30
31	9116,6561	9996,2519	9120,4042	10879,5957	29
32	9117,6125	9996,2352	9121,3773	10878,6227	28
33	9118,5667	9996,2185	9122,3482	10877,6517	27
34	9119,5188	9996,2017	9123,3171	10876,6829	26
35	9120,4688	9996,1849	9124,2838	10875,7161	25
36	9121,4166	9996,1681	9125,2485	10874,7514	24
37	9122,3624	9996,1512	9126,2112	10873,7888	23
38	9123,3061	9996,1343	9127,1717	10872,8282	22
39	6124,2476	9996,1173	9128,1303	10871,8696	21
40	9125,1871	9996,1003	9129,0868	10870,9131	20
41	9126,1246	9996,0833	9130,0412	10869,9587	19
42	9127,0600	9996,0663	9130,9937	10869,0062	18
43	9127,9933	9996,0492	9131,9441	10868,0558	17
44	6128,9246	9996,0320	9132,8926	10867,1073	16
45	9129,8539	9996,0148	9133,8390	10866,1609	15
46	9130,7812	9995,9976	9134,7835	10865,2164	14
47	9131,7064	9995,9804	9135,7260	10864,2739	13
48	9132,6296	9995,9631	9136,6665	10863,3334	12
49	9133,5509	9965,9458	9137,6051	10862,3948	11
50	9134,4702	9995,9284	9138,5417	10861,4582	10
51	9135,3874	9995,9110	9139,4764	10860,5235	9
52	9136,3027	9995,8936	9140,4091	10859,5908	8
53	9137,2161	9995,8761	9141,3399	10858,6600	7
54	9138,1275	9995,8586	9142,2688	10857,7311	6
55	9139,0369	9995,8410	9143,1959	10856,8040	5
56	9139,9445	9995,8235	9144,1210	10855,8790	4
57	9140,8500	9995,8058	9145,0441	10854,9558	3
58	9141,7537	9995,7882	9145,9654	10854,0345	2
59	9142,6554	9995,7705	9146,8849	10853,1150	1
60	9143,5553	9995,7527	9047,8025	10852,1974	0
[illegible]	[illegible]	Sin. 82.		Tan. 82.	M

M	*Sin.* 8.		*Tan.* 8.		
0	9143,5553	9995,7527	9147,8025	10852,1974	60
1	9144,4532	9995,7350	9148,7182	10851,2817	59
2	9145,3493	9995,7172	9149,6321	10850,3679	58
3	9146,2434	9995,6993	9150,5441	10849,4558	57
4	9147,1358	9995,6814	9151,4543	10848,5456	56
5	9148,0262	9995,6635	9152,3627	10847,6373	55
6	9148,9148	9995,6455	9153,2692	10846,7307	54
7	9149,8015	9995,6275	9154,1739	10845,8260	53
8	9150,6863	9995,6095	9155,0768	10844,9231	52
9	9151,5694	9995,5914	9155,9779	10844,0220	51
10	9152,4506	9995,5733	9156,8773	10843,1227	50
11	9153,3300	9995,5552	9157,7748	10842,2251	49
12	9154,2076	9995,5370	9158,6706	10841,3293	48
13	9155,0834	9995,5188	9159,5646	10840,4353	47
14	9155,9574	9995,5005	9160,4568	10839,5431	46
15	9156,8295	9995,4822	9161,3473	10838,6526	45
16	9157,6999	9995,4639	9162,2361	10837,7639	44
17	9158,5686	9995,4455	9163,1230	10836,8769	43
18	9159,4354	9995,4271	9164,0083	10835,9916	42
19	9160,3005	9995,4086	9164,8918	10835,1081	41
20	9161,1638	9995,3901	9165,7732	10834,2263	40
21	9162,0254	9995,3716	9166,6537	10833,3462	39
22	9162,8852	9995,3531	9167,5321	10832,4678	38
23	9163,7433	9995,3345	9168,4088	10831,5911	37
24	9164,5997	9995,3158	9169,2839	10830,7161	36
25	9165,4544	9995,2972	9170,1572	10829,8427	35
26	9166,3073	9995,2784	9171,0288	10828,9711	34
27	9167,1585	9995,2597	9171,8988	10828,1011	33
28	9168,0081	9995,2409	9172,7671	10827,2328	32
29	9168,8559	9995,2221	9173,6338	10826,3661	31
30	9169,7020	9995,2032	9174,4988	10825,5011	30
		Sin. 81		*Tan.* 81.	M

M	Sin. 8.		Tan. 8.		
30	9169,7020	9995,2032	9174,4988	10825,5011	30
31	9170,5465	9995,1843	9175,3622	10824,6377	29
32	9171,3893	9995,1654	9176,2239	10823,7760	28
33	9172,2304	9995,1464	9177,0840	10822,9159	27
34	9173,0699	9995,1274	9177,9424	10822,0575	26
35	9173,9077	9995,1084	9178,7993	10821,2006	25
36	9174,7438	9995,0893	9179,6545	10820,3454	24
37	9175,5783	9995,0702	9180,5081	10819,4918	23
38	9176,4112	9995,0510	9181,3602	10818,6398	22
39	9177,2424	9995,0318	9182,2106	10817,7893	21
40	9178,0721	9995,0125	9183,0595	10816,9404	20
41	9178,9000	9994,9933	9183,9068	10816,0932	19
42	9179,7264	9994,9739	9184,7524	10815,2475	18
43	9180,5512	9994,9546	9185,5965	10814,4034	17
44	9181,3744	9994,9352	9186,4391	10813,5608	16
45	9182,1959	9994,9158	9187,2801	10812,7198	15
46	9183,0160	9994,8963	9188,1196	10811,8803	14
47	9183,8344	9994,8768	9188,9575	10811,0424	13
48	9184,6512	9994,8573	9189,7939	10810,2061	12
49	6185,4664	9994,8377	9190,6287	10809,3712	11
50	9186,2801	9994,8181	9191,4620	10808,5379	10
51	9187,0923	9994,7984	9192,2938	10807,7061	9
52	9187,9029	9994,7787	9193,1241	10806,8758	8
53	9188,7119	9994,7590	9193,9529	10806,0470	7
54	9189,5194	9994,7393	9194,7801	10805,2198	6
55	9190,3254	9994,7195	9195,6059	10804,3940	5
56	9191,1398	9994,6996	9196,4402	10803,5597	4
57	9191,9327	9994,6797	9197,2530	10802,7470	3
58	9192,7341	9994,6598	9198,0743	10801,9256	2
59	9193,5340	9994,6399	9198,8941	10801,1058	1
60	9194,3324	9994,6199	9199,7125	10800,2874	0
		Sin. 81.		Tan 81.	M

M	Sin. 9.		Tan. 9.		
0	9194,3324	9994,6199	9199,7125	10800,2874	60
1	9195,1293	9994,5998	9200,5294	10799,4705	59
2	9195,9246	9994,5798	9201,3448	10798,6551	58
3	9196,7185	9994,5597	9202,1588	10797,8411	57
4	9197,5109	9994,5396	9202,9713	10797,0286	56
5	9198,3019	9994,5194	9203,7825	10796,2175	55
6	9199,0913	9994,4992	9204,5921	10795,4078	54
7	9199,8793	9994,4789	9205,4004	10794,5996	53
8	9200,6658	9994,4586	9206,2072	10793,7928	52
9	9201,4509	9994,4383	9207,0125	10792,9874	51
10	9202,2345	9994,4179	9207,8165	10792,1834	50
11	9203,0166	9994,3975	9208,6191	10791,3809	49
12	9203,7973	9994,3771	9209,4202	10790,5797	48
13	9204,5766	9994,3566	9210,2200	10789,7799	47
14	9205,3544	9994,3361	9211,0183	10788,9816	46
15	9206,1309	9994,3155	9211,8153	10788,1846	45
16	9206,9058	9994,2949	9212,6109	10787,3890	44
17	9207,6794	9994,2743	9213,4051	10786,5948	43
18	9208,4516	9994,2536	9214,1979	10785,8020	42
19	9209,2224	9994,2329	9214,9894	10785,0105	41
20	9209,9917	9994,2121	9215,7795	10784,2204	40
21	9210,7597	9994,1914	9216,5682	10783,4317	39
22	9211,5262	9994,1706	9217,3556	10782,6443	38
23	9212,2914	9994,1497	9218,1416	10781,8583	37
24	9213,0552	9994,1288	9218,9263	10781,0736	36
25	9213,8176	9994,1079	9219,7097	10780,2902	35
26	9214,5787	9994,0869	9220,4917	10779,5082	34
27	9215,3383	9994,0659	9221,2724	10778,7275	33
28	9216,0966	9994,0449	9222,0518	10777,9482	32
29	9216,8536	9994,0238	9222,8298	10777,1701	31
30	9217,6092	9994,0027	9223,6065	10776,3934	30
		Sin. 80.		Tan. 80.	M

M	Sin 9.		Tan. 9.		
30	9217,6092	9994,0027	9223,6065	10776,3934	30
31	9218,3634	9993,9815	9224,3819	10775,6180	29
32	9219,1163	9993,9602	9225,1560	10774,8439	28
33	9219,8679	9993,9391	9225,9288	10774,0711	27
34	9220,6182	9993,9178	9226,7003	10773,2996	26
35	9221,3671	9993,8965	9227,4705	10772,5294	25
36	9222,1146	9993,8751	9228,2395	10771,7605	24
37	9222,8609	9993,8537	9229,0071	10770,9928	23
38	9223,6058	9993,8323	9229,7735	10770,2265	22
39	9224,3494	9993,8109	9230,5386	10769,4614	21
40	9225,0918	9993,7893	9231,3024	10768,6975	20
41	9225,8328	9993,7678	9232,0649	10767,9350	19
42	9226,5725	9993,7462	9232,8262	10767,1737	18
43	9227,3109	9993,7246	9233,5862	10766,4137	17
44	9228,0480	9993,7030	9234,3450	10765,6549	16
45	9228,7839	9993,6813	9235,1026	10764,8974	15
46	9229,5184	9993,6596	9235,8588	10764,1411	14
47	9230,2517	9993,6378	9236,6139	10763,3860	13
48	9230,9838	9993,6160	9237,3677	10762,6322	12
49	9231,7145	9993,5942	9238,1203	10761,8796	11
50	9232,4440	9993,5723	9238,8717	10761,1283	10
51	9233,1722	9993,5504	9239,6218	10760,3781	9
52	9233,8992	9993,5284	9240,3707	10759,6292	8
53	9234,6249	9993,5064	9241,1184	10758,8815	7
54	9235,3494	9993,4844	9241,8649	10758,1350	6
55	9236,0725	9993,4623	9242,6102	10757,3897	5
56	9236,7946	9993,4402	9243,3543	10756,6456	4
57	9237,5153	9993,4181	9244,0972	10755,9027	3
58	9138,8348	9993,3959	9244,8389	10755,1610	2
59	9238 9531	9993,3737	9245,5794	10754,4205	1
60	9239,6702	9993,3514	9246,3187	10753,6812	0
		Sin. 80.		Tan. 80.	M

M	Sin.10.		Tan.10.		
0	9239,6702	9993,3514	9246,3187	10753,6812	60
1	9240,3861	9993,3291	9247,0569	10752,9430	59
2	9241,1007	9993,3068	9247,7939	10752,2061	58
3	9241,8141	9993,2844	9248,5296	10751,4703	57
4	9242,5263	9993,2620	9249,2643	10750,7356	56
5	9243,2373	9993,2396	9249,9977	10750,0022	55
6	9243,9472	9993,2171	9250,7300	10749,2699	54
7	9244,6558	9993,1946	9251,4612	10748,5387	53
8	9245,3632	9993,1720	9252,1912	10747,8087	52
9	9246,0695	9993,1494	9252,9200	10747,0799	51
10	9246,7745	9993,1268	9253,6477	10746,3522	50
11	9247,4784	9993,1041	9254,3743	10745,6257	49
12	9248,1811	9993,0814	9255,0997	10744,9002	48
13	9248,8826	9993,0586	9255,8240	10744,1759	47
14	9249,5830	9993,0358	9256,5471	10743,4528	46
15	9250,2822	9993,0130	9257,2691	10742,7308	45
16	9250,9802	9992,9902	9257,9900	10742,0099	44
17	9251,6771	9992,9673	9258,7098	10741,2901	43
18	9252,3729	9992,9443	9259,4285	10740,5714	42
19	9253,0674	9992,9213	9260,1461	10739,8538	41
20	9253,7609	9992,8983	9260,8625	10739,1374	40
21	9254,4532	9992,8753	9261,5779	10738,4221	39
22	9255,1443	9992,8522	9262,2921	10737,7078	38
23	9255,8343	9992,8290	9263,0053	10736,9947	37
24	9256,5232	9992,8059	9263,7173	10736,2826	36
25	9257,2110	9992,7827	9264,4283	10735,5716	35
26	9257,8976	9992,7594	9265,1382	10734,8618	34
27	9258,5831	9992,7362	9265,8469	10734,1530	33
28	9259,2675	9992,7128	9266,5547	10733,4452	32
29	9259,9508	9992,6895	9267,2613	10732,7386	31
30	9260,6330	9992,6661	9267,9669	10732,0330	30
		Sin. 79.		Tan.79.	M

M	Sin. 10.		Tan. 10.		
30	9260,6330	9992,6661	9267,9609	10732,0330	30
31	9261,3141	9992,6427	9268,6714	10731,3285	29
32	9261,9940	9992,6192	9269,3748	10730,6251	28
33	9262,6729	9992,5957	9270,0772	10729,9227	27
34	9263,3507	9992,5721	9270,7785	10729,2214	26
35	9264,0274	9992,5485	9271,4788	10728,5211	25
36	9264,7029	9992,5249	9272,1780	10727,8219	24
37	9265,3775	9992,5013	9272.8762	10727,1238	23
38	9266,0509	9992,4776	9273,5733	10726,4266	22
39	9266,7232	9992,4538	9274,2694	10725,7306	21
40	9267,3945	9992,4300	9274,9644	10725,0355	20
41	9268,0646	9992,4062	9275,6584	10724,3415	19
42	9268,7338	9992,3824	9276,3514	10723,6485	18
43	9269,4019	9992,3585	9277,0433	10722.9566	17
44	9270,0689	9992,3346	9277,7343	10722,2657	16
45	9270,7348	9992,3106	9278,4241	10721,5758	15
46	9271,3996	9992,2866	9279,1130	10720,8869	14
47	9272,0634	9992,2625	9279,8009	10720,1991	13
48	9272,7262	9992,2385	9280,4877	10719,5122	12
49	9273,3880	9992,2143	9281,1736	10718,8263	11
50	9274,0487	9992,1902	9282,8584	10718,1415	10
51	9274,7083	9992,1660	9282,5423	10717,4577	9
52	9275,3669	9992,1418	9283,2251	10716,7748	8
53	9276,0245	9992,1175	9283,9070	10716,0930	7
54	9276,6810	9992,0932	9284,5878	10715,4121	6
55	9277,3365	9992,0688	9285,2677	10714,7322	5
56	9277,9910	9992,0445	9285.9465	10714.0534	4
57	9278,6445	9992,0200	9286,6244	10713,3755	3
58	9279,2969	9991,9956	9287,3013	10712,6986	2
59	9279,9484	9991,9711	9287,9773	10712,0227	1
60	9280,5988	9991,9465	9288,6522	10711,3477	0
		Sin. 79.		Tan. 79.	M

M	Sin. 11.		Tan. 11.		
0	9280,5988	9991,9465	9288,6522	10711,3477	60
1	9281,2482	9991,9220	9289,3262	10710,6737	59
2	9281,8966	9991,8973	9289,9992	10710,0007	58
3	9282,5440	9991,8727	9290,6713	10709,3286	57
4	9283,1904	9991,8480	9291,3424	10708,6575	56
5	9283,8359	9991,8233	9292,0125	10707,9874	55
6	9284,4803	9991,7985	9292,6817	10707,3182	54
7	9285,1237	9991,7737	9293,3499	10706,6500	53
8	9285,7661	9991,7489	9294,0172	10705,9827	52
9	9286,4075	9991,7240	9294,6835	10705,3164	51
10	9287,0480	9991,6991	9295,3489	10704,6510	50
11	9287,6875	9991,6741	9296,0134	10703,9866	49
12	9288,3260	9991,6491	9296,6768	10703,3231	48
13	9288,9635	9991,6241	9297,3394	10702,6605	47
14	9289,6001	9991,5990	9298,0010	10701,9989	46
15	5290,2357	9991,5739	9298,6617	10701,3382	45
16	9290,8703	9991,5487	9299,3215	10700,6784	44
17	9291,5040	9991,5236	9299,9804	10700,0195	43
18	9292,1367	9991,4983	9300,6383	10699,3616	42
19	9292,7684	9991,4731	9301,2953	10698,7046	41
20	9293,3992	9991,4478	9301,9514	10698,0485	40
21	9294,0291	9991,4224	9302,6066	10697,3933	39
22	9294,6580	9991,3971	9303,2609	10696,7390	38
23	9295,2859	9991,3716	9303,9142	10696,0857	37
24	9295,9129	9991,3462	9304,5667	10695,4332	36
25	9296,5390	9991,3207	9305,2182	10694,7817	35
26	9297,1641	9991,2952	9305,8689	10694,1310	34
27	9297,7883	9991,2696	9306,5186	10693,4813	33
28	9298,4116	9991,2440	9307,1675	10692,8324	32
29	9299,0339	9991,2183	9307,8155	10692,1844	31
30	9299,6559	9991,1927	9308,4626	10691,5374	30
		Sin.78.		Tang. 78.	M

M	Sin. 11.		Tan. 11.		
30	9299,6553	9991,1927	9308,4626	10691,5374	30
31	9300,2757	9991,1669	9309,1088	10690,8912	29
32	9300,8953	9991,1412	9309,7541	10690,2459	28
33	9301,5139	9991,1154	9310,3985	10689,6014	27
34	9302,1317	9991,0895	9311,0421	10688,9578	26
35	9302,7485	9991,0637	9311,6847	10688,3152	25
36	9303,3643	9991,0378	9312,3265	10687,6734	24
37	9303,9793	9991,0118	9312,9675	10687,0324	23
38	9304,5934	9990,9858	9313,6076	10686,3924	22
39	9305,2066	9990,9598	9314,2468	10685,7532	21
40	9305,8189	9990,9337	9314,8851	10685,1148	20
41	9306,4302	9990,9076	9315,5226	10684,4774	19
42	9307,0407	9990,8815	9316,1592	10683,8407	18
43	9307,6503	9990,8553	9316,7950	10683,2050	17
44	9308,2590	9990,8291	9317,4299	10682,5701	16
45	9308,8668	9990,8028	9318,0639	10681,9360	15
46	9309,4737	9990,7765	9318,6971	10681,3028	14
47	9310,0797	9990,7502	9319,3295	10680,6704	13
48	9310,6849	9990,7238	9319,9610	10680,0389	12
49	9311,2892	9990,6974	9320,5917	10679,4082	11
50	9311,8926	9990,6710	9321,2216	10678,7784	10
51	9312,4951	9990,6445	9321,8506	10678,1493	9
52	9313,0967	9990,6179	9322,4788	10677,5212	8
53	9313,6975	9990,5914	9323,1061	10676,8938	7
54	9314,2974	9990,5648	9323,7326	10676,2673	6
55	9314,8965	9990,5381	9324,3583	10675,6416	5
56	9315,4947	9990,5115	9324,9832	10675,0167	4
57	9316,0920	9990,4847	9325,6072	10674,3927	3
58	9316,6885	9990,4580	9326,2305	10673,7694	2
59	9317,2841	9990,4312	9326,8529	10673,1470	1
60	9317,8789	9990,4044	9327,4745	10672,5254	0
		Sin. 78.		Tan. 78.	M

M	Sin.12.		Tan.12.		
0	9317,8789	9990,4044	9327,4745	10672.5254	60
1	9318,4728	9990,3775	9328,0953	10671,9046	59
2	9319,0659	9990,3506	9328,7153	10671.2847	58
3	9319,6581	9990,3236	9329,3344	10670,6655	57
4	9320,2495	9990,2966	9329.9528	10670,0471	56
5	9320,8400	9990,2690	9330,5704	10669,4296	55
6	9321,4297	9990,2425	9331,1871	10668,8128	54
7	9322,0186	9990,2154	9331,8031	10668,1968	53
8	9322,6066	9990,1883	9332,4183	10667.5817	52
9	9323,1938	9990,1611	9333,0326	10666.9673	51
10	9323,7802	9990,1339	9333,6462	10666,3537	50
11	9324,3657	9990,1066	9334,2590	10665,7409	49
12	9324,9504	9990,0794	9334,8710	10665,1289	48
13	9325,5343	9990,0520	9335,4823	10664,5177	47
14	9326,1174	9990,0247	9336,0927	10663,9072	46
15	9326,6996	9989,9972	9336,7024	10663,2976	45
16	9327,2811	9989,9698	9337,3112	10662.6887	44
17	9327,8617	9989,9423	9337,9194	10662,0806	43
18	9328,4415	9989.9148	9338,5267	10661,4732	42
19	9329,0205	9989,8872	9339.1333	10660,8666	41
20	9329,5987	9989,8596	9339,7391	10660,2608	40
21	9330,1761	9989,8320	9340.3441	10659,6558	39
22	9330,7527	9989,8043	9340,9483	10659,0516	38
23	9331,3285	9989.7766	9341,5518	10658,4481	37
24	9331,9035	9989.7488	6342,1546	10657,8453	36
25	9332,4777	9989,7210	9342,7566	10657,2433	35
26	9333,0511	9989,6932	9343,3578	10656,6421	34
27	9333,6236	9989,6653	9343,9583	10656,0416	33
28	9334,1955	9989,6374	9344,5580	10655,4419	32
29	9334.7665	9989,6095	9345,1570	10654,8429	31
30	9335,3367	9989,5815	9345,7552	10654,2447	30
		Sin.77.		Tang.77	M

M	Sin.12.		Tan.12.		
30	9335,3367	9989,5815	9345,7552	10654,2447	30
31	9335,9062	9989,5534	9346,3527	10653,6472	29
32	9336,4748	9989,5254	9346,9494	10653,0505	28
33	9337,0427	9989,4973	9347,5454	10652,4545	27
34	9337,6098	9989,4691	9348,1407	10651,8593	26
35	9338,1762	9989,4410	9348,7352	10651,2647	25
36	9338,7417	9989,4127	9349,3290	10650,6710	24
37	9339,3065	9989,3845	9349,9220	10650,0779	23
38	9339,8705	9989,3562	9350,5143	10649,4856	22
39	9340,4338	9989,3278	9351,1059	10648,8940	21
40	9340,9963	9989,2995	9351,6968	10648,3032	20
41	9341,5580	9989,2711	9352,2869	10647,7130	19
42	9342,1189	9989,2426	9352,8763	10647,1236	18
43	9342,6791	9989,2141	9353,4650	10646,5349	17
44	9343,2386	9989,1856	9354,0529	10645,9470	16
45	9343,7972	9989,1570	9354,6402	10645,3597	15
46	9344,3552	9989,1284	9355,2267	10644,7732	14
47	9344,9123	9989,0998	9355,8125	10644,1874	13
48	9345,4688	9989,0711	9356,3976	10643,6023	12
49	9346,0244	9989,0424	9356,9820	10643,0179	11
50	9346,5794	9989,0136	9357,5657	10642,4342	10
51	9347,1336	9988,9848	9358,1487	10641,8512	9
52	9347,6870	9988,9560	9358,7310	10641,2689	8
53	9348,2397	9988,9271	9359,3126	10640,6874	7
54	9348,7917	9988,8982	9359,8934	10640,1065	6
55	9349,3429	9988,8692	9360,4736	10639,5263	5
56	9349,8934	9988,8402	9361,0531	10638,9468	4
57	9350,4431	9988,8112	9361,6319	10638,3681	3
58	9350,9921	9988,7821	9362,2100	10637,7900	2
59	9351,5404	9988,7530	9362,7874	10637,2126	1
60	9352,0880	9988,7239	9363,3641	10636,6358	0
		Sin.77.		Tan.77.	M

M	Sin. 13.		Tan. 13.		
0	9352,0880	9988,7239	9363,3641	10636,6359	60
1	9352,6348	9988,6947	9363,9401	10636,0598	59
2	9353,1809	9988,6655	9364,5154	10635,4845	58
3	9353,7263	9988,6362	9365,0901	10634,9098	57
4	9354,2710	9988,6069	9365,6640	10634,3359	56
5	9354,8150	9988,5776	9366,2373	10633,7626	55
6	9355,3582	9988,5482	9366,8099	10633,1900	54
7	9355,9007	9988,5188	9367,3819	10632,6180	53
8	9356,4425	9988,4893	9367,9532	10632,0468	52
9	9356,9836	9988,4598	9368,5237	10631,4762	51
10	9357,5240	9988,4303	9369,0937	10630,9063	50
11	9358,0637	9988,4007	9369,6629	10630,3370	49
12	9358,6026	9988,3711	9370,2315	10629,7684	48
13	9359,1409	9988,3415	9370,7994	10629,2005	47
14	9359,6785	9988,3118	9371,3666	10628,6333	46
15	9360,2153	9988,2820	9371,9332	10628,0667	45
16	9360,7515	9988,2523	9372,4992	10627,5008	44
17	9361,2869	9988,2225	9373,0644	10626,9355	43
18	9361,8217	9988,1926	9373,6290	10626,3709	42
19	9362,3558	9988,1627	9374,1930	10625,8069	41
20	9362,8892	9988,1328	9374,7563	10625,2436	40
21	9363,4218	9988,1029	9375,3189	10624,6810	39
22	9363,9538	9988,0729	9375,8809	10624,1190	38
23	9364,4852	9988,0428	9376,4423	10623,5576	37
24	9365,0158	9988,0128	9377,0030	10622,9969	36
25	9365,5457	9987,9826	9377,5631	10622,4369	35
26	9366,0750	9987,9525	9378,1225	10621,8775	34
27	9366,6036	9987,9223	9378,6812	10621,3187	33
28	9367,1315	9987,8921	9379,2394	10620,7605	32
29	9367,6587	9987,8618	9379,7969	10620,2031	31
30	9368,1852	9987,8315	9380,3537	10619,6462	30
		Sin. 76.		Tan. 76.	M

M	Sin. 13.		Tan. 13.		
30	9368,1852	9987,8315	9380,3537	10619,6462	30
31	9368,7111	9987,8011	9380,9099	10619,0900	29
32	9369.2363	9987,7707	9381,4655	10618,5344	28
33	9369,7608	9987,7403	9382,0205	10617,9795	27
34	9370,2847	9987,7098	9382,5748	10617,4251	26
35	9370,8079	9987.6793	9383,1285	10616,8714	25
36	9371,3304	9987,6488	9383,6815	10616,3184	24
37	9371,8522	9987,6182	9384,2340	10615,7659	23
38	9372,3734	9987,5876	9384,7858	10615,2141	22
39	9372,8940	9987,5569	9385,3370	10614,6629	21
40	9373,4138	9987,5262	9385,8876	10614,1124	20
41	9373,9331	9987,4955	9386,4375	10613,5624	19
42	9374,4516	9987,4647	9386,9869	10613,0131	18
43	9374.9695	9987,4339	9387,5356	10612,4643	17
44	9375,4868	9987,4031	9388,0837	10611,9162	16
45	9376,0034	9987,3722	9388,6312	10611,3687	15
46	9376,5193	9987,3412	9389,1781	10610,8219	14
47	9377.0346	9987.3103	9389.7243	10610,2756	13
48	9377,5493	9987,2792	9390,2700	10609,7299	12
49	9378,0633	9987,2482	9390,8150	10609,1849	11
50	9378,5766	9987,2171	9391,3595	10608,6404	10
51	9379,0893	9987,1860	9391,9033	10608,0966	9
52	9379,6014	9987,1548	9392,4466	10607,5533	8
53	9380,1129	9987,1236	9392,9892	10607,0107	7
54	9380,6237	9987,0924	9393,5313	10606,4687	6
55	9381,1338	9987,0611	9394,0727	10605,9272	5
56	9381,6434	9987,0298	9394,6136	10605,3863	4
57	9382,1522	9986,9984	9395,1538	10604,8461	3
58	9382,6605	9986,9670	9395,6935	10604,3064	2
59	9383,1681	9986,9356	9396,2325	10603,7674	1
60	9383,6751	9986,9041	9396,7710	10603,2289	0
		Sin. 76.		Tan. 76.	M

M	Sin.14.	[illegible]	Tan. 14.	[illegible]	
0	9383,6751	9986,9041	9396,7710	10603,2289	60
1	9384,1815	9986,8726	9397,3089	10602,6910	59
2	9384,6873	9986,8410	9397,8462	10602,1537	58
3	9385,1924	9986,8094	9398,3829	10601,6170	57
4	9385,6969	9986,7778	9398,9191	10601,0808	56
5	9386,2008	9986,7461	9399,4546	10600,5453	55
6	9386,7040	9986,7144	9399,9896	10600,0103	54
7	9387,2066	9986,6826	9400,5240	10599,4760	53
8	9387,7087	9986,6508	9401,0578	10598,9421	52
9	9388,2101	9986,6190	9401,5910	10598,4089	51
10	9388,7108	9986,5872	9402,1237	10597,8763	50
11	9389,2110	9986,5552	9402,6557	10597,3442	49
12	9389,7106	9986,5233	9403,1873	10596,8127	48
13	9390,2095	9986,4913	9403,7182	10596,2817	47
14	9390,7079	9986,4593	9404,2486	10595,7514	46
15	9391,2056	9986,4272	9404,7784	10595,2216	45
16	9391,7027	9986,3951	9405,3076	10594,6923	44
17	9392,1993	9986,3630	9405,8363	10594,1637	43
18	9392,6952	9986,3308	9406,3644	10593,6356	42
19	9393,1905	9986,2986	9406,8919	10593,1080	41
20	9393,6852	9986,2663	9407,4189	10592,5810	40
21	9394,1793	9986,2340	9407,9453	10592,0546	39
22	9394,6728	9986,2017	9408,4711	10591,5288	38
23	9395,1658	9986,1693	9408,9964	10591,0035	37
24	9395,6581	9986,1369	9409,5212	10590,4787	36
25	9396,1498	9986,1044	9410,0454	10589,9545	35
26	9396,6410	9986,0719	9410,5690	10589,4309	34
27	9397,1315	9986,0394	9411,0921	10588,9078	33
28	9397,6215	9986,0068	9411,6146	10588,3853	32
29	9398,1108	9985,9742	9412,1366	10587,8633	31
30	9398,5996	9985,9416	9412,6580	10587,3419	30
[illegible]	[illegible]	Sin.75.	[illegible]	Tan.75.	M

M	Sin. 14.		Tan. 14.		
30	9398,5996	9985,9416	9412,6580	90587,3419	30
31	9399,0878	9985,9089	9413,1789	10586,8210	29
32	9399,5754	9985,8761	9413,6992	10586,3007	28
33	9400,0624	9985,8434	9414,2190	10585,7809	27
34	9400,5489	9985,8100	9414,7383	10585,2616	26
35	9401,0347	9985,7777	9415,2570	10584,7429	25
36	9401,5200	9985,7448	9415,7752	10584,2248	24
37	9402,0047	9985,7119	9416,2928	10583,7071	23
38	9402,4889	9985,6789	9416,8099	10583,1900	22
39	9402,9724	9985,6459	9417,3264	10582,6735	21
40	9403,4554	9985,6129	9417,8425	10582,1575	20
41	9403,9378	9985,5798	9418,3579	10581,6420	19
42	9404,4196	9985,5467	9418,8729	10581,1270	18
43	9404,9009	9985,5135	9419,3873	10580,6126	17
44	9405,3816	9985,4803	9419,9012	10580,0987	16
45	9405,8617	9985,4471	9420,4146	10579,5853	15
46	9406,3412	9985,4138	9420,9274	10579,0725	14
47	9406,8202	9985,3805	9421,4397	10578,5602	13
48	9407,2987	9985,3471	9421,9515	10578,0484	12
49	9407,7765	9985,3137	9422,4628	10577,5372	11
50	9408,2538	9985,2803	9422,9735	10577,0264	10
51	9408,7306	9985,2468	9423,4837	10576,5162	9
52	9409,2067	9985,2133	9423,9934	10576,0065	8
53	9409,6824	9985,1797	9424,5026	10575,4973	7
54	9410,1574	9985,1461	9425,0113	10574,9887	6
55	9410,6319	9985,1125	9425,5194	10574,4805	5
56	9411,1059	9985,0788	9426,0270	10573,9729	4
57	9411,5793	9985,0451	9426,5341	10573,4658	3
58	9412,0521	9985,0114	9427,0408	10572,9592	2
59	9412,5244	9984,9776	9427,5468	10572,4531	1
60	9412,9962	9984,9437	9428,0524	10571,9475	0
		Sin. 75.		Tan. 75.	M

M	Sin.15.		Tan.15.		
0	9412,9962	9984,9437	9428,0524	10571,9475	60
1	9413,4674	9984,9099	9428,5575	10571,4424	59
2	9413,9380	9984,8760	9429,0620	10570,9379	58
3	9414,4082	9984,8420	9429,5661	10570,4338	57
4	9414,8777	9984,8080	9430,0697	10569,9303	56
5	9415,3467	9984,7740	9430,5727	10569,4272	55
6	9415,8152	9984,7399	9431,0752	10568,9247	54
7	9416,2831	9984,7058	9431,5773	10568,4226	53
8	9416,7505	9984,6717	9432,0788	10567,9211	52
9	9417,2174	9984,6375	9432,5799	10567,4201	51
10	9417,6837	9984,6033	9433,0804	10566,9195	50
11	9418,1495	9984,5690	9433,5805	10566,4195	49
12	9418,6147	9984,5347	9434,0800	10565,9199	48
13	9419,0794	9984,5003	9434,5791	10565,4209	47
14	9419,5436	9984,4660	9435,0776	10564,9223	46
15	9420,0073	9984,4315	9435,5757	10564,4242	45
16	9420,4704	9984,3971	9436,0732	10563,9267	44
17	9420,9329	9984,3626	9436,5703	10563,4296	43
18	9421,3950	9984,3280	9437,0669	10562,9330	42
19	9421,8565	9984,2935	9437,5630	10562,4369	41
20	9422,3175	9984,2588	9438,0586	10561,9413	40
21	9422,7780	9984,2242	9438,5538	10561,4461	39
22	9423,2380	9984,1895	9439,0484	10560,9515	38
23	9423,6974	9984,1548	9439,5426	10560,4573	37
24	9424,1563	9984,1200	9440,0363	10559,9637	36
25	9424,6147	9984,0852	9440,5295	10559,4705	35
26	9425,0725	9984,0503	9441,0222	10558,9777	34
27	9425,5299	9984,0154	9441,5144	10558,4855	33
28	9425,9867	9983,9805	9442,0062	10557,9937	32
29	9426,4430	9983,9455	9442,4975	10557,5024	31
30	9426,8988	9983,9105	9442,9883	10557,0116	30
		Sin.74.		Tan.74.	M

M	Sin. 9.		Tan. 15.		
30	9426,8988	9983,9105	9442,9883	10557,0116	30
31	9427,3541	9983,8754	9443,4786	10556,5213	29
32	9427,8088	9983,8403	9443,9685	10556,0314	28
33	9428,2631	9983,8052	9444,4579	10555,5421	27
34	9428,7168	9983,7700	9444,9468	10555,0531	26
35	9429,1701	9983,7348	9445,4352	10554,5647	25
36	9429,6228	9983,6996	9445,9232	10554,0767	24
37	9430,0750	9983,6643	9446,4107	10553,5892	23
38	9430,5267	9983,6289	9446,8977	10553,1022	22
39	9430,9779	9983,5936	9447,3843	10552,6156	21
40	9431,4286	9983,5581	9447,8704	10552,1295	20
41	9431,8788	9983,5227	9448,3561	10551,6439	19
42	9432,3285	9983,4872	9448,8412	10551,1587	18
43	9432,7777	9983,4517	9449,3259	10550,6740	17
44	9433,2263	9983,4161	9449,8102	10550,1897	16
45	9433,6745	9983,3805	9450,2940	10549,7059	15
46	9434,1222	9983,3449	9450,7773	10549,2226	14
47	9434,5694	9983,3092	9451,2602	10548,7397	13
48	9435,0162	9983,2734	9451,7426	10548,2573	12
49	9435,4623	9983,2377	9452,2246	10547,7753	11
50	9435,9080	9983,2019	9452,7061	10547,2938	10
51	9436,3532	9983,1660	9453,1872	10546,8128	9
52	9436,7979	9983,1301	9453,6677	10546,3322	8
53	9437,2421	9983,0942	9454,1479	10545,8520	7
54	9437,6859	9983,0582	9454,6276	10545,3723	6
55	9438,1291	9983,0222	9455,1069	10544,8930	5
56	9438,5719	9982,9862	9455,5857	10544,4142	4
57	9439,0141	9982,9501	9456,0640	10543,9359	3
58	9439,4559	9982,9140	9456,5419	10543,4580	2
59	9439,8972	9982,8778	9457,0194	10542,9805	1
60	9440,3380	9982,8416	9457,4964	10542,5035	0
		Sin. 74		Tan. 47.	M

M	Sin.16.		Tan.16.		
0	9440,3380	9982,8416	9457,4964	10542,5035	60
1	9440,7784	9982,8053	9457,9730	10542,0269	59
2	9441,2182	9982,7691	9458,4491	10541,5508	58
3	9441,6576	9982,7327	9458,9248	10541,0751	57
4	9442,0964	9982,6964	9459,4000	10540,5999	56
5	9442,5348	9982,6600	9459,8748	10540,1251	55
6	9442,9728	9982,6235	9460,3492	10539,6507	54
7	9443,4102	9982,5870	9460,8231	10539,1768	53
8	9443,8472	9982,5505	9461,2966	10538,7033	52
9	9444,2837	9902,5140	9461,7697	10538,2302	51
10	9444,7197	9982,4774	9462,2423	10537,7576	50
11	9445,1552	9982,4407	9462,7145	10537,2854	49
12	9445,5903	9982,4040	9463,1862	10536,8137	48
13	9446,0249	9083,3673	9463,6576	10536,3423	47
14	9446,4590	9982,3305	9464,1285	10535,8715	46
15	9446,8927	9982,2937	9464,5989	10535,4010	45
16	9447,3259	9982,2569	9465,0690	10534,9310	44
17	9447,7586	9982,2200	9465,5386	10534,4614	43
18	9448,1909	9982,1831	9466,0077	10533,9922	42
19	9448,6226	9982,1461	9466,4765	10533,5234	41
20	9449,0540	9982,1091	9466,9448	10533,0551	40
21	9449,4848	9982,0721	9467,4127	10532,5872	39
22	9449,9152	9982,0350	9467,8802	10532,1198	38
23	9450,3451	9981,9979	9468,3472	10531,6527	37
24	9450,7746	9981,9607	9468,8138	10531,1861	36
25	9451,2036	9981,9235	9469,2801	10530,7199	35
26	9451,6322	9981,8863	9469,7459	10530,2541	34
27	9452,0603	9981,8490	9470,2112	10529,7887	33
28	9452,4879	9981,8117	9470,6762	10529,3238	32
29	9452,9150	9981,7743	9471,1407	10528,8592	31
30	9453,3418	9981,7369	9471,6048	10528,3951	30
		Sin.73.		Tan.73.	M

M	Sin.16.		Tan.16.		
30	9453,3418	9981,7369	9471,6048	10528,3951	30
31	9453,7680	9981,6995	9472,0685	10527,9314	29
32	9454,1938	9981,6620	9472,5318	10527,4681	28
33	9454,6192	9981,6245	9472,9947	10527,0053	27
34	9455,0441	9981,5869	9473,4571	10526,5428	26
35	9455,4685	9981,5493	9473,9192	10526,0807	25
36	9455,8925	9981,5117	9474,3808	10525,6191	24
37	9456,3161	9981,4740	9474,8420	10525,1579	23
38	9456,7392	9981,4363	9475,3029	10524,6971	22
39	9457,1618	9981,3985	9475,7633	10524,2367	21
40	9457,5840	9981,3607	9476,2233	10523,7767	20
41	9458,0058	9981,3229	9476,6828	10523,3171	19
42	9458,4271	9981,2850	9477,1420	10522,8579	18
43	9458,8479	9981,2471	9477,6008	10522,3991	17
44	9459,2683	9981,2091	9478,0592	10521,9407	16
45	9459,6883	9981,1711	9478,5172	10521,4827	15
46	9460,1078	9981,1331	9478,9747	10521,0252	14
47	9460,5269	9981,0950	9479,4319	10520,5680	13
48	9460,9456	9981,0569	9479,8887	10520,1112	12
49	9461,3638	9981,0187	9480,3451	10519,6549	11
50	9461,7816	9980,9805	9480,8010	10519,1989	10
51	9462,1989	9980,9423	9481,2566	10518,7433	9
52	9462,6158	9980,9040	9481,7118	10518,2881	8
53	9463,0323	9980,8657	9482,1666	10517,8334	7
54	9463,4483	9980,8273	9482,6209	10517,3790	6
55	9463,8638	9980,7889	9483,0749	10516,9250	5
56	9464,2790	9980,7504	9483,5285	10516,4714	4
57	9464,6937	9980,7120	9483,9817	10516,0182	3
58	9465,1080	9980,6734	9484,4345	10515,5654	2
59	9465,5219	9980,6349	9484,8870	10515,1130	1
60	9465,9353	9980,5963	9485,3390	10514,6609	0
		Sin. 73.		Tan.73.	M

M	Sin. 17.		Tan. 17.		
0	9465,9353	9980,5963	9485,3390	10514,6609	60
1	9466,3483	9980,5576	9485,7906	10514,2093	59
2	9466,7609	9980,5189	9486,2419	10513,7580	58
3	9467,1730	9980,4802	9486,6927	10513,3072	57
4	9467,5847	9980,4415	9487,1432	10512,8567	56
5	9457,9960	9980,4027	9487,5933	10512,4066	55
6	9468,4069	9980,3638	9488,0430	10511,9569	54
7	9468,8173	9980,3249	9488,4923	10511,5076	53
8	9469,2273	9980,2860	9488,9412	10511,0587	52
9	9469,6369	9980,2470	9489,3898	10510,6101	51
10	9470,0460	9980,2080	9489,8380	10510,1619	50
11	9470,4548	9980,1690	9490,2858	10509,7142	49
12	9470,8631	9980,1299	9490,7332	10509,2668	48
13	9471,2710	9980,0908	9491,1802	10508,8197	47
14	9471,6785	9980,0516	9491,6268	10508,3731	46
15	9472,0856	9980,0124	9492,0731	10507,9268	45
16	9472,4922	9979,9732	9492,5109	10507,4809	44
17	9472,8984	9979,9339	9492,9645	10507,0354	43
18	9473,3042	9979,8945	9493,4097	10506,5903	42
19	9473,7096	9979,8552	9493,8544	10506,1455	41
20	9474,1146	9979,8158	9494,2988	10505,7011	40
21	9474,5192	9979,7763	9494,7428	10505,2571	39
22	9474,9233	9979,7368	9495,1865	10504,8135	38
23	9475,3271	9979,6973	9495,6297	10504,3702	37
24	9475,7304	9979,6577	9496,0726	10503,9273	36
25	9476,1333	9979,6181	9496,5152	10503,4848	35
26	9476,5358	9979,5785	9496,9573	10503,0426	34
27	9476,9379	9979,5388	9497,3991	10502,6008	33
28	9477,3396	9979,4990	9497,8405	10502,1594	32
29	9477,7409	9979,4593	9498,2816	10501,7183	31
30	9478,1418	9979,4195	9498,7223	10501,2777	30
		Sin. 72.		Tan. 72.	M

M	Sin. 17.		Tan. 17.		
30	9478,1418	9979,4195	9498,7223	10501,2777	30
31	9478,5422	9979,3796	9499,1626	10500,8373	29
32	9478,9423	9979,3397	9499,6025	10500,3974	28
33	9479,3420	9979,2998	9500,0421	10499,9578	27
34	9479,7412	9979,2598	9500,4814	10499,5186	26
35	9480,1401	9979,2198	9500,9202	10499,0797	25
36	9480,5385	9979,1797	9501,3587	10498,6412	24
37	9480,9365	9979,1396	9501,7969	10498,2030	23
38	9481,3342	9979,0995	9502,2346	10497,7653	22
39	9481,7314	9979,0593	9502,6721	10497,3279	21
40	9482,1283	9979,0191	9503,1091	10496,8908	20
41	9482,5247	9978,9789	9503,5458	10496,4541	19
42	9482,9208	9978,9386	9503,9822	10496,0177	18
43	9483,3164	9978,8982	9504,4182	10495,5818	17
44	9483,7117	9978,8579	9504,8538	10495,1461	16
45	9484,1065	9978,8174	9505,2891	10494,7109	15
46	9484,5010	9978,7770	9505,7240	10494,2759	14
47	9484,8951	9978,7365	9506,1585	10493,8414	13
48	9485,2887	9978,6959	9506,5928	10493,4072	12
49	9485,6820	9978,6553	9507,0266	10492,9733	11
50	9486,0749	9978,6147	9507,4601	10492,5398	10
51	9486,4674	9978,5741	9507,8933	10492,1066	9
52	9486,8595	9978,5333	9508,3261	10491,6738	8
53	9487,2512	9978,4926	9508,7586	10491,2414	7
54	9487,6425	9978,4518	9509,1906	10490,8093	6
55	9488,0335	9978,4110	9509,6224	10490,3775	5
56	9488,4240	9978,3701	9510,0538	10489,9461	4
57	9488,8142	9978,3292	9510,4849	10489,5150	3
58	9489,2039	9978,2883	9510,9156	10489,0843	2
59	9489,5933	9978,2473	9511,3460	10488,6539	1
60	9489,9823	9978,2063	9511,7760	10488,2239	0
		Sin. 72.		Tan. 72.	M

M	Sin.18.		Tan.18.		
0	9489,9823	9978,2063	9511,7760	10488,2239	60
1	9490,3709	9978,1652	9512,2057	10487,7942	59
2	9490,7592	9978,1241	9512,6350	10487,3649	58
3	9491,1470	9978,0830	9513,0640	10486,9359	57
4	9491,5345	9978,0418	9513,4927	10486,5073	56
5	9491,9216	9978,0005	9513,9210	10486,0789	55
6	9492,3083	9977,9593	9514,3490	10485,6510	54
7	9492,6946	9977,9180	9514,7766	10485,2233	53
8	9493,0805	9977,8766	9515,2039	10484,7960	52
9	9493,4661	9977,8352	9515,6308	10484,3691	51
10	9493,8513	9977,7938	9516,0575	10483,9424	50
11	9494,2361	9977,7523	9516,4838	10483,5162	49
12	9494,6205	9977,7108	9516,9097	10483,0902	48
13	9495,0046	9977,6692	9517,3353	10482,6046	47
14	9495,3882	9977,6276	9517,7606	10482,2394	46
15	9495,7715	9977,5860	9518,1855	10481,8144	45
16	9496,1544	9977,5443	9518,6101	10481,3898	44
17	9496,5370	9977,5026	9519,0344	10480,9656	43
18	9496,9192	9977,4608	9519,4583	10480,5416	42
19	9497,3010	9977,4190	9519,8819	10480,1180	41
20	9497,6824	9977,3772	9520,3052	10479,6947	40
21	9498,0635	9977,3353	9520,7281	10479,2718	39
22	9498,4442	9977,2934	9521,1507	10478,8492	38
23	9498,8245	9977,2514	9521,5730	10478,4269	37
24	9499,2044	9977,2094	9521,9950	10478,0050	36
25	9499,5840	9977,1674	9522,4166	10477,5833	35
26	9499,9632	9977,1253	9522,8379	10477,1620	34
27	9500,3421	9977,0832	9523,2589	10476,7411	33
28	9500,7205	9977,0410	9523,6795	10476,3204	32
29	9501,0987	9976,9988	9524,0998	10475,9001	31
30	9501,4764	9976,9565	9524,5198	10475,4801	30
		Sin.71.		Tang.71	M

M	Sin. 18.		Tan. 18.		
30	9501,4764	9976,9565	9524,5198	10475,4801	30
31	9501,8538	9976,9143	9524,9395	10475,0604	29
32	9502,2308	9976,8719	9525,3588	10474,6411	28
33	9502,6075	9976,8296	9525,7779	10474,2221	27
34	9502,9837	9976,7871	9526,1966	10473,8033	26
35	9503,3597	9976,7447	9526,6150	10473,3850	25
36	9503,7352	9976,7022	9527,0330	10472,9669	24
37	9504,1104	9976,6597	9527,4508	10472,5492	23
38	9504,4853	9975,6171	9527,8682	10472,1318	22
39	9504,8598	9976,5745	9528,2853	10471,7146	21
40	9505,2339	9976,5318	9528,7021	10471,2979	20
41	9505,6077	9976,4891	9529,1185	10470,8814	19
42	9505,9811	9976,4464	9529,5347	10470,4652	18
43	9506,3541	9976,4036	9529,9505	10470,0494	17
44	9506,7268	9976,3608	9530,3660	10469,6339	16
45	9507,0992	9976,3179	9530,7812	10469,2187	15
46	9507,4711	9976,2750	9531,1961	10468,8038	14
47	9507,8428	9976,2320	9531,6107	10468,3892	13
48	9508,2140	9976,1891	9532,0249	10467,9750	12
49	9508,5850	9976,1460	9532,4389	10467,5610	11
50	9508,9555	9976,1030	9532,8525	10467,1474	10
51	9509,3257	9976,0599	9533,2658	10466,7341	9
52	9509,6956	9976,0167	9533,6789	10466,3211	8
53	9510,0651	9975,9735	9534,0916	10465,9084	7
54	9510,4343	9975,9303	9534,5039	10465,4960	6
55	9510,80[illegible]1	9975,8870	9534,9160	10465,0839	5
56	9511,1715	9975,8437	9535,3278	10464,6721	4
57	9511,[illegible]395	9975,8003	9535,7393	10464,2607	3
58	9511,9[illegible]74	9975,7569	9536,1504	10463,8495	2
59	9512,2748	9975,7135	9536,5613	10463,4386	1
60	9512,6419	9975,6700	6536,9718	10463,0281	0
		Sin. 71.		Tan. 71.	M

M	Sin. 19.		Tan. 19.		
0	9512,6419	9975,6700	9536,9718	10463,0281	60
1	9513,0086	9975,6265	9537,3820	10462,6179	59
2	9513,3750	9975,5829	9537,7920	10462,2079	58
3	9513,7410	9975,5393	9538,2016	10461,7983	57
4	9514,1067	9975,4957	9538,6109	10461,3890	56
5	9514,4720	9975,4520	9539,0200	10460,9800	55
6	9514,8370	9975,4083	9539,4287	10460,5712	54
7	9515,2017	9975,3645	9539,8371	10460,1628	53
8	9515,5660	9975,3207	9540,2452	10459,7547	52
9	9515,9299	9975,2769	9540,6530	10459,3469	51
10	9516,2936	9975,2330	9541,0606	10458,9394	50
11	9516,6569	9975,1890	9541,4678	10458,5321	49
12	9517,0198	9975,1451	9541,8747	10458,1252	48
13	9517,3824	9975,1010	9542,2813	10457,7186	47
14	9517,7447	9975,0570	9542,6876	10457,3123	46
15	9518,1066	9975,0129	9543,0936	10457,9063	45
16	9518,4682	9974,9688	9543,4994	10456,5006	44
17	9518,8294	9974,9246	9543,9048	10456,0951	43
18	9519,1903	9974,8804	9544,3099	10455,6900	42
19	9519,5509	9974,8361	9544,7148	10455,2851	41
20	9519,9112	9974,7918	9545,1193	10454,8806	40
21	9520,2711	9974,7475	9545,5236	10454,4764	39
22	9520,6306	9974,7031	9545,9275	10454,0724	38
23	9520,9899	9974,6587	9546,3312	10453,6687	37
24	9521,3488	9974,6142	9546,7346	10453,2654	36
25	9521,7073	9974,5697	9547,1376	10452,8623	35
26	9522,0656	9974,5251	9547,5404	10452,4595	34
27	9522,4235	9974,4805	9547,9429	10452,0570	33
28	9522,7811	9974,4359	9548,3451	10451,6548	32
29	9523,1383	9974,3912	9548,7470	10451,2529	31
30	9523,4952	9974,3465	9549,1487	10450,8513	30
		Sin. 70.		Tan. 70.	M

M	Sin. 19.		Tan. 19.		
30	9523,4952	9974,3465	9549,1487	10450.8513	30
31	9523,8518	9974,3018	9549,5500	10450,4499	29
32	9524,2080	9974,2570	9549,9511	10450,0489	28
33	9524,5640	9974,2121	9550,3518	10449,6481	27
34	9524,9196	9974,1672	9550,7523	10449,2476	26
35	9525,2748	9974,1223	9551,1525	14448.8474	25
36	9525,6298	9974,0774	9551,5524	10448,4475	24
37	9525,9844	9974,0324	9551,9520	10448,0479	23
38	9526,3387	9973,9873	9552,3514	10447,6486	22
39	9526,6927	9973,9422	9552,7504	18447,2495	21
40	9527,0463	9973,8971	9553,1492	10446,8507	20
41	9527,3996	9973,8519	9553,5477	10446,4522	19
42	9527,7526	9973,8067	9553,9459	10446,0540	18
43	9728,1053	9973,7614	9554,3438	10445,6561	17
44	9528,4576	9973,7161	9554,7414	10445,2585	16
45	9528,8096	9973,6708	9555,1388	10444,8611	15
46	9529,1613	9973,6254	9555,5359	10444,4641	14
47	9529,5127	9973,5800	9555,9327	10444,0673	13
48	9529,8638	9973,5346	9556,3292	10443,6707	12
49	6530,2145	9973,4890	9556,7254	10443,2745	11
50	9530,5649	9973,4435	9557,121[illegible]	10442,8785	10
51	9530,9150	9973,3979	9557,5171	10442,4828	9
52	9531,2648	9973,3523	9557,9125	10442,0874	8
53	9531,6143	9973,3066	9558,3076	10441,6923	7
54	9531,9634	9973,2609	9558,7025	10441,2974	6
55	9532,3123	9973,2152	9559,0971	10440,9029	5
56	9532,6608	9973,1694	9559,4914	10440,5086	4
57	9533,0090	9973,1235	9559,8854	10440,1145	3
58	9533,3568	9973,0777	9560,279[illegible]	10439,7208	2
59	9533,7044	9973,0317	9560,6726	10439,3273	1
60	9534,0516	9972,9858	9561,0658	10438,9341	0
		Sin. 70.		Tan. 70.	M

M	Sin.20.		Tan.16		
0	9534,0516	9972,9858	9561,0658	10438,9341	60
1	9534,3986	9972,9398	9561,4588	10438,5411	59
2	9534,7452	9972,8937	9561,8514	10438,1485	58
3	9535,0915	9972,8476	9562,2438	10437,7561	57
4	9535,4375	9972,8015	9562,6359	10437,3640	56
5	9535,7832	9972,7554	9563,0278	10436,9721	55
6	9536,1286	9972,7091	9563,4[illegible]94	10436,5895	54
7	9536,4736	9972,6629	9563,8107	10436,1892	53
8	9536,8184	9972,6166	9564,2017	10435,7982	52
9	9537,1628	9972,5703	9564,5925	10435,4074	51
10	9537,5069	9972,5239	9564,9830	10435,0169	50
11	9537,8508	9972,4775	9565,3733	10434,6267	49
12	9538,1943	9972,4310	9565,7632	10434,2367	48
13	9538,5375	9972,3845	9566,1530	10433,8470	47
14	9538,8804	9972,3380	9566,5424	10433,4575	46
15	9539,2230	9972,2914	9566,9316	10433,0684	45
16	9539,5653	9972,2448	9567,3205	10432,6795	44
17	9539,9072	9972,1981	9567,7091	10432,2908	43
18	9540,2489	9972,1514	9568,0975	10431,9024	42
19	9540,5903	9972,1046	9568,4856	10431,5143	41
20	9540,9313	9972,0578	9568,8735	10431,1265	40
21	9541,2721	9972,0110	9569,2610	10430,7389	39
22	9541,6125	9971,9641	9569,6484	10430,3515	38
23	9541,9527	9971,9172	9570,0354	10429,9645	37
24	9542,2925	9971,8702	9570,4222	10429,5777	36
25	9542,6321	9971,8232	9570,8088	10429,1911	35
26	9542,9713	9971,7762	9571,1951	10428,8048	34
27	9543,3102	9971,7291	9571,5811	10428,4188	33
28	9543,6489	9971,6820	9571,9669	10428,0330	32
29	9543,9872	9971,6348	9572,3524	10427,6475	31
30	9544,3253	9971,5876	9572,7376	10427,2623	30
		Sin.69.		Tan.60.	M

M	Sin.20.		Tan.20.		
30	9544,3253	9971,5876	9572,7376	10427,2623	30
31	9544,6630	9971,5403	9573,1226	10426,8773	29
32	9545,0004	9971,4930	9573,5074	10426,4925	28
33	9545,3375	9971,4457	9573,8918	10426,1081	27
34	9545,6741	9971,3983	9574,2761	10425,7239	26
35	9545,0110	9971,3509	9574,6500	10425,3399	25
36	9546,3472	9971,3034	9575,0437	10424,9562	24
37	9546,6832	9971,2559	9575,4272	10424,5727	23
38	9547,0188	9971,2084	9575,8104	10424,1895	22
39	9547,3542	9971,1608	9576,1933	10423,8066	21
40	9547,6892	9971,1131	9576,5760	10423,4239	20
41	9548,0240	9971,0655	9576,9585	10423,0414	19
42	9548,3585	9971,0178	9577,3407	10422,6592	18
43	9548,6926	9970,9700	9577,7226	10422,2773	17
44	9549,0265	9970,9222	9578,1043	10421,8956	16
45	9549,3601	9970,8744	9578,4857	10421,5142	15
46	9549,6934	9970,8265	9578,8669	10421,1330	14
47	9550,0264	9970,7786	9579,2478	10420,7521	13
48	9550,3591	9970,7306	9579,6285	10420,3714	12
49	9550,6916	9970,6826	9580,0090	10419,9910	11
50	9551,0237	9970,6345	9580,3891	10419,6108	10
51	9551,3555	9970,5864	9580,7691	10419,2308	9
52	9551,6871	9970,5383	9581,1488	10418,8512	8
53	9552,0184	9970,4901	9581,5282	10418,4717	7
54	9552,3493	9970,4419	9581,9074	10418,0925	6
55	9552,6800	9970,3936	9582,2864	10417,7136	5
56	9553,0104	9970,3453	9582,6651	10417,3349	4
57	9553,3405	9970,2970	9583,0435	10416,9564	3
58	9553,6703	9970,2486	9583,4217	17416,5782	2
59	9553,9999	9970,2002	9583,7997	10416,2002	1
60	9554,3291	9970,1517	9584,1774	10415,8225	0
		Sin. 69.		Tan.69.	M

M	Sin. 21.		Tan. 21.		
0	9554,3291	9970,1517	9584,1774	10415,8225	60
1	9554,6581	9970,1032	9584,5549	10415,4451	59
2	9554,9868	9970,0546	9584,932[illegible]	10415,0678	58
3	9555,3151	9970,0060	9585,3091	10414,6908	57
4	9555,6433	9969,9574	9585,6858	10414,3141	56
5	9555,9711	9969,9087	9586,0623	10413,9376	55
6	9556,2986	9969,8600	9586,4386	10413,5613	54
7	9556,6259	9969,8112	9586,8146	10413,1853	53
8	9556,9528	9969,7624	9587,1904	10412,8095	52
9	9557,2795	9969,7135	9587,5659	10412,4340	51
10	9557,6059	9969,6647	9587,9412	10412,0587	50
11	9557,9320	9969,6157	9588,3163	10411,6836	49
12	9558,2579	9969,5667	9588,6911	10411,3088	48
13	9558,5835	9969,5177	9589,0657	10410,9342	47
14	9558,9087	9969,4686	9589,4401	10410,5599	46
15	9559,2337	9969,4195	9589,8142	10410,1858	45
16	9559,5585	9969,3704	9590,1880	10409,8119	44
17	9559,8829	9969,3212	6590,5617	10409,4383	43
18	9560,2071	9969,2720	9590,9351	10409,0649	42
19	9560,5309	9969,2227	9591,3082	10408,6917	41
20	9560,8546	9969,1734	9591,6811	10408,3188	40
21	9561,1779	9969,1240	9592,0538	10407,9461	39
22	9561,5009	9969,0746	9592,4263	10407,5736	38
23	9561,8237	9969,0252	9592,7985	10407,2014	37
24	9562,1462	9968,9757	9593,1705	10406,8294	36
25	9562,4684	9968,9261	9593,5422	10406,4577	35
26	9562,7904	9968,8766	9593,9137	10406,0862	34
27	9563,1120	9968,8270	9594,2850	10405,7149	33
28	9563,4334	9968,7773	9594,6561	10405,3438	32
29	9563,7565	9968,7276	9595,0269	10404,9730	31
30	9564,0754	9968,6779	9595,3975	10404,6024	30
		Sin. 68.		Tan. 68.	M

M	Sin. 21.		Tan. 21.		
30	9564,0754	9968,6779	9595,3975	10404,6024	30
31	9564,3960	9968,6281	9595,7678	10404,2321	29
32	9564,7163	9968,5783	9596,1380	10403,8619	28
33	9565,0363	9968,5284	9596,5079	10403,4920	27
34	9565,3560	9968,4785	9596,8776	10403,1224	26
35	9565,6755	9968,4285	9597,2470	10402,7529	25
36	9565,9947	9968,3785	9597,6162	10402,3837	24
37	9566,3137	9968,3285	9597,9852	10402,0147	23
38	9566,6324	9968,2784	9598,3539	10401,6460	22
39	9566,9508	9968,2283	9598,7224	10401,2775	21
40	9567,2689	9968,1781	9599,0907	10400,9092	20
41	9567,5867	9968,1279	9599,4588	10400,5412	19
42	9567,9043	9968,0776	9599,8267	10400,1733	18
43	9568,2217	9968,0274	9600,1943	10399,8057	17
44	9568,5387	9967,9770	9600,5617	10399,4383	16
45	9568,8555	9967,9266	9600,9288	10399,0711	15
46	9569,1720	9967,8762	9601,2958	10398,7041	14
47	9569,4883	9967,8257	9601,6625	10398,3374	13
48	9569,8042	9967,7752	9602,0290	10397,9709	12
49	9570,1200	9967,7247	9602,3952	10397,6047	11
50	9570,4354	9967,6741	9602,7613	10397,2386	10
51	9570,7506	9967,6235	9603,1271	10396,8728	9
52	9571,0655	9967,5728	9603,4927	10396,5072	8
53	9571,3802	9967,5221	9603,8581	10396,1418	7
54	9571,6946	9967,4713	9604,2232	10395,7767	6
55	9572,0087	9967,4205	9604,5881	10395,4118	5
56	9572,3225	9967,3697	9604,9529	10395,0471	4
57	9572,6362	9967,3188	9605,3174	10394,6826	3
58	9572,9495	9967,2678	9605,6816	10394,3183	2
59	9573,2626	9967,2168	9606,0457	10393,9542	1
60	9573,5754	9967,1658	9606,4095	10393,5904	0
		Sin. 68.		Tan. 68.	M

M	Sin.22.		Tan.22.		
0	9573,5754	9967,1658	9606,4095	10393,5904	60
1	9573,8879	9967,1147	9606,7731	10393,2268	59
2	9574,2002	9967,0636	9607,1365	10392,8634	58
3	9574,5122	9967,0125	9607,4997	10392,5002	57
4	9574,8240	9966,9613	9607,8627	10392,1372	56
5	9575,1355	9966,9101	9608,2254	10391,7745	55
6	9575,4468	9966,8588	9608,5879	10391,4120	54
7	9575,7578	9966,8075	9608,9502	10391,0497	53
8	9576,0685	9965,7561	9609,3123	10390,6876	52
9	9576,3789	9966,7047	9609,6742	10390,3257	51
10	9576,6892	9966,6533	9610,0359	10389,9641	50
11	9576,9991	9966,6018	9610,3973	10389,6026	49
12	9577,3088	9966,5502	9610,7585	10389,2414	48
13	9577,6182	9966,4987	9611,1195	10388,8804	47
14	9577,9274	9966,4470	9611,4803	10388,5196	46
15	9578,2363	9966,3954	9611,8409	10388,1590	45
16	9578,5450	9966,3437	9612,2013	10387,7986	44
17	9578,8534	9966,2919	9612,5614	10387,4385	43
18	9579,1616	9966,2401	9612,9214	10387,0785	42
19	9579,4695	9966,1883	9613,2811	10386,7188	41
20	9579,7771	9966,1364	9613,6407	10386,3593	40
21	9580,0845	9966,0845	9614,0000	10386,0000	39
22	9580,3916	9965,0325	9614,3591	10385,6409	38
23	9580,6985	9965,9805	9614,7179	10385,2820	37
24	9581,0051	9965,9285	9615,0766	10384,9233	36
25	9581,3115	9965,8764	9615,4351	10384,5648	35
26	9581,6176	9965,8243	9615,7933	10384,2066	34
27	9581,9235	9965,7721	9616,1514	10383,8485	33
28	9582,2291	9965,7199	9616,5092	10383,4907	32
29	9582,5345	9965,6676	9616,8669	10383,1331	31
30	9582,8396	9965,6153	9617,2243	10382,7756	30
		Sin.67.		Tang.67	M

M	Sin.22.		Tan.22.		
30	9582,8396	9965,6153	9617,2243	10382,7756	30
31	9583,1445	9965,5629	9617,5815	10382,4184	29
32	9583,4491	9965,5106	9617,9385	10382,0614	28
33	9583,7533	9965,4581	9618,2953	10381,7046	27
34	9584,0576	9965,4057	9618,6519	10381,3480	26
35	9584,3614	9995,3531	9619,0083	10380,9917	25
36	9584,6650	9965,3006	9619,3644	10380,6355	24
37	9584,9684	9965,2480	9619,7204	10380,2795	23
38	9585,2715	9965,1953	9620,0762	10379,9237	22
39	9585,5744	9965,1426	9620,4318	10379,5681	21
40	9585,8770	9965,0899	9620,7871	10379,2128	20
41	9586,1794	9965,0371	9621,1423	10378,8576	19
42	9586,4815	9964,9843	9621,4972	10378,5027	18
43	9586,7834	9964,9314	9621,8520	10378,1479	17
44	9587,0851	9964,8785	9622,2065	10377,7934	16
45	9587,3864	9964,8255	9622,5609	10377,4390	15
46	9587,6876	9964,7725	9622,9150	10377,0849	14
47	9587,9885	9964,7195	9623,2690	10376,7310	13
48	9588,2891	9964,6664	9623,6227	10376,3772	12
49	9588,5895	9964,6133	9623,9762	10376,0237	11
50	9588,8897	9964,5601	9624,3296	10375,6704	10
51	9589,1896	9964,5069	9624,6827	10375,3172	9
52	9589,4893	9964,4536	9625,0356	10374,9643	8
53	9589,7887	9964,4003	9625,3883	10374,6116	7
54	9590,0879	9964,3470	9625,7409	10374,2590	6
55	9590,3869	9964,2936	9626,0932	10373,9067	5
56	9590,6856	9964,2402	9626,4453	10373,5546	4
57	9590,9840	9964,1867	9626,7973	10373,2026	3
58	9591,2823	9964,1332	9627,1490	10372,8509	2
59	9591,5802	9964,0796	9627,5006	10372,4994	1
60	9591,8780	9964,0260	9627,8519	10372,1480	0
		Sin.67.		Tan.67.	M

M	Sin. 23.		Tan. 23.		
0	9591,8780	9964,0260	9627,8519	10372,1480	60
1	9592,1755	9963,9724	9628,2030	10371,7969	59
2	9592,4727	9963,9187	9628,5540	10371,4459	58
3	9592,7697	9963,8650	9628,9047	10371,0952	57
4	9593,0665	9963,8112	9629,2553	10370,7446	56
5	9593,3631	9963,7574	9629,6057	10370,3943	55
6	9593,6594	9963,7035	9629,9558	10370,0441	54
7	9593,9554	9963,6496	9630,3057	10369,6942	53
8	9594,2512	9963,5957	9630,6555	10369,3444	52
9	9594,5468	9963,5417	9631,0051	10368,9948	51
10	9594,8422	9963,4876	9631,3545	10368,6454	50
11	9595,1373	9963,4335	9631,7037	10368,2962	49
12	9595,4321	9963,3794	9632,0527	10367,9472	48
13	9595,7268	9963,3253	9632,4015	10367,5984	47
14	9596,0212	9963,2710	9632,7501	10367,2498	46
15	9596,3153	9963,2168	9633,0985	10366,9014	45
16	9596,6093	9963,1625	9633,4467	10366,5532	44
17	9596,9030	9963,1081	9633,7948	10366,2051	43
18	9597,1964	9963,0538	9634,1426	10365,8573	42
19	9597,4896	9962,9993	9634,4903	10365,5097	41
20	9597,7826	9962,9449	9634,8377	10365,1622	40
21	9598,0754	9962,8904	9635,1850	10364,8149	39
22	9598,3679	9962,8358	9635,5321	10364,4679	38
23	9598,6602	9962,7812	9635,8790	10364,1210	37
24	9598,9522	9962,7265	9636,2256	10363,7743	36
25	9599,2440	9962,6718	9636,5722	10363,4278	35
26	9599,5356	9962,6171	9636,9185	10363,0815	34
27	9599,8270	9962,5623	9637,2646	10362,7353	33
28	9600,1181	9962,5075	9637,6105	10362,3894	32
29	9600,4090	9962,4527	9637,9563	10362,0436	31
30	9600,6997	9962,3977	9638,3019	10361,6981	30
		Sin. 66.		Tan. 66.	M

M	Sin.23.		Tan.23.		
30	9600,6997	9962,3977	9638,3019	10361,6981	30
31	9600,9901	9962,3428	9638,6472	10361,3527	29
32	9601,2803	9962,2878	9638,9924	10261,0075	28
33	9601,5702	9962,2328	9639,3374	10360,6625	27
34	9601,8600	9962,1777	9639,6823	10360,3177	26
35	9602,1495	9962,1225	9640,0269	10359,9730	25
36	9602,4387	9962,0674	9640,3713	10359,6286	24
37	9602,7278	9962,0122	9640,7156	10359,2843	23
38	9603,0166	9961,9569	9641,0597	10358,9402	22
39	9603,3052	9961,9016	9641,4036	10358,5964	21
40	9603,5936	9961,8463	9641,7473	10358,2527	20
41	9603,8817	9961,7909	9642,0908	10357,9091	19
42	9604,1696	9961,7354	9642,4341	10357,5658	18
43	9604,4573	9961,6799	9642,7773	10357,2226	17
44	9604,7447	9961,6244	9643,1203	10356,8797	16
45	9605,0319	9961,5689	9643,4630	10356,5369	15
46	9605,3189	9961,5133	9643,8056	10356,1943	14
47	9605,6057	9961,4576	9644,1481	10355,8519	13
48	9605,8922	9961,4019	9644,4903	10355,5096	12
49	9606,1786	9961,3462	9644,8324	10355,1676	11
50	9606,4646	9961,2904	9645,1742	10354,8257	10
51	9606,7505	9961,2346	9645,5159	10354,4840	9
52	9607,0362	9961,1787	9645,8575	10354,1425	8
53	9607,3216	9961,1228	9646,1988	10353,8011	7
54	9607,6068	9961,0668	9646,5399	10353,4600	6
55	9607,8918	9961,0108	9646,8809	10353,1190	5
56	9608,1765	9960,9548	9647,2217	10352,7782	4
57	9608,4610	9960,8987	9647,5623	10352,4376	3
58	9608,7453	9960,8425	9647,9028	10352,0972	2
59	9609,0294	9960,7863	9648,2430	10351,7569	1
60	9609,3133	9960,7301	9648,5831	10351,4168	0
		Sin. 66.		Tan.66.	M

M	Sin.24.		Tan.24.		
0	9609,3133	9960,7301	9648,5831	10351,4168	60
1	9609,5969	9960,6738	9648,9230	10351,0769	59
2	9609,8803	9960,6175	9649,2627	10350,7372	58
3	9610,1635	9960,5612	9649,6023	10350,3976	57
4	9610,4465	9960,5048	9649,9416	10350,0583	56
5	9610,7292	9960,4483	9650,2808	10349,7191	55
6	9611,0117	9960,3919	9650,6198	10349,3801	54
7	9611,2940	9960,3353	9650,9587	10349,0412	53
8	9611,5751	9960,2787	9651,2974	10348,7026	52
9	9611,8589	9960,2221	9651,6358	10348,3641	51
10	9612,1396	9960,1655	9651,9742	10348,0258	50
11	9612,4211	9960,1087	9652,3123	10347,6876	49
12	9612,7023	9960,0520	9652,6503	10347,3497	48
13	9612,9833	9959,9952	9652,9881	10347,0119	47
14	9613,2641	9959,9383	9653,3257	10346,6742	46
15	9613,5446	9959,8815	9653,6631	10346,3368	45
16	9613,8249	9959,8245	9654,0004	10345,9995	44
17	9614,1051	9959,7675	9654,3375	10345,6624	43
18	9614,3850	9959,7105	9654,6744	10345,3255	42
19	9614,6646	9959,6535	9655,0111	10344,9888	41
20	9614,9441	9959,5964	9655,3477	10344,6522	40
21	9615,2234	9959,5392	9655,6841	10344,3158	39
22	9615,5024	9959,4820	9656,0203	10343,9796	38
23	9615,7812	9959,4248	9656,3564	10343,6435	37
24	9616,0598	9959,3675	9656,6923	10343,3077	36
25	9616,3382	9959,3102	9657,0280	10342,9719	35
26	9616,6164	9959,2528	9657,3635	10342,6364	34
27	9616,8943	9959,1954	9657,6989	10342,3010	33
28	9617,1721	9959,1379	9658,0341	10341,9658	32
29	9617,4496	9959,0804	9658,3692	10341,6307	31
30	9617,7269	9959,0229	9658,7040	10341,2959	30
		Sin.65.		Tan.65.	M

M	Sin.24.		Tan.24.		
30	9617,7269	9959,0229	9658,7040	10341,2959	30
31	9618,0040	9958,9653	9659,0387	10340,9612	29
32	9618,2809	9958,9076	9659,3732	10340,6267	28
33	9618,5576	9958,8500	9659,7076	10340,2923	27
34	9618,8340	9958,7922	9660,0418	10339,9581	26
35	9619,1103	9958,7344	9660,3758	10339,6241	25
36	9619,3863	9958,6766	9660,7097	10339,2903	24
37	9619,6622	9958,6188	9661,0434	10338,9566	23
38	9619,9378	9958,5609	9661,3769	10338,6230	22
39	9620,2132	9958,5029	9661,7102	10338,2897	21
40	9620,4884	9958,4449	9662,0434	10337,9565	20
41	9620,7633	9958,3869	9662,3764	10337,6235	19
42	9621,0381	9958,3288	9662,7093	10337,2906	18
43	9621,3127	9958,2707	9663,0420	10336,9579	17
44	9621,5870	9958,2125	9663,3745	10336,6254	16
45	9621,8612	9958,1543	9663,7068	10336,2931	15
46	9622,1351	9958,0960	9664,0390	10335,9609	14
47	9622,4088	9958,0377	9664,3711	10335,6289	13
48	9622,6823	9957,9794	9664,7029	10335,2970	12
49	9622,9556	9957,9210	9665,0346	10334,9653	11
50	9623,2287	9957,8625	9665,3661	10334,6338	10
51	9623,5016	9957,8040	9665,6975	10334,3024	9
52	9623,7743	9957,7455	9666,0287	10333,9712	8
53	9624,0467	9957,6869	9666,3598	10333,6401	7
54	9624,3190	9957,6283	9666,6906	10333,3093	6
55	9624,5911	9957,5697	9667,0214	10332,9786	5
56	9624,8629	9957,5109	9667,3519	10332,6480	4
57	9625,1345	9957,4522	9667,6823	10332,3175	3
58	9625,4060	9957,3934	9668,0125	10331,9874	2
59	9625,6772	9957,3340	9668,3426	10331,6573	1
60	9625,9482	9957,2757	9668,6725	10331,3274	0
		Sin.65.		Tan.65.	M

M	Sin. 25.		Tan. 25.		
0	9625,9482	9957,2757	9668,6725	10331,3274	60
1	9626,2190	9957,2167	9669,0023	10330,9976	59
2	9626,4897	9957,1578	9669,3319	10330,6681	58
3	9626,7601	9957,0987	9669,6613	10330,3386	57
4	9627,0303	9957,0397	9669,9905	10330,0094	56
5	9627,3002	9956,9806	9670,3196	10329,6803	55
6	9627,5700	9956,9214	9670,6486	10329,3513	54
7	9627,8396	9956,8622	9670,9774	10329,0225	53
8	9628,1090	9956,8030	9671,3060	10328,6939	52
9	9628,3782	9956,7437	9671,6345	10328,3655	51
10	9628,6471	9956,6843	9671,9628	10328,0371	50
11	9628,9159	9956,6250	9672,2909	10327,7090	49
12	9629,1845	9956,5655	9672,6189	10327,3810	48
13	9629,4529	9956,5061	9672,9468	10327,0532	47
14	9629,7210	9956,4465	9673,2744	10326,7255	46
15	9629,9890	9956,3870	9673,6020	10326,3980	45
16	9630,2567	9956,3274	9673,9293	10326,0706	44
17	9630,5243	9956,2677	9674,2565	10325,7434	43
18	9630,7917	9956,2080	9674,5836	10325,4163	42
19	9631,0588	9956,1483	9674,9105	10325,0894	41
20	9631,3258	9956,0885	9675,2372	10324,7627	40
21	9631,5925	9956,0278	9675,5638	10324,4361	39
22	9631,8591	9955,9688	9675,8902	10324,1097	38
23	9632,1254	9955,9089	9676,2165	10323,7834	37
24	9632,3916	9955,8489	9676,5426	10323,4573	36
25	9632,6575	9955,7889	9676,8689	10323,1313	35
26	9632,9233	9955,7289	9677,1944	10322,8055	34
27	9633,1888	9955,6688	9677,5200	10322,4799	33
28	9633,4542	9955,6086	9677,8455	10322,1544	32
29	9633,7194	9955,5484	9678,1709	10321,8290	31
30	9633,9843	9955,4882	9678,4961	10321,5038	30
		Sin. 64.		Tan. 64.	M

M	Sin. 25.		Tan. 25.		
30	9633,9843	9955,4882	9678,4961	10321,5038	30
31	9634,2491	9955,4279	9678,8211	10321,1788	29
32	9634,5136	9955,3676	9679,1460	10320,8539	28
33	9634,7780	9955,3072	9679,4707	10320,5292	27
34	9635,0422	9955,2468	9679,7953	10320,2046	26
35	9635,3061	9955,1864	9680,1197	10319,8802	25
36	9635,5699	9955,1259	9680,4440	10319,5559	24
37	9635,8335	9955,0653	9680,7682	10319,2318	23
38	9636,0969	9955,0047	9681,0921	10318,9078	22
39	9636,3600	9954,9441	9681,4159	10318,5840	21
40	9636,6230	9954,8834	9681,7396	10318,2603	20
41	9636,8858	9954,8226	9682,0631	10317,9368	19
42	9637,1484	9954,7619	9682,3865	10317,6134	18
43	9637,4108	9954,7011	9682,7097	10317,2902	17
44	9637,6730	9954,6402	9682,0328	10316,9671	16
45	9637,9350	9954,5793	9683,3557	10316,6442	15
46	9638,1968	9954,5183	9683,6785	10316,3214	14
47	9638,4585	9954,4573	9684,0011	10315,9988	13
48	9638,7199	9954,3963	9684,3236	10315,6763	12
49	9638,9811	9954,3352	9684,6459	10315,3540	11
50	9639,2422	9954,2740	9684,9681	10315,0318	10
51	9639,5030	9954,2128	9685,2901	10314,7098	9
52	9639,7636	9954,1516	9685,6120	10314,3879	8
53	9640,0241	9954,0903	9685,9337	10314,0662	7
54	9640,2844	9954,0290	9686,2553	10313,7446	6
55	9640,5444	9953,9677	9686,5767	10313,4232	5
56	9640,8043	9953,9062	9686,8980	10313,1019	4
57	9641,0640	9953,8448	9687,2192	10312,7807	3
58	9641,3235	9953,7833	9687,5402	10312,4597	2
59	9641,5828	9953,7217	9687,8610	10312,1389	1
60	9641,8419	9953,6601	9688,1817	10311,8182	0
		Sin. 64.		Tan. 64.	M

M	Sin.26		Tan.26.		
0	9641,8419	9953,6601	9688,1817	10311,8182	60
1	9642,1008	9953,5985	9688,5023	10311,4976	59
2	9642,3596	9953,5368	9688,8227	10311,1772	58
3	9642,6181	9953,4751	9689,1430	10310,8569	57
4	9642,8765	9953,4133	9689,4631	10310,5368	56
5	9643,1346	9953,3515	9689,7831	10310,2168	55
6	9643,3926	9953,2896	9690,1029	10309,8970	54
7	9643,6504	9953,2277	9690,4226	10309,5773	53
8	9643,9080	9953,1658	9690,7422	10309,2578	52
9	9644,1654	9953,1038	9691,0616	10308,9384	51
10	9644,4226	9953,0417	9691,3808	10308,6191	50
11	9644,6796	9952,9796	9691,6999	10308,3000	49
12	9644,9364	9952,9175	9692,0189	10307,9810	48
13	9645,1931	9952,8553	9692,3378	10307,6622	47
14	9645,4495	9952,7931	9692,6564	10307,3435	46
15	9645,7058	9952,7308	9692,9750	10307,0249	45
16	9645,9619	9952,6685	9693,2934	10306,7065	44
17	9646,2178	9952,6061	9693,6116	10306,3883	43
18	9646,4735	9952,5437	9693,9298	10306,0702	42
19	9646,7290	9952,4812	9694,2478	10305,7522	41
20	9646,9843	9952,4187	9694,5656	10305,4343	40
21	9647,2395	9952,3562	9694,8833	10305,1167	39
22	9647,4944	9952,2936	9695,2008	10304,7991	38
23	9647,7492	9952,2309	9695,5182	10304,4817	37
24	9648,0038	9952,1682	9695,8355	10304,1644	36
25	9648,2582	9952,1055	9696,1526	10303,8473	35
26	9648,5124	9952,0427	9696,4696	10303,5303	34
27	9648,7664	9951,9799	9696,7865	10303,2134	33
28	9649,0203	9951,9170	9697,1032	10302,8967	32
29	9649,2739	9951,8541	9697,4198	10302,5801	31
30	9649,5274	9951,7911	9697,7362	10302,2637	30
		Sin.63.		Tang.63	M

M	Sin.26.		Tan.26.		
30	9649,5274	9951,7911	9697,7362	10302,2637	30
31	9609,7807	9951,7281	9698,0525	10301,9474	29
32	9650,0338	9951,6651	9698,3687	10301,6312	28
33	9650,2867	9951,6020	9698,6847	10301,3152	27
34	9650,5394	9988,5388	9699,0006	10300,9993	26
35	9650,7920	9951,4756	9699,3163	10300,6836	25
36	9651,0444	9951,4124	9699,6319	10300,3680	24
37	9651,2965	9951,3491	9699,9474	10300,0525	23
38	9651,5485	9951,2858	9700,2627	10299,7372	22
39	9651,8004	9951,2224	9700,5779	10299,4220	21
40	9652,0520	9951,1590	9700,8930	10299,1069	20
41	9652,3035	9951,0955	9701,2079	10298,7920	19
42	9652,5547	9951,0320	9701,5227	10298,4773	18
43	9652,8058	9950,9684	9701,8373	10298,1626	17
44	9653,0567	9950,9048	9702,1518	10297,8481	16
45	9653,3075	9950,8412	9702,4663	10297,5337	15
46	9653,5580	9950,7775	9702,7805	10297,2194	14
47	9653,8084	9950,7137	9703,0946	10296,9053	13
48	9654,0586	9950,6499	9703,4086	10296,5913	12
49	9654,3086	9950,5861	9703,7224	10296,2775	11
50	9654,5584	9950,5222	9704,0361	10295,9638	10
51	9654,8080	9950,4583	9704,3497	10295,6502	9
52	9655,0575	9950,3943	9704,6631	10295,3368	8
53	9655,3068	9950,3303	9704,9764	10295,0235	7
54	9655,5559	9950,2662	9705,2896	10294,7103	6
55	9655,8048	9950,2021	9705,6026	10294,3973	5
56	9656,0535	9950,1380	9705,9155	10294,0844	4
57	9656,3021	9950,0737	9706,2283	10293,7716	3
58	9656,5505	9950,0095	9706,5410	10293,4 90	2
59	9656,7987	9949,9452	9706,8535	10293,1464	1
60	9657,0467	9949,8808	9707,1658	10292,8341	0
		Sin.63.		Tan.63.	M

M	Sin. 27.		Sin. 27.		
0	9657.0467	9949,8808	9707,1658	10292,8341	60
1	9657.2946	9949,8165	9707,4781	10292.5218	59
2	9657.5422	9949,7520	9707,7902	10292.2097	58
3	9657.7897	9949,6875	9708.1022	10291,8977	57
4	9658.0371	9949,6230	9708.4140	10291,5859	56
5	9658.2842	9949,5584	9708,7257	10291.2742	55
6	9658,5312	9949,4938	9709,0373	10290,9626	54
7	9658,7779	9949.4291	9709,3488	10290,6511	53
8	9659.0246	9949,3644	9709.6601	10290.3398	52
9	9659,2710	9949.2996	9709.9713	10290,0286	51
10	9659.5172	9949 2348	9710.2824	10289,7176	50
11	9659.7633	9949,1700	9710,5933	10289,4066	49
12	9660,0092	9949.1051	9710,9041	10289,0958	48
13	9660.2549	9949.0401	9711,2148	10288.7851	47
14	9960.5005	9948.9751	9711,5253	10288.4746	46
15	9660,7459	9948,9101	9711.8357	10288.1642	45
16	9660.9911	9948.8450	9712.1460	10287.8539	44
17	9661,2361	9948.7799	9712,4562	10287,5438	43
18	9661,4809	9948,7147	9712,7662	10287,2337	42
19	9661,7256	9948.6495	9713.0761	10286,9238	41
20	9661,9701	9948,5842	9713.3859	10286,6140	40
21	9662,2144	9948,5189	9713,6955	10286,3044	39
22	9662,4586	9948.4535	9714,0051	10285,9949	38
23	9662,7026	9948,3881	9714.3144	10285.6855	37
24	9662.9464	9948.3226	9714.6237	10285,3762	36
25	9663.1900	9948.2571	9714,9329	10285,0671	35
26	9663,4335	9948,1916	9715.2419	10284,7581	34
27	9663,6767	9948.1260	9715,5507	10284.4492	33
28	9663,9199	9948.0603	9715,8595	10284,1404	32
29	9664.1628	9947,9946	9716.1681	10243.8318	31
30	9664,4056	9947.9289	9716,4766	10283,5233	30
		Tan. 62.		Tan. 62.	M

M	Sin.27.		Tan.27.		
30	9664,4056	9947,9289	9716,4766	10283,5233	30
31	9664,6482	9947,8631	9716,7850	10283,2149	29
32	9664,8906	9947,7973	9717,0933	10282,9060	28
33	9665,1328	9947,7314	9717,4014	10282,5985	27
34	9665,3749	9947,6655	9717,7094	10282,2905	26
35	9665,6168	9947,5995	9718,0173	10281,9826	25
36	9665,8585	9947,5335	9718,3250	10281,6749	24
37	9666,1001	9947,4674	9718,6327	10281,3672	23
38	9666,3415	9947,4013	9718,9402	10281,0597	22
39	9666,5827	9947,3351	9719,2476	10280,7523	21
40	9666,8238	9947,2689	9719,5548	10280,4451	20
41	9667,0647	9947,2027	9719,8620	10280,1380	19
42	9667,3054	9947,1364	9720,1690	10279,8309	18
43	9667,5459	9947,0700	9720,4759	10279,5240	17
44	9667,7863	9947,0036	9720,7826	10279,2173	16
45	9668,0265	9946,9372	9721,0893	10278,9106	15
46	9668,2665	9946,8707	9721,3958	10278,6041	14
47	9668,5064	9946,8041	9721,7022	10278,2977	13
48	9668,7461	9946,7376	9722,0085	10277,9914	12
49	9668,9856	9946,6709	9722,3146	10277,6853	11
50	9669,2249	9946,6043	9722,6200	10277,3793	10
51	9669,4641	9946,5375	9722,9266	10277,0734	9
52	9669,7031	9946,4708	9723,2324	10276,7676	8
53	9669,9420	9946,4039	9723,5380	10276,4619	7
54	9670,1807	9946,3371	9723,8436	10276,1563	6
55	9670,4192	9946,2702	9724,1490	10275,8509	5
56	9670,6575	9946,2032	9724,4543	10275,5456	4
57	9670,8957	9946,1362	9724,7595	10275,2404	3
58	9671,1337	9946,0691	9725,0646	10274,9354	2
59	9671,3716	9946,0020	9725,3695	10274,6304	1
60	9671,6093	9945,9349	9725,6743	10274,3256	0
		Sin.62.		Tan.62.	M

M	Sin.28.		Tan.29.		
0	9671,6093	9945,9349	9725,6743	10274,3256	60
1	9671,8468	9945,8677	9725,9790	10274,0209	59
2	9672,0841	9945,8005	9726,2836	10273,7163	58
3	9672,3213	9945,7332	9726,5881	10273,4118	57
4	9672,558[illegible]	9945,6658	9726,8924	10273,1075	56
5	9672,7952	9945,5984	9727,1967	10272,8032	55
6	9673,0318	9945,5310	9727,5008	10272,4991	54
7	9673,2683	9945,4635	9727,8048	10272,1951	53
8	9673,5047	9945,3960	9728,1087	10271,8913	52
9	9673,7409	9945,3284	9728,4124	10271,5875	51
10	9673,9769	9945,2608	9728,7160	10271,2839	50
11	9674,2128	9945,1932	9729,0196	10270,9804	49
12	9674,4484	9945,1254	9729,3230	10270,6770	48
13	9674,6840	9945,0577	9729,6262	10270,3737	47
14	9674,9193	9944,9899	9729,9294	10270,0705	46
15	9675,1545	9944,9220	9730,2325	10269,7674	45
16	9675,3895	9944,8541	9730,5354	10269,4645	44
17	9675,6244	9944,7862	9730,8382	10269,1617	43
18	9675,8591	9944,7182	9731,1409	10268,8590	42
19	9676,0937	9944,6501	9731,4435	10268,5564	41
20	9676,3280	9944,5820	9731,7460	10268,2539	40
21	9676,5622	9944,5139	9732,0483	10267,9516	39
22	9676,7963	9944,4457	9732,3506	10267,6493	38
23	9677,0302	9944,3774	9732,6527	10267,3472	37
24	9677,2639	9944,3092	9732,9547	10267,0452	36
25	9677,4975	9944,2408	9733,2566	10266,7433	35
26	9677,7309	9944,1724	9733,5584	10266,4415	34
27	9677,9641	9944,1040	9733,8601	10266,1399	33
28	9678,1972	9944,0356	9734,1616	10265,8383	32
29	9678,4301	9943,9670	9734,4630	10265,5369	31
30	9678,6629	9943,8985	9734,7644	10265,2356	30
		Sin.61.		Tang.61	M

M	Sin. 28.		Tan. 28.		
30	9678,6629	9943,8985	9734,7644	10265,2356	30
31	9678,8955	9943,8298	9735,0656	10264,9343	29
32	9679,1279	9943,7612	9735,3667	10264,6333	28
33	9679,3602	9943,6925	9735,6676	10264,3323	27
34	9679,5923	9943,6237	9735,9685	10264,0314	26
35	9679,8242	9943,5549	9736,2693	10263,7306	25
36	9680,0560	9943,4861	9736,5699	10263,4300	24
37	9680,2876	9943,4172	9736,8704	10263,1295	23
38	9680,5191	9943,3482	9737,1709	10262,8291	22
39	9680,7504	9943,2792	9737,4712	10262,5288	21
40	9680,9816	9943,2102	9737,7714	10262,2286	20
41	9681,2125	9943,1411	9738,0714	10261,9285	19
42	9681,4434	9943,0719	9738,3714	10261,6285	18
43	9681,6740	9943,0027	9738,6713	10261,3287	17
44	9681,9045	9942,9335	9738,9710	10261,0289	16
45	9682,1349	9942,8642	9739,2706	10260,7293	15
46	9682,3651	9942,7949	9739,5702	10260,4298	14
47	9682,5951	9942,7255	9739,8696	10260,1304	13
48	9682,8250	9942,6561	9740,1689	10259,8310	12
49	9683,0547	9942,5866	9740,4681	10259,5318	11
50	9683,2843	9942,5171	9740,7672	10259,2328	10
51	9683,5137	9942,4475	9741,0661	10258,9338	9
52	9683,7429	9942,3779	9741,3650	10258,6349	8
53	9683,9720	9942,3082	9741,6637	10258,3362	7
54	9684,2009	9942,2385	9741,9624	10258,0375	6
55	9684,4297	9942,1688	9742,2609	10257,7390	5
56	9684,6583	9942,0989	9742,5593	10257,4406	4
57	9684,8868	9942,0291	9742,8576	10257,1423	3
58	9685,1151	9941,9592	9743,1559	10256,8441	2
59	9685,3432	9941,8892	9743,4540	10256,5460	1
60	9685,5712	9941,8192	9743,7520	10256,1480	0
		Sin. 45.		Tan. 28.	M

M	Sin. 29.		Tan. 29.		
0	9685,5712	9941,8192	9743,7519	10256,2480	60
	9685,7990	9941,7492	9744,0498	10255,9501	59
2	9686,0267	9941,6791	9744,3476	10255,6523	58
3	9686,2542	9941,6089	9744,6452	10255,3547	57
4	9686,4816	9941,5387	9744,9428	10255,0571	56
5	9686,7088	9941,4685	9745,2403	10254 7597	55
6	9686,9358	9941,3982	9745,5376	10254,4623	54
7	9687,1627	9941,3279	9745,8348	10254,1651	53
8	9687,3895	9941,2575	9746,1319	10253,8680	52
9	9687,6160	9941,1870	9746,4290	10253,5709	51
10	9687,8425	9941,1165	9746,7259	10253,2740	50
11	9688,0687	9941,0460	9747,0227	10252,9772	49
12	9688,2949	9940,9754	9747,3194	10252,6805	48
13	9688,5208	9940,9048	9747,6160	10252,3839	47
14	9688,7466	9940,8341	9747,9125	10252,0874	46
15	9688,9723	9940,7634	9748,2089	10251,7911	45
16	9689,1978	9940,6926	9748,5051	10251,4948	44
17	9689,4232	9940,6218	9748,8013	10251,1986	43
18	9689,6484	9940,5509	9749,0974	10250,9025	42
19	9689,8734	9940,4800	9749,3933	10250,6066	41
20	9690,0983	9940,4091	9749,6892	10250,3107	40
21	9690,3230	9940,3380	9749,9849	10250,0150	39
22	9690,5476	9940,2670	9750,2806	10249,7193	38
23	9690,7720	9940,1959	9750,5761	10249,4238	37
24	9690,9963	9940,1247	9750,8716	10249,1284	36
25	9691,2204	9940,0535	9751,1669	10248,8330	35
26	9691,4444	9939,9822	9751,4621	10248,5378	34
27	9691,6682	9939,9109	9751,7573	10248,2427	33
28	9691,8919	9939,8396	9752,0523	10247,9476	32
29	9692,1154	9939,7682	9752,3472	10247,6527	31
30	9692,3388	9939,6967	9752,6420	10247,3579	30
		Sin. 60.		Tan. 60.	M

M	Sin. 29.		Tan. 29.		
30	9692,3388	9939,6967	9752,6420	10247,3579	30
31	9692,5620	9939,6252	9752,9367	10247,0632	29
32	9692,7851	9939,5537	9753,2313	10246,7686	28
33	9693,0080	9939,4821	9753,5258	10246,4741	27
34	9693,2307	9939,4104	9753,8203	10246,1797	26
35	9693,4533	9939,3388	9754,1146	10245,8854	25
36	9693,6758	9939,2670	9754,4087	10245,5912	24
37	9693,8981	9939,1952	9754,7028	10245,2971	23
38	9694,1203	9939,1234	9754,9968	10245,0031	22
39	9694,3423	9939,0515	9755,2907	10244,7092	21
40	9694,5641	9938,9796	9755,5845	10244,4154	20
41	9694,7858	9938,9076	9755,8782	10244,1217	19
42	9695,0074	9938,8355	9756,1718	10243,8281	18
43	9695,2288	9938,7635	9756,4653	10243,5346	17
44	9695,4501	9938,6913	9756,7587	10243,2412	16
45	9695,6712	9938,6191	9757,0520	10242,9479	15
46	9695,8921	9938,5469	9757,3452	10242,6548	14
47	9696,1129	9938,4746	9757,6383	10242,3617	13
48	9696,3336	9938,4023	9757,9312	10242,0687	12
49	9696,5541	9938,3299	9758,2241	10241,7758	11
50	9696,7745	9938,2575	9758,5169	10241,4830	10
51	9696,9947	9938,1850	9758,8096	10241,1903	9
52	9697,2148	9938,1125	9759,1022	10240,8977	8
53	9697,4347	9938,0400	9759,3947	10240,6052	7
54	9697,6544	9937,9673	9759,6871	10240,3128	6
55	9697,8741	9937,8947	9759,9794	10240,0206	5
56	9698,0935	9937,8220	9760,2715	10239,7284	4
57	9698,3129	9937,7492	9760,5636	10239,4363	3
58	9698,5320	9937,6764	9760,8556	10239,1443	2
59	9698,7511	9937,6035	9761,1475	10238,8524	1
60	9698,9700	9937,5306	9761,4393	10238,5606	0
		Sin. 60.		Tan. 60.	M

M	Sin.30.		Tan.30.		
0	9698,9700	9937,5306	9761,4393	10238,5606	60
1	9699,1887	9937,4576	9761,7310	10238,2689	59
2	9699,4073	9937,3845	9762,0226	10237,9773	58
3	9699,6257	9937,3116	9762,3141	10237,6858	57
4	9699,8440	9937,2385	9762,6056	10237,3944	56
5	9700 0622	9937,1653	9762,8969	10237,1031	55
6	9700,2802	9937,0921	9763,1881	10236,8118	54
7	9700,4981	9937,0188	9763,4792	10236,5207	53
8	9700,7158	9936,9455	9763,7702	10236,2297	52
9	9700,9333	9936,8722	9764 0611	10235,9388	51
10	9701,1508	9936,7988	9764,3520	10235,6480	50
11	9701,3680	9936,7253	9764,6427	10235,3572	49
12	9701,5852	9936,6518	9764,9333	10235,0666	48
13	9701,8021	9936,5783	9765,2238	10234,7761	47
14	9702,0190	9936,5047	9765,5143	10234,4856	46
15	9702,2357	9936,4310	9765,8046	10234,1953	45
16	9702,4522	9936,3573	9766,0949	10233,9050	44
17	9702,6686	9936,2836	9766,3850	10233,6149	43
18	9702,8849	9936,2098	9766,6751	10233,3248	42
19	9703,1010	9936,1359	6766,9651	10233,0349	41
20	9703,3170	9936,0620	9767,2549	10232,7450	40
21	9703,5328	9935,9881	9767,5447	10232,4552	39
22	9703,7485	9935,9141	9767,8344	10232,1655	38
23	9703,9641	9935,8400	9768,1240	10231,8759	37
24	9704,1795	9935,7659	9768,4135	10231,5864	36
25	9704,3947	9935,6918	9768,7029	10231,2971	35
26	9704,6098	9935,6176	9768,9922	10231,0078	34
27	9704,8248	9935,5434	9769,2814	10230,7185	33
28	9705,0396	9935,4691	9769,5705	10230,4294	32
29	9705,2543	9935,3947	9769,8595	10230,1404	31
30	9705,4688	9935,3203	9770,1485	10229,8515	30
M		Sin.59.		Tan.59.	M

M	Sin. 30.		Tan. 30.		
30	9705,4688	9935,3203	9770,1485	10229,8515	30
31	9705,6832	9935,2459	9770,4373	10229,5626	29
32	9705,8975	9935,1714	9770,7260	10229,2732	28
33	9706,1116	9935,0959	9771,0147	10228,9852	27
34	9706,3256	9935,0223	9771,3032	10228,6967	26
35	9706,5394	9934,9477	9771,5917	10228,4082	25
36	9706,7531	9934,8730	9771,8801	10228,1198	24
37	9706,9666	9934,7982	9772,1684	10227,8316	23
38	9707,1800	9934,7234	9772,4565	10227,5434	22
39	9707,3933	9944,6486	9772,7446	10227,2553	21
40	9707,6064	9934,5737	9773,0327	10226,9673	20
41	9707,8194	9934,4988	9773,3206	10226,6794	19
42	9708,0322	9934,4238	9773,6084	10226,3915	18
43	9708,2449	9934,3488	9773,8961	10226,1038	17
44	9708,4575	9934,2737	9774,1838	10225,8162	16
45	9708,6699	9934,1986	9774,4713	10225,5286	15
46	9708,8822	9934,1233	9774,7588	10225,2411	14
47	9709,0943	9934,0481	9775,0461	10224,9538	13
48	9709,3063	9933,9728	9775,3334	10224,6665	12
49	9709,5181	9933,8975	9775,6206	10224,3793	11
50	9709,7298	9933,8221	9775,9077	10224,0922	10
51	9709,9414	9933,7467	9776,1947	10223,8052	9
52	9710,1528	9933,6712	9776,4816	10223,5183	8
53	9710,3641	9933,5957	9776,7684	10223,2315	7
54	9710,5753	9933,5201	9777,0552	10222,9448	6
55	9710,7863	9933,4445	9777,3418	10222,6581	5
56	9710,9972	9933,3688	9777,6284	10222,3715	4
57	9711,2079	9933,2931	9777,9148	10222,0851	3
58	9711,4185	9933,2173	9778,2012	10221,7987	2
59	9711,6290	9933,1414	9778,4875	10221,5124	1
60	9711,8393	9933,0656	9778,7737	10221,2262	0
		Sin. 59.		Tan. 59.	M

M	Sin. 31.		Tang. 31.		
0	9711,8393	9933,0656	9778.7737	10221,2262	60
1	9712.0495	9932,9896	9779.0598	10220.9401	59
2	9712,2595	9932.9136	9779,3458	10220,6541	58
3	9712.4694	9932,8376	9779.6318	10220,3681	57
4	9712,6792	9932.7615	9779.9176	10220,0823	56
5	9712,8888	9932.6854	9780.2034	10219,7965	55
6	9713,0983	9932.6092	9780,4891	10219.5109	54
7	9713.3077	9932,5330	9780.7746	10219,2253	53
8	9713.5169	9932.4567	9781.0602	10218,9398	52
9	9713,7260	9932,3804	9781,3456	10218,6544	51
10	9713,9349	9932,3040	9781,6309	10218,3670	50
11	9714,1437	9932,2275	9781.9161	10218,0838	49
12	9714.3524	9932.1511	9782,2013	10217.7989	48
13	9714,5609	9932,0745	9782,4863	10217.5136	47
14	9714,7693	9931,9979	9782.7713	10217.2286	46
15	9714,9775	9931,9213	9783.0562	10216.9437	45
16	9715,1857	9931,8446	9783,3410	10216,6589	44
17	9715.3936	9931.7679	9783,6257	10216,3742	43
18	9715.6015	9931,6911	9783.9103	10216.0896	42
19	9715.8092	9931.6143	9784,1949	10215,8050	41
20	9716.0168	9931.5374	9784,4794	10215.5205	40
21	9716.2242	9931.4604	9784,7637	10215,2362	39
22	9716.4315	9931,3835	9785.0480	10214,9519	38
23	9716,6387	9931.3064	9785.3322	10214.6677	37
24	9716.8457	9931,2293	9785,6163	10214,3836	36
25	9717,0526	9931.1522	9785.9004	10214.0995	35
26	9717,2594	9931.0750	9786.1843	10213,8156	34
27	9717.4660	9930.9978	9786,4682	10213,5317	33
28	9717.6725	9930,9205	9786.7520	10213.2479	32
29	9717,8788	9930,8431	9787.0357	10212,9642	31
30	9718,0851	9930.7657	9787.3193	10212,6806	30
		Tan. 58.		Sin. 58.	M

M	Sin. 31.		Tan. 31.		
30	9718,0851	9930,7657	9787,3193	10212,6806	30
31	9718,2912	9930,6883	9787,6028	10212,3971	29
32	9718,4971	9930,6108	9787,8863	10212,1137	28
33	9718,7029	9930,5333	9788,1696	10211,8303	27
34	9718,9086	9930,4557	9788,4529	10211,5470	26
35	9719,1142	9930,3780	9788,7361	10211,2638	25
36	9719,3196	9930,3003	9789,0192	10210,9807	24
37	9719,5249	9930,2226	9789,3022	10210,6977	23
38	9719,7300	9930,1448	9789,5852	10210,4147	22
39	9719,9350	9930,0670	9789,8680	10210,1319	21
40	9720,1399	9929,9891	9790,1508	10209,8491	20
41	9720,3446	9929,9111	9790,4335	10209,5664	19
42	9720,5493	9929,8331	9790,7161	10209,2838	18
43	9720,7537	9929,7551	9790,9986	10209,0013	17
44	9720,9581	9929,6770	9791,2811	10208 7188	16
45	9721,1623	9929,5988	9791,5634	10208,4365	15
46	9721,3664	9929,5206	9791,8457	10208,1542	14
47	9721'5703	9929,4424	9792,1279	10207,8720	13
48	9721,7741	9929,3641	9792,4101	10207,5899	12
49	9721,9778	9929,2857	9792,6921	10207,3078	11
50	9722,1814	9929,2073	9792,9741	10207,0259	10
51	9722,3848	9929,1288	9793,2559	10206,7440	9
52	9722,5881	9929,0503	9793,5377	10206,4622	8
53	9722,7913	9928,9718	9793,8194	10206,1805	7
54	9722,9943	9928,8932	9794,1011	10205,8988	6
55	9723,1972	9928,8145	9794,3826	10205,6173	5
56	9723,3999	9928,7358	9794,6641	10205,3358	4
57	9723,6026	9928,6570	9794,9455	10205,0544	3
58	9723,8051	9928,5782	9795,2268	10204,7731	2
59	9724,0074	9928,4994	9795,5080	10204,4919	1
60	9724,2097	9928,4204	9795,7892	10204,2107	0
		Sin. 58.		Tan. 58.	M

M	Sin.32		Tan.32.		
0	9724,2097	9928,4204	9795,7892	10204,2107	60
1	9724,4118	9928,3415	9796,0703	10203,9297	59
2	9724,6138	9928,2625	9796,3513	10203,6487	58
3	9724,8156	9928,1834	9796,6322	10203,3678	57
4	9725,0173	9928,1043	9796,9130	10203,0869	56
5	9725,2189	9928,0251	9797,1938	10202,8062	55
6	9725,4203	9927,9459	9797,4744	10202,5255	54
7	9725,6217	9927,8666	9797,7550	10202,2449	53
8	9725,8229	9927,7873	9798,0356	10201,9644	52
9	9726,0239	9927,7079	9798,3160	10201,6839	51
10	9726,2249	9927,6285	9798,5964	10201,4036	50
11	9726,4257	9927,5490	9798,8766	10201,1233	49
12	9726,6263	9927,4695	9799,1568	10200,8431	48
13	9726,8269	9927,3899	9799,4370	10200,5629	47
14	9727,0273	9927,3103	9799,7170	10200,2829	46
15	9727,2276	9927,2306	9799,9970	10200,0029	45
16	9727,4278	9927,1508	9800,2769	10199,7230	44
17	9727,6278	9927,0711	9800,5567	10199,4432	43
18	9727,8277	9926,9912	9800,8364	10199,1635	42
19	9728,0275	9926,9113	9801,1161	10198,8838	41
20	9728,2271	9926,8314	9801,3957	10198,6042	40
21	9728,4266	9926,7514	9801,6752	10198,3247	39
22	9728,6260	9926,6713	9801,9546	10198,0453	38
23	9728,8253	9926,5913	9802,2340	10197,7659	37
24	9729,0244	9926,5111	9802,5132	10197,4867	36
25	9729,2234	9926,4309	9802,7924	10197,2075	35
26	9729,4223	9926,3507	9803,0716	10196,9283	34
27	9729,6210	9926,2704	9803,3506	10196,6403	33
28	9729,8196	9926,1900	9803,6296	10196,3703	32
29	9730,0181	9926,1096	9803,9085	10196,0914	31
30	9730,2165	9926,0292	9804,1873	10195,8126	30
		Sin.57.		Tang.75	M

M	Sin.32.		Tan.32.		
30	9730,2165	9926,0292	9804,1873	10195,8126	30
31	9730,4147	9925,9486	9804,4660	10195,5339	29
32	9730,6128	9925,8681	9804,7447	10195,2552	28
33	9730,8108	9925,7875	9805,0233	10194,9766	27
34	9731,0087	9925,7068	9805,3018	10194,6981	26
35	9731,2064	9925,6261	9805,5803	10194,4196	25
36	9731,4040	9925,5453	9805,8586	10194,1413	24
37	9731,6015	9925,4645	9806,1369	10193,8630	23
38	9731,7988	9925,3836	9806,4151	10193,4848	22
39	9731,9960	9925,3027	9806,6933	10193,3066	21
40	9732,1931	9925,2118	9806,9714	10193,0285	20
41	9732,3901	9925,1407	9807,2494	10192,7506	19
42	9732,5870	9925,0596	9807,5273	10192,4726	18
43	9732,7837	9924,9785	9807,8051	10192,1948	17
44	9732,9803	9924,8973	9808,0829	10191,9170	16
45	9733,1767	9924,8161	9808,3606	10191,6393	15
46	9733,3731	9924,7348	9808,6382	10191,3617	14
47	9733,5693	9924,6535	9808,9158	10191,0842	13
48	9733,7654	9924,5721	9809,1932	10190,8067	12
49	9733,9613	9924,4906	9809,4707	10190,5293	11
50	9734,1572	9924,4092	9809,7480	10190,2519	10
51	9734,3529	9924.3276	9810,0252	10189,9747	9
52	9734,5485	9924,2460	9810,3024	10189,6975	8
53	9734,7440	9924,1644	9810,5796	10189,4204	7
54	9734,9393	9924,0827	9810,8566	10189,1433	6
55	9735,1345	9924,0009	9811,1336	10188,8663	5
56	9735,3296	9923,9191	9811,4105	10188,5894	4
57	9735,5246	9923,8372	9811,6873	10188,3126	3
58	9735,7194	9923,7553	9811,9640	10188,0359	2
59	9735,9141	9923,6734	9812,2407	10187,7592	1
60	9736,1087	9923,5914	9812,5173	10187,4826	0
		Sin.57.		Tan.57.	M

M	Sin. 33.		Tan. 33.		
0	9736,1087	9923,5914	9822,5173	10187,4826	60
1	9736,3032	9923,5093	9812,7939	10187,2061	59
2	9736,4975	9923,4272	9813,0703	10186,9296	58
3	9736,6918	9923,3450	9813,3467	10186,6532	57
4	9736,8859	9923,2628	9813,6231	10186,3769	56
5	9737,0798	9923,1805	9813,8993	10186,1006	55
6	9737,2737	9923,0982	9814,1755	10185,8244	54
7	9737,4674	9923,0158	9814,4516	10185,5483	53
8	9737,6610	9922,9334	9814,7276	10185,2723	52
9	9737,8545	9922,8509	9815,0036	10184,9963	51
10	9738,0479	9922,7683	9815,2795	10184,7204	50
11	9738,2411	9922,6858	9815,5553	10184,4446	49
12	9738,4342	9922,6031	9815,8311	10184,1688	48
13	9738,6272	9922,5204	9816,1068	10183,8931	47
14	9738,8201	9922,4377	9816,3824	10183,6175	46
15	9739,0128	9922,3549	9816,6579	10183,3420	45
16	9739,2055	9922,2720	9816,9334	10183,0665	44
17	9739,3980	9922,1891	9817,2088	10182,7911	43
18	9739,5904	9922,1062	9817,4842	10182,5158	42
19	9739,7826	9922,0231	9817,7594	10182,2405	41
20	9739,9748	9921,9401	9818,0346	10181,9653	40
21	9740,1668	9921,8570	9818,3098	10181,6902	39
22	9740,3587	9921,7738	9818,5848	10181,4151	38
23	9740,5504	9921,6906	9818,8598	10181,1401	37
24	9740,7421	9921,6073	9819,1347	10180,8652	36
25	9740,9336	9921,5240	9819,4095	10180,5903	35
26	9741,1250	9921,4406	9819,6844	10180,3155	34
27	9741,3163	9921,3572	9819,9591	10180,0408	33
28	9741,5075	9921,2737	9820,2338	10179,7662	32
29	9741,6985	9921,1901	5820,5084	10179,4916	31
30	9741,8895	9921,1066	9820,7829	10179,2171	30
				Tan. 46.	M

M	Sin. 33.		Tan. 33.		
30	9741,8895	9921,1066	9820,7829	10179,2171	30
31	9742,0803	9921,0229	9821,0573	10178,9426	29
32	9742,2710	9920,9392	9821,3317	10178,6682	28
33	9742,4615	9920,8555	9821,6060	10178,3939	27
34	9742,6520	9920,7717	9821,8803	10178,1197	26
35	9742,8423	9920,6878	9822,1544	10177,8455	25
36	9743,0325	9920,6039	9822,4286	10177,5714	24
37	9743,2226	9920,5199	6822,7026	10177,2973	23
38	9743,4125	9920,4359	9822,9766	10177,0233	22
39	9743,6024	9920,3519	9823,2505	10176,7494	21
40	9743,7921	9920,2677	9823,5243	10176,4756	20
41	9743,9817	9920,1836	9823,7981	10176,2018	19
42	9744,1712	9920,0993	9824,0718	10175,9281	18
43	9744,3606	9920,0151	9824,3455	10175,6545	17
44	9744,5498	9919,9307	9824,6190	10175,3809	16
45	9744,7389	9919,8465	9824,8925	10175,1074	15
46	9744,9279	9919,7619	9825,1660	10174,8339	14
47	9745,1168	9919,6774	9825,4394	10174,5605	13
48	9745,3056	9919,5929	9825,7127	10174,2872	12
49	9745,4942	9919,5083	9825,9859	10174,0140	11
50	9745,6828	9919,4236	9826,2591	10173,7408	10
51	9745,8712	9919,3389	9826,5322	10173,4677	9
52	9746,0595	9919,2542	9826,8053	10173,1945	8
53	9746,2477	9919,1693	9827,0783	10172,9216	7
54	9746,4357	9919,0845	9827,3512	10172,6487	6
55	9746,6237	9918,9996	9827,6241	10172,3758	5
56	9746,8115	9918,9146	9827,8969	10172,1030	4
57	9746,9992	9918,8296	9828,1696	10171,8303	3
58	9747,1868	9918,7445	9828,4423	10171,5576	2
59	9747,3743	9918,6594	9828,7149	10171,2850	1
60	9747,5616	9918,5742	9828,9874	10171,0125	0
		Sin. 56.		Tan. 56.	M

M	Sin. 34.		Tan. 34.		
0	9747,5616	9918,5742	9828,9874	10171,0125	60
1	9747,7489	9918,4889	9829,2599	10170,7400	59
2	9747,9360	9918,4036	9829,5323	10170,4676	58
3	9748,1230	9918,3183	9829,8046	10170,1953	57
4	9748,3099	9918,2329	5830,0769	10169,9230	56
5	9748,4966	9918,1474	9830,3491	10169,6508	55
6	9748,6833	9918,0619	9830,6213	10169,3786	54
7	9748,8698	9917,9764	9830,8934	10169,1065	53
8	9749,0562	9917,8908	9831,1654	10168,8345	52
9	9749,2425	9917,8051	9831,4374	10168,5626	51
10	9749,4287	9917,7194	9831,7093	10168,2906	50
11	9749,6148	9917,6336	9831,9811	10168,0188	49
12	9749,8007	9917,5478	9832,2529	10167,7470	48
13	9749,9865	9917,4619	9832,5246	10167,4753	47
14	9750,1723	9917,3760	9832,7963	10167,2037	46
15	9750,3579	9917,2900	9833,0678	10166,9321	45
16	9750,5433	9917,2039	9833,3394	10166,6605	44
17	9750,7287	9917,1178	9833,6108	10166,3891	43
18	9750,9140	9917,0317	9833,8822	10166,1177	42
19	9751,0991	9916,9455	9834,1536	10165,8463	41
20	9751,2841	9916,8592	9834,4249	10165,5751	40
21	9751,4690	9916,7729	9834,6961	10165,3039	39
22	9751,6538	9916,6865	9834,9672	10165,0327	38
23	9751,8385	9916,6001	9835,2383	10164,7616	37
24	9752,0230	9916,5137	9835,5094	10164,4906	36
25	9752,2075	9916,4271	9835,7803	10164,2196	35
26	9752,3918	9916,3405	9836,0513	10763,9487	34
27	9752,5760	9916,2539	9836,3221	10163,6778	33
28	9752,7601	9916,1672	9836,5929	10163,4070	32
29	9752,9441	9916,0805	9836,8636	10163,1363	31
30	9753,1280	9915,9937	9837,1343	10162,8656	30
		Sin. 55.		Tan. 58.	M

M	Sin.34.		Tan.34.		
30	9753,1280	9915,9937	9837,1343	10162,8656	30
31	9753,3117	9915,9068	9837,4049	10162,5950	29
32	9753,4954	9915,8199	9837,6754	10162,3245	28
33	9753,6789	9915,7330	9837,9459	10162,0540	27
34	9753,8623	9915,6460	9838,2163	10161,7836	26
35	9754,0456	9915,5589	9838,4867	10161,5132	25
36	9754,2288	9915,4718	9838,7570	10161,2429	24
37	9754,4119	9915,3846	9839,0273	10160,9726	23
38	9754,5948	9915,2973	9839,2975	10160,7025	22
39	9754,7777	9915,2101	9839,5676	10160,4323	21
40	9754,9604	9915,1227	9839,8377	10160,1623	20
41	9755,1430	9915,0353	9840,1077	10159,8922	19
42	9755,3255	9914,9479	9840,3776	10159,6223	18
43	9755,5079	9914,8604	9840,6475	10159,3524	17
44	9755,6902	9914,7728	9840,9173	10159,0826	16
45	9755,8723	9914,6852	9841,1871	10158,8128	15
46	9756,0544	9914,5975	9841,4568	10158,5431	14
47	9756,2363	9914,5098	9841,7265	10158,2734	13
48	9756,4182	9914,4220	9841,9961	10158,0038	12
49	9756,5999	9914,3342	9842,2656	10157,7343	11
50	9756,7815	9914,2463	9842,5351	10157,4648	10
51	9756,9630	9914,1584	9842,8045	10157,1954	9
52	9757,1443	9914,0704	9843,0739	10156,9260	8
53	9757,3256	9913,9823	9843,3432	10156,6567	7
54	9757,5067	9913,8942	9843,6125	10156,3875	6
55	9757,6878	9913,8061	9843,8817	10156,1183	5
56	9757,8687	9913,7179	9844,1508	10155,8491	4
57	9758,0495	9913,6296	9844,4199	10155,5801	3
58	9758,2302	9913,5413	9844,6889	10155,3110	2
59	9758,4108	9913,4529	9844,9579	10155,0421	1
60	9758,5913	9913,3645	9845,2267	10154,7732	0
		Sin. 55.		Tan.55.	M

M	Sin. 35.		Tan. 35.		
0	9758,5913	9913.3645	9845,2267	10154,7732	60
1	9758,7716	9913,2760	9845,4956	10154,5043	59
2	9758,9519	9913,1875	9845,7644	10154,2355	58
3	9759,1320	9913,0989	9846,0331	10153,9668	57
4	9759,3120	9913,0102	9846,3018	10153,6981	56
5	9759,4920	9912,9215	9846.5704	10153,4295	55
6	9759,6718	9912,8327	9846,8390	10153,1609	54
7	9759,8515	9912,7439	9847.1075	10152,8924	53
8	9760.0310	9912,6551	9847,3759	10152,6240	52
9	9760,2105	9912,5601	9847.6444	10152,3556	51
10	9760,3899	9912,4772	9847.9127	10152,0872	50
11	9760,5691	9912,3881	9848,1810	10151.8189	49
12	9760,7483	9912,2990	9848,4492	10151,5507	48
13	9760,9273	9912,2099	9848,7174	10151,2825	47
14	9761.1062	9912,1207	9848,9855	10151,0144	46
15	9761,2850	9912,0314	9849,2536	10150,7463	45
16	9761,4637	9911,9421	9849.5216	10150,4783	44
17	9761,6423	9911,8528	9849,7895	10150,2104	43
18	9761,8208	9911,7633	9850,0574	10149,9425	42
19	9761,9992	9911,6739	9850,3253	10149,6747	41
20	9762,1774	9911,5843	9850.5931	10149,4069	40
21	9762,3556	9911,4948	9850,8608	10149,1391	39
22	9762,5336	9911,4051	9851,1285	10148,8714	38
23	9762,7116	9911,3154	9851,3961	10148,6038	37
24	9762.8894	9911,2254	9851.6637	10148.3362	36
25	9763.0671	9911,1359	9851,9312	10148,0687	35
26	9763,2447	9911,0460	9852,1987	10147,8013	34
27	9763.4222	9910,9561	9852,4661	10147,5339	33
28	9763,5995	9910,8661	9852.7334	10147,2665	32
29	9763,7768	9910,7761	9853,0007	10146,9992	31
30	9763,9540	9910,6860	9853,2680	10146,7320	30
		Sin. 54.		Sin. 54.	M

M	Sin.35.		Tan.35.		
30	9763,9540	9910,6860	9853.2680	10146,7320	30
31	9764.1311	9910,5959	9853,5352	10146,4648	29
32	9764,3080	9910,5057	9853,8023	10146,1976	28
33	9764.4848	9910.4154	9854,0694	10145,9305	27
34	9764,6616	9910.3251	9854,3364	10145,6635	26
35	9764,8382	9910,2347	9854,6034	10145,3965	25
36	9765,0147	9910.1443	9854,8703	10145,1296	24
37	9765.1911	9910,0539	9855,1372	10144,8627	23
38	9765,3674	9909.9633	9855,4040	10144,5959	22
39	9765,5436	9909,8727	9855,6708	10144,3291	21
40	9765,7197	9909,7821	9855,9375	10144,0624	20
41	9765,8956	9909,6914	9856,2042	10143,7957	19
42	9766.0715	9909,6007	9856,4708	10143,5291	18
43	9766,2473	9909.5099	9856,7374	10143,2626	17
44	9766.4229	9909,4190	9857,0039	10142,9960	16
45	9766,5984	9909,3281	9857,2703	10142,7296	15
46	9766,7739	9909,2371	9857,5367	10142,4632	14
47	9766,9492	9909,1461	9857,8031	10142,1968	13
48	9767.1244	9909.0550	9858,0694	10141,9305	12
49	9767,2995	9908,9638	9858,3356	10141,6643	11
50	9767,4745	9908,8726	9858,6018	10141,3981	10
51	9767,6494	9908.7814	9858,8680	10141,1319	9
52	9767,8242	9908,6901	9859,1341	10140,8658	8
53	9767,9989	9908,5987	9859,4001	10140,5998	7
54	9768,1734	9908,5073	9859,6661	10140.3338	6
55	9768,3479	9908,4158	9859,9321	10140,0679	5
56	9768,5223	9908,3243	9860,1979	10139,8020	4
57	9768,6965	9908,2327	9860,4638	10139,5361	3
58	9768.8707	9908,1411	9860,7296	10139.2703	2
59	9769,0447	9908,0494	9860,9953	10139,0046	1
60	9769,2186	9907,9576	9861,2610	10138.7389	0
		Sin.54.		Tan.54.	M

M	Sin.36.		Tan.36.		
0	9769,2186	9907,9576	9861,2610	10138,7389	60
1	9769,3925	9907,8658	9861,5266	10138,4733	59
2	9769,5661	9907,7739	9861,7922	10138,2077	58
3	9769,7398	9907,6820	9862,0578	10137,9421	57
4	9769,9133	9907,5900	9862,3232	10137,6767	56
5	9770,0867	9907,4980	9862,5887	10137,4112	55
6	9770,2600	9907,4059	9862,8541	10137,1458	54
7	9770,4332	9907,3137	9863,1194	10136,8805	53
8	9770,6063	9907,2215	9863,3847	10136,6152	52
9	9770,7793	9907,1293	9863,6500	10136,3500	51
10	9770,9521	9907,0369	9863,9152	10136,0848	50
11	9771,1249	9906,9446	9864,1803	10135,8196	49
12	9771,2976	9906,8521	9864,4454	10135,5545	48
13	9771,4701	9906,7596	9864,7104	10135,2895	47
14	9771,6426	9906,6671	9864,9754	10135,0245	46
15	9771,8149	9906,5745	9865,2404	10134,7595	45
16	9771,9872	9906,4818	9865,5053	10134,4946	44
17	9772,1593	9906,3891	9865,7701	10134,2298	43
18	9772,3313	9906,2964	9866,0349	10133,9650	42
19	9772,5033	9906,2035	9866,2997	10133,7002	41
20	9772,6751	9906,1106	9866,5644	10133,4355	40
21	9772,8468	9906,0177	9866,8291	10133,1709	39
22	9773,0184	9905,9247	9867,0937	10132,9062	38
23	9773,1899	9905,8317	9867,3582	10132,6417	37
24	9773,3613	9905,7385	9867,6227	10132,3772	36
25	9773,5326	9905,6454	9867,8872	10132,1127	35
26	9773,7038	9905,5521	9868,1516	10131,8483	34
27	9773,8749	9905,4589	9868,4160	10131,5839	33
28	9774,0459	9905,3655	9868,6803	10131,3196	32
29	9774,2168	9905,2721	9868,9446	10131,0553	31
30	9774,3876	9905,1787	9869,2088	10130,7911	30
		Sin.53.		Tang.53	M

M	Sin.36.		Tan.36.		
30	9774,3876	9905,1787	9869,2088	10130,7911	30
31	9774,5582	9905,0852	9869,4730	10130,5269	29
32	9774,7288	9904,9916	9869,7372	10130,2627	28
33	9774,8993	9904,8980	9870,0013	10129,9987	27
34	9775,0696	9904,8043	9870,2653	10129,7346	26
35	9775,2399	9904,7106	9870,5293	10129,4706	25
36	9775,4101	9904,6168	9870,7932	10129,2067	24
37	9775,5801	9904,5229	9871,0572	10128,9428	23
38	9775,7501	9904,4290	9871,3210	10128,6789	22
39	9775,9199	9904,3351	9871,5848	10128,4151	21
40	9776,0896	9904,2410	9871,8486	10128,1513	20
41	9776,2593	9904,1470	9872,1123	10127,8876	19
42	9776,4288	9904,0528	9872,3759	10127,6240	18
43	9776,5983	9903,9586	9872,6396	10127,3603	17
44	9776,7676	9903,8644	9872,9032	10127,0968	16
45	9776,9368	9903,7701	9873,1667	10126,8332	15
46	9777,1059	9903,6757	9873,4302	10126,5697	14
47	9777,2750	9903,5813	9873,6937	10126,3063	13
48	9777,4439	9903,4868	9873,9571	10126,0429	12
49	9777,6127	9903,3923	9874,2204	10125,7795	11
50	9777,7814	9903,2977	9874,4837	10125,5162	10
51	9777,9500	9903,2030	9874,7470	10125,2529	9
52	9778,1186	9903,1083	9875,0102	10124,9897	8
53	9778,2870	9903,0135	9875,2734	10124,7265	7
54	9778,4553	9902,9187	9875,5365	10124,4634	6
55	9778,6235	9902,8238	9875,7996	10124,2003	5
56	9778,7916	9902,7289	9876,0626	10123,9373	4
57	9778,9596	9902,6339	9876,3256	10123,6743	3
58	9779,1275	9902,5389	9876,5886	10123,4113	2
59	9779,2953	9902,4437	9876,8515	10123,1484	1
60	9779,4630	9902,3486	9877,1144	10122,8855	0
		Sin.53.		Tan.53.	M

M	Sin. 37.		Tan. 37.		
0	9779,4630	9902,3486	9877,1144	10122,8855	60
1	9779,6306	9902,2533	9877,3772	10122,6227	59
2	9779,7981	9902,1581	9877,6400	10122,3599	58
3	9779,9655	9902,0627	9877,9027	10122,0972	57
4	9780,1328	9901,9673	9878,1654	10121,8345	56
5	9780,3000	9901,8719	9878,4280	10121,5719	55
6	9780,4670	9901,7764	9878,6907	10121,3093	54
7	9780,6340	9901,6808	9878,9532	10121,0467	53
8	9780,8009	9901,5852	9879,2157	10120,7842	52
9	9780,9677	9901,4895	9879,4782	10120,5217	51
10	9781,1344	9901,3937	9879,7406	10120,2593	50
11	9781,3010	9901,2979	9880,0030	10119,9969	49
12	9781,4675	9901,2021	9880,2654	10119,7345	48
13	9781,6339	9901,1061	9880,5277	10119,4722	47
14	9781,8001	9901,0102	9880,7899	10119,2100	46
15	9781,9663	9900,9141	9881,0522	10118,9478	45
16	9782,1324	9900,8180	9881,3143	10118,6856	44
17	9782,2984	9900,7219	9881,5765	10118,4234	43
18	9782,4643	9900,6257	9881,8386	10118,1613	42
19	9782,6301	9900,5294	9882,1006	10117,8993	41
20	9782,7957	9900,4331	9882,3626	10117,6373	40
21	9782,9613	9900,3367	9882,6246	10117,3753	39
22	9783,1268	9900,2402	9882,8865	10117,1134	38
23	9783,2922	9900,1437	9883,1484	10116,8515	37
24	9783,4575	9900,0472	9883,4103	10116,5897	36
25	9783,6227	9899,9506	9883,6721	10116,3279	35
26	9783,7877	9899,8539	9883,9338	10116,0661	34
27	9783,9527	9899,7572	9884,1955	10115,8044	33
28	9784,1176	9899,6604	9884,4572	10115,5427	32
29	9784,2824	9899,5635	9884,7188	10115,2811	31
30	9784,4471	9899,4666	9884,9804	10115,0195	30
		Sin. 52.		Tan. 52.	

M	Sin. 56.		Tan. 37.		
30	9784,4471	9899,4666	9884,9804	10115,0195	30
31	9784,6117	9899,3696	9885,2420	10114,7579	29
32	9784,7762	9899,2726	9885,5035	10114,4964	28
33	9784,9406	9899,1755	9885,7650	10114,2349	27
34	9785,1048	9899,0784	9886,0264	10113,9735	26
35	9785,2690	9898,9812	9886,2878	10113,7121	25
36	9785,4331	9898,8839	9886,5492	10113,4508	24
37	9785,5971	9898,7866	9886,8105	10113,1894	23
38	9785,7610	9898,6892	9887,0717	10112,9282	22
39	9785,9248	9898,5918	9887,3330	10112,6669	21
40	9786,0885	9898,4943	9887,5942	10112,4057	20
41	9786,2521	9898,3968	9887,8553	10112,1446	19
42	9786,4156	9898,2992	9888,1164	10111,8835	18
43	9786,5790	9898,2015	9888,3775	10111,6224	17
44	9786,7423	9898,1038	9888,6385	10111,3614	16
45	9786,9055	9898,0060	9888,8995	10111,1004	15
46	9787,0687	9897,9081	9889,1605	10110,8394	14
47	9787,2317	9897,8102	9889,4214	10110,5785	13
48	9787,3946	9897,7123	9889,6823	10110,3176	12
49	9787,5574	9897,6142	9889,9431	10110,0568	11
50	9787,7201	9897,5162	9890,2039	10109,7960	10
51	9787,8827	9897,4180	9890,4647	10109,5352	9
52	9788,0453	9897,3198	9890,7254	10109,2745	8
53	9788 2077	9897,2216	9890,9861	10109,0138	7
54	9788,3700	9897,1232	9891,2467	10108,7532	6
55	9788,5322	9897,0249	9891,5073	10108,4926	5
56	9788,6944	9896,9264	9891,7679	10108,2320	4
57	9788,8564	9896,8279	9892,0284	10107,9715	3
58	9789,0184	9896,7294	9892,2889	10107,7110	2
59	9789,1802	9896,6308	9892,5494	10107,4505	1
60	9789,3419	9896,5321	9892,8098	10107,1901	0
		Sin. 33.		Tan. 52.	M

M	Sin. 38.		Tan. 38.		
0	9789,3419	9896,5321	9892,8098	10107,1901	60
1	9789,5036	9896,4334	9893,0702	10106,9297	59
2	9789,6651	9896,3346	9893,3305	10106,6694	58
3	9789,8266	9896,2357	9893,5908	10106,4091	57
4	9789,9880	9896,1368	9893,8511	10106,1488	56
5	9790,1492	9896,0379	9894,1113	10105,8886	55
6	9790,3104	9895,9388	9894,3715	10105,6284	54
7	9790,4714	9895,8398	9894,6317	10105,3683	53
8	9790,6324	9895,7406	9894,8918	10105,1081	52
9	9790,7933	9895,6414	9895,1519	10104,8481	51
10	9790,9541	9895,5421	9895,4119	10104,5880	50
11	9791,1148	9895,4428	9895,6719	10104,3280	49
12	9791,2753	9895,3434	9895,9319	10104,0680	48
13	9791,4358	9895,2440	9896,1918	10103,8081	47
14	9791,5962	9895,1445	9896,4517	10103,5482	46
15	9791,7565	9895,0449	9896,7116	10103,2884	45
16	9791,9167	9894,9453	9896,9714	10103,0285	44
17	9792,0768	9894,8456	9897,2312	10102,7687	43
18	9792,2368	9894,7459	9897,4909	10102,5090	42
19	9792,3968	9894,6461	9897,7507	10102,2493	41
20	9792,5566	9894,5462	9898,0103	10101,9896	40
21	9792,7163	9894,4463	9898,2700	10101,7300	39
22	9792,8759	9894,3463	9898,5296	10101,4704	38
23	9793,0355	9894,2463	9898,7891	10101,2108	37
24	9793,1949	9894,1462	9899,0487	10100,9512	36
25	9793,3542	9894,0460	9899,3082	10100,6917	35
26	9793,5135	9893,9458	9899,5677	10100,4323	34
27	9793,6726	9893,8455	9899,8271	10100,1728	33
28	9793,8317	9893,7452	9900,0865	10099,9134	32
29	9793,9907	9893,6448	9900,3458	10099,6541	31
30	9794,1495	9893,5443	9900,6052	10099,3948	30
		Sin. 51.		Tan. 51.	M

M	Sin.38.		Tan.38.		
30	9794,1495	9893,5443	9900,6052	10099,3948	30
31	9794,3083	9893,4438	9900,8644	10099,1355	29
32	9794,4670	9893,3432	9901,1237	10098,8762	28
33	9794,6256	9893,2426	9901,3829	10098,6170	27
34	9794,7840	9893,1419	9901,6421	10098,3578	26
35	9794,9424	9893,0411	9901,9013	10098,0987	25
36	9795,1007	9892,9403	9902,1604	10097,8395	24
37	9795,2590	9892,8394	9902,4195	10097,5804	23
38	9795,4171	9892,7385	9902,6785	10097,3214	22
39	9795,5751	9892,6375	9902,9375	10097,0624	21
40	9795,7330	9892,5364	9903,1965	10096,8034	20
41	9795,8908	9892,4353	9903,4555	10096,5445	19
42	9796,0486	9892,3341	9903,7144	10096,2855	18
43	9796,2062	9892,2329	9903,9733	10096,0267	17
44	9796,3637	9892,1316	9904,2321	10095,7678	16
45	9796,5212	9892,0302	9904,4909	10095,5090	15
46	9796,6785	9891,9288	9904,7497	10095,2502	14
47	9796,8358	9891,8273	9905,0084	10094,9915	13
48	9796,6930	9891,7258	9905,2672	10094,7328	12
49	9797,1501	9891,6242	9905,5258	10094,4741	11
50	9797,3070	9891,5225	9905,7845	10094,2154	10
51	9797,4639	9891,4208	9906,0431	10093,9568	9
52	9797,6207	9891,3190	9906,3017	10093,6982	8
53	9797,7774	9891,2172	9906,5602	10093,4397	7
54	9797,9341	9891,1153	9906,8188	10093,1812	6
55	9798,0906	9891,0133	9907,0772	10092,9227	5
56	9798,2470	9890,9113	9907,3357	10092,6642	4
57	9798,4033	9890,8092	9907,5941	10092,4058	3
58	9798,5596	9890,7070	9907,8525	10092,1474	2
59	9798,7157	9890,6048	9908,1109	10091,8891	1
60	9798,8718	9890,5026	9908,3692	10091,6307	0
		Sin.51.		Tan.51.	M

M	Sin.35.		Tan.39.		
0	9789,8718	9890.5026	9908,3692	10091,6307	60
1	9799,0277	9890,4002	9908,6275	10091,3724	59
2	9799,1836	9890.2978	9908,8857	10091,1142	58
3	9799,3394	9890,1954	9909.1440	10090,8560	57
4	9799,4950	9890,0929	9909,4022	10090,5978	56
5	9799,6506	9889,9903	9909,6603	10090,3396	55
6	9799,8061	9889,8877	9909,9184	10090,0815	54
7	9799,9615	9889,7850	9910,1765	10089,8234	53
8	9800,1168	9889,6822	9910,4346	10089,5653	52
9	9800,2721	9889,5794	9910,6927	10089.3073	51
10	9800,4272	9889,4765	9910,9507	10089,0493	50
11	9800,5822	9889,3736	9911,2086	10088,7913	49
12	9800,7372	9889,2706	9911,4666	10088,5333	48
13	9800,8920	9889.1675	9911,7245	10088,2754	47
14	9801,0468	9889,0644	9911,9824	10088,0175	46
15	9801,2015	9888,9612	9912,2402	10087,7597	45
16	9801,3560	9888,8579	9912,4981	10087,5019	44
17	9801,5105	9888,7546	9912,7558	10087,2441	43
18	9801,6649	9888,6513	9913,0136	10086,9863	42
19	9801,8192	9888,5478	9913,2714	10086,7286	41
20	9801,9734	9888,4443	9913,5291	10086,4709	40
21	9802,1275	9888,3408	9913,7867	10086,2132	39
22	9802.2816	9888,2372	9914,0444	10085,9559	38
23	9802.4355	9888,1335	9914.3020	10085,6980	37
24	9802.5893	9888,0297	9914,5596	10085,4404	36
25	9802.7431	9887,9259	9914,8171	10085,1828	35
26	9802,8967	9887,8221	9915,0746	10084,9253	34
27	9803,0503	9887,7182	9915,3321	10084,6678	33
28	9803,2038	9887,6142	9915,5896	10084,4103	32
29	9803,3572	9887,5101	9915,8470	10084,1529	31
30	9803,5105	9887,4060	9916,1044	10083,8955	30
		Sin.54.	Tang. 50		M

M	Sin. 39.		Tan. 39.		
30	9803,5105	9887,4060	9916,1044	10083,8955	30
31	9803,6637	9887,3018	9916,3618	10083,6381	29
32	9803,8168	9887,1976	9916,6192	10083,3808	28
33	9803,9698	9887,0933	9916,8765	10083,1234	27
34	9804,1228	9886,9890	9917,1338	10082,8661	26
35	9804,2756	9886,8845	9917,3910	10082,6089	25
36	9804,4284	9886,7801	9917,6483	10082,3517	24
37	9804,5810	9886,6755	9917,9055	10082,0944	23
38	9804,7336	9886,5709	9918,1626	10082,8373	22
39	9804,8861	9886,4663	9918,4198	10081,5801	21
40	9805,0385	9886,3615	9918,6769	10081,3230	20
41	9805,1908	9886,2567	9918,9340	10081,0659	19
42	9805,3430	9886,1519	9919,1911	10080,8089	18
43	9805,4951	9886,0470	9919,4481	10080,5518	17
44	9805,6471	9885,9420	9919,7051	10080,2948	16
45	9805,7991	9885,8370	9919,9621	10080,0378	15
46	9805,9509	9885,7319	9920,2190	10079,7809	14
47	9806,1027	9885,6267	9920,4760	10079,5240	13
48	9806,2544	9885,5215	9920,7328	10079,2671	12
49	9806,4060	9885,4162	9920,9897	10079,0102	11
50	9806,5575	9885,3108	9921,2466	10078,7533	10
51	9806,7088	9885,2054	9921,5034	10078,4965	9
52	9806,8602	9885,1000	9921,7602	10078,2397	8
53	9807,0114	9884,9944	9922,0169	10077,9830	7
54	9807,1625	9884,8888	9922,2737	10077,7263	6
55	9807,3136	9884,7832	9922,5304	10077,4695	5
56	9807,4645	9884,6774	9922,7870	10077,2129	4
57	9807,6154	9884,5717	9923,0437	10076,9562	3
58	9807,7662	9884,4658	9923,3003	10076,6996	2
59	9807,9169	9884,3599	9923,5569	10076,4430	1
60	9808,0675	9884,2539	9923,8135	10076,1864	0
		Sin. 50.		Tan. 50.	M

M	*Sin.*40.		*Tan.*40.		
0	9808,0675	9884,2539	9923,8135	10076,1864	60
1	9808,2180	9884,1479	9924,0700	10075,9299	59
2	9808,3684	9884,0418	9924,3266	10075,6734	58
3	9808,5187	9883,9356	9924,5831	10075,4169	57
4	9808,6690	9883,8294	9924,8395	10075,1604	56
5	9808,8191	9883,7231	9925,0960	10074,9039	55
6	9808,9692	9883,6168	9925,3524	10074,6475	54
7	9809,1192	9883,5104	9925,6088	10074,3911	53
8	9809,2691	9883,4039	9925,8651	10074,1348	52
9	9809,4189	9883,2973	9926,1215	10073,8784	51
10	9809,5686	9883,1907	9926,3778	10073,6221	50
11	9809,7182	9883,0841	9926,6341	10073,3658	49
12	9809,8677	9882,9774	9926,8903	10073,1096	48
13	9810,0172	9882,8706	9927,1466	10072,8533	47
14	9810,1665	9882,7637	9927,4028	10072,5971	46
15	9810,3158	9882,6568	9927,6590	10072,3409	45
16	9810,4650	9882,5498	9927,9152	10072,0848	44
17	9810,6141	9882,4428	9928,1713	10071,8286	43
18	9810,7631	9882,3357	9928,4274	10071,5725	42
19	9810,9120	9882,2285	9928,6835	10071,3164	41
20	9811,0609	9882,1213	9928,9396	10071,0604	40
21	9811,2096	9882,0140	9929,1956	10070,8043	39
22	9811,3583	9881,9066	9929,4516	10070,5483	38
23	9811,5068	9881,7992	9929,7076	10070,2923	37
24	9811,6553	9881,6917	9929,9636	10070,0364	36
25	9811,8037	9881,5842	9930,2195	10069,7804	35
26	9811,9520	9881,4766	9930,4754	10069,5245	34
27	9812,1002	9881,3689	9930,7313	10069,2686	33
28	9812,2484	9881,2611	9930,9872	10069,0127	32
29	9812,3964	9881,1533	9931,2430	10068,7569	31
30	9812,5444	9881,0455	9931,4989	10068,5011	30
		*Sin.*49.		*Tang.*49	M

M	Sin.40.		Tan.40.		
30	9812,5444	9881,0455	9931,4989	10068,5011	30
31	9812,6922	9880,9375	9931,7547	10068,2452	29
32	9812,8400	9880,8295	9932,0104	10067,9895	28
33	9812,9877	9880,7215	9932,2662	10067,7337	27
34	9813,1353	9880,6134	9932,5219	10067,4780	26
35	9813,2829	9880,5052	9932,7776	10067,2223	25
36	9813,4303	9880,3969	9933,0333	10066,9666	24
37	9813,5776	9880,2886	9933,2890	10066,7109	23
38	9813,7249	9880,1803	9933,5446	10066,4553	22
39	9813,8721	9880,0718	9933,8002	10066,1997	21
40	9814,0192	9879,9633	9934,0558	10065,9441	20
41	9814,1661	9879,8547	9934,3114	10065,6885	19
42	9814,3131	9879,7461	9934,5669	10065,4330	18
43	9814,4599	9879,6374	9934,8225	10065,1775	17
44	9814,6067	9879,5287	9935,0780	10064,9220	16
45	9814,7533	9879,4199	9935,3334	10064,6665	15
46	9814,8999	9879,3110	9935,5889	10064,4110	14
47	9815,0464	9879,2020	9935,8443	10064,1556	13
48	9815,1928	9879,0930	9936,0997	10063,9002	12
49	9815,3391	9878,9839	9936,3552	10063,6448	11
50	9815,4853	9878,8748	9936,6105	10063,3894	10
51	9815,6315	9878,7656	9936,8659	10063,1341	9
52	9815,7775	9878,6563	9937,1212	10062,8787	8
53	9815,9235	9878,5470	9937,3765	10062,6234	7
54	9816,0694	9878,4376	9937,6318	10062,3682	6
55	9816,2152	9878,3281	9937,8870	10062,1129	5
56	9816,3609	9878,2186	9938,1423	10061,8576	4
57	9816,5065	9878,1090	9938,3975	10061,6024	3
58	9816,6521	9877,9993	9938,6527	10061,3472	2
59	9816,7975	9877,8896	9938,9079	10061,0921	1
60	9816,9429	9877,7798	9939,1630	10060,8369	0
		Sin.49.		Tan.49.	M

M	Sin. 41.		Tan. 41.		
0	9816,9929	9877,7798	9939,1630	10060,8369	60
1	9817,0882	9877,6700	9939,4182	10060,5818	59
2	9817,2334	9877,5601	9939,6733	10060,3266	58
3	9817,3785	9877,4501	9939,9284	10060,0716	57
4	9817,5235	9877,3400	9940,1834	10059,8165	56
5	9817,6684	9877,2299	9940,4385	10059,5614	55
6	9817,8133	9877,1198	9940,6935	10059,3064	54
7	9817,9581	9877,0095	9940,9485	10059,0514	53
8	9818,1028	9876,8992	9941,2035	10058,7964	52
9	9818,2474	9876,7888	9941,4585	10058,5414	51
10	9818,3919	9876,6784	9941,7134	10058,2865	50
11	9818,5363	9876,5679	9942,9684	10058,0316	49
12	9818,6807	9876,4574	9942,2233	10057,7766	48
13	9818,8249	9876,3467	9942,4782	10057,5217	47
14	9818,9691	9876,2360	9942,7331	10057,2669	46
15	9819,1132	9876,1253	9942,9879	10057,0120	45
16	9819,2572	9876,0145	9943,2427	10056,7572	44
17	9819,4012	9875,9036	9943,4976	10056,5024	43
18	9819,5450	9875,7926	9943,7523	10056,2476	42
19	9819,6888	9875,6816	9944,0071	10055,9928	41
20	9819,8324	9875,5705	9944,2619	10055,7380	40
21	9819,9760	9875,4594	9944,5166	10055,4833	39
22	9820,1195	9875,3481	9944,7713	10055,2286	38
23	9820,2629	9875,2369	9945,0260	10054,9739	37
24	9820,4063	9875,1255	9945,2807	10054,7192	36
25	9820,5495	9875,0141	9945,5354	10054,4645	35
26	9820,6927	9874,9027	9945,7900	10054,2099	34
27	9820,8358	9874,7911	9946,0446	10053,9553	33
28	9820,9788	9874,6795	9946,2992	10053,7007	32
29	9821,1217	9874,5678	9946,5538	10053,4461	31
30	9821,2645	9874,4561	9946,8084	10053,1915	30
		Sin. 48.		Tan. 48.	M

M	Sin. 41.		Tan. 41.		
30	9821,2645	9874,4561	9946,8084	10053,1915	30
31	9821,4073	9874,3443	9947,0672	10052,9370	29
32	9821,5500	9874,2324	9947,3175	10052,6824	28
33	9821,6925	9874,1205	9947,5720	10052,4279	27
34	9821,8350	9874,0085	9947,8265	10052,1734	26
35	9821,9774	9873,8964	9948,0810	10051,9190	25
36	9822,1198	9873,7843	9948,3354	10051,6645	24
37	9822,2620	9873,6721	9948,5899	10051,4101	23
38	9822,4042	9873,5599	9948,8443	10051,1556	22
39	9822,5463	9873,4475	9949,0987	10050,9012	21
40	9822,6883	9873,3351	9949,3531	10050,6468	20
41	9822,8302	9873,2227	9949,6075	10050,3925	19
42	9822,9720	9873,1102	9949,8618	10050,1381	18
43	9823,1138	9872,9976	9950,1162	10049,8837	17
44	9823,2554	9872,8849	9950,3705	10049,6294	16
45	9823,3970	9872,7722	9950,6248	10049,3752	15
46	9823,5385	9872,6594	9950,8791	10049,1208	14
47	9823,6799	9872,5465	9951,1334	10048,8566	13
48	9823,8213	9872,4336	9951,3870	10048,6123	12
49	9823,9625	9872,3206	9951,6418	10048,3581	11
50	9824,1037	9872,2076	9951,8961	11048,1039	10
51	9824,2448	9872,0945	9952,1503	10047,8496	9
52	9824,3858	9871,9813	9952,4045	10047,5955	8
53	9824,5267	9871,8680	9952,6580	10047,3413	7
54	9824,6675	9871,7547	9952,9128	10047,0871	6
55	9824,8083	9871,6413	9953,1669	10046,8330	5
56	9824,9490	9871,5279	9953,4211	10046,5789	4
57	9825,0896	9871,4144	9953,6752	10046,3247	3
58	9825,2301	9871,3008	9953,9293	10046,0707	2
59	9825,3701	9871,1871	9954,1833	10045,8166	1
60	9825,5109	9871,0734	9954,4374	10045,5625	0
		Sin. 48		Tan. 48.	M

M	Sin.42.		Tan.42.		
0	9825,5109	9871,0734	9954,4374	10045,5625	60
1	9825,6511	9470,9596	9954,6914	10045,3085	59
2	9825,7913	9870,8458	9954,9455	10045,0545	58
3	9825,9314	9870,7319	9955,1995	10044,8004	57
4	9826,0714	9870,6179	9955,4535	10044,5464	56
5	9826,2114	9870,5038	9955,7075	10044,2924	55
6	9826,3512	9870,3897	9955,9615	10044,0385	54
7	9826,4910	9870,2755	9956,2154	10043,7845	53
8	9826,6307	9870,1613	9956,4693	10043,5306	52
9	9826,7703	9870,0470	9956,7233	10043,2766	51
10	9826,9098	9869,9326	9956,9772	10043,0227	50
11	9827,0493	9869,8181	9957,2311	10042,7688	49
12	9827,1886	9869,7036	9957,4850	10042,5150	48
13	9827,3279	9869,5890	9957,7388	10042,2611	47
14	9827,4671	9869,4744	9957,9927	10042,0072	46
15	9827,6062	9869,3597	9958,2465	10041,7534	45
16	9827,7453	9869,2449	9958,5003	10041,4996	44
17	9827,8842	9869,1300	9958,7541	10041,2458	43
18	9828,0231	9869,0151	9959,0079	10040,9920	42
19	9828,1619	9868,9001	9959,2617	10040,7382	41
20	9828,3006	9868,7851	9959,5155	10040,4844	40
21	9828,4392	9868,6700	9959,7692	10040,2307	39
22	9828,5778	9868,5548	9960,0230	10039,9769	38
23	9828,7163	9868,4395	9960,2767	10039,7232	37
24	9828,8547	9868,3242	9960,5304	10039,4695	36
25	9828,9930	9868,2088	9960,7841	10039,2158	35
26	9829,1312	9868,0934	9961,0378	10038,9621	34
27	9829,2693	9867,9778	9961,2915	10038,7084	33
28	9829,4074	9867,8622	9961,5451	10038,4548	32
29	9829,5454	9867,7466	9961,7988	10038,2011	31
30	9829,6833	9867,6308	9962,0524	10037,9475	30
		Sin.74.		Tan.47.	M

M	Sin.42.		Tan.42.		
30	9829,6833	9867,6308	9962,0524	10037,9475	30
31	9829,8211	9867,5150	9962,3061	10037,6939	29
32	9829,9589	9867,3992	9962,5597	10037,4403	28
33	9830,0965	9867,2833	9962,8133	10037,1867	27
34	9830,2341	9867,1673	9963,0668	10036,9331	26
35	9830,3716	9867,0512	9963,3204	10036,6795	25
36	9830,5091	9866,9351	9963,5740	10036,4260	24
37	9830,6464	9866,8189	9963,8275	10036,1724	23
38	9830,7837	9866,7026	9964,0810	10035,9189	22
39	9830,9208	9866,5863	9964,3346	10035,6654	21
40	9831,0579	9866,4698	9964,5881	10035,4119	20
41	9831,1950	9866,3534	9964,8416	10035,1584	19
42	9831,3319	9866,2368	9965,0950	10034,9049	18
43	9831,4688	9866,1202	9965,3485	10034,6514	17
44	9831,6056	9866,0036	9965,6020	10034,3979	16
45	9831,7423	9865,8868	9965,8554	10034,1445	15
46	9831,8789	9865,7700	9966,1089	10033,8910	14
47	9832,0154	9865,6531	9966,3623	10033,6376	13
48	9832,1519	9865,5362	9966,6157	10033,3842	12
49	9832,2883	9865,4191	9966,8691	10033,1308	11
50	9832,4246	9865,3021	9967,1225	10032,8774	10
51	9832,5608	9865,1849	9967,3759	10032,6240	9
52	9832,6970	9865,0677	9967,6293	10032,3707	8
53	9832,8330	9864,9504	9967,8826	10032,1173	7
54	9832,9690	9864,8330	9968,1360	10031,8640	6
55	9833,1049	9864,7156	9968,3893	19031,6106	5
56	9833,2408	9864,5981	9968,6426	10031,3573	4
57	9833,3765	9864,4805	9968,8959	10031,1040	3
58	9833,5122	9864,3629	9969,1493	10030,8507	2
59	9833,6478	9864,2452	9969,4026	10030,5974	1
60	9833,7833	9864,1274	9969,6558	10030,3441	0
		Sin.47.		Tan.47.	M

M	Sin. 43.		Tan. 43.		
0	9833,7833	9864,1274	9969,6558	10030,3441	60
1	9833,9187	9864,0096	9969,9091	10030,0908	59
2	9834,0541	9563,8917	9970,1624	10029,8376	58
3	9834,1894	9863,7737	9970,4156	10029,5843	57
4	9834,3246	9863,6556	9970,6689	10029,3310	56
5	9834,4597	9863,5375	9970,9221	10029,0778	55
6	9834,5947	9863,4194	9971,1753	10028,8246	54
7	9834,7297	9863,3011	9971,4285	10028,5714	53
8	9834,8646	9863,1828	9971,6817	10028,3182	52
9	9834,9994	9863,0644	9971,9349	10028,0650	51
10	9835,1341	9862,9459	9972,1881	10027,8118	50
11	9835,2687	9862,8274	9972,4413	10027,5586	49
12	9835,4033	9862,7088	9972,6945	10027,3055	48
13	9835,5378	9862,5901	9972,9476	10027,0523	47
14	9835,6722	9862,4714	9973,2008	10026,7991	46
15	9835,8065	9862,3526	9973,4539	10026,5460	45
16	9835,9408	9862,2337	9973,7071	10026,2929	44
17	9836,0750	9862,1148	9973,9602	10026,0397	43
18	9836,2091	9861,9958	9974,2133	10025,7866	42
19	9836,3431	9861,8767	9974,4664	10025,5335	41
20	9836,4770	9861,7575	9974,7195	10025,2804	40
21	9836,6109	9861,6383	9974,9726	10025,0274	39
22	9836,7447	9861,5190	9975,2256	10024,7743	38
23	9836,8784	9861,3996	9975,4787	10024,5212	37
24	9837,0120	9861,2802	9975,7318	10024,2682	36
25	9837,1456	9861,1607	9975,9848	10024,0151	35
26	9837,2790	9861,0411	9976,2379	10023,7621	34
27	9837,4124	9860,9215	9976,4909	10023,509[illegible]	33
28	9837,5457	9860,8018	9976,7439	10023,2560	32
29	9837,6790	9860,6820	9976,9969	10023,0030	31
30	9837,8122	9860,5622	9977,2500	10022,7500	30
		Sin. 46.		Tan. 46.	M

M	*Sin.* 43.		*Tan.* 43.		
30	9837,8122	9860,5622	9977,2500	10022,7500	30
31	9837,9453	9860,4423	9977,5030	10022,4970	29
32	9838,0783	9860,3223	9977,7560	10022,2440	28
33	9838,2112	9860,2022	9978,0090	10021,9910	27
34	9838,3441	9860,0821	9978,2619	10021,7380	26
35	9838,4768	9859,9619	9978,5149	10021,4850	25
36	9838,6095	9859,8416	9978,7679	10021,2320	24
37	9838,7421	9859,7213	9979,0208	10020,9791	23
38	9838,8747	9859,6009	9979,2738	10020,7261	22
39	9839,0072	9859,4804	9979,5267	10020,4732	21
40	9839,1396	9859,3598	9979,7797	10020,2202	20
41	9839,2719	9859,2392	9980,0326	10019,9673	19
42	9839,4041	9859,1185	9980,2855	10019,7144	18
43	9839,5363	9858,9978	9980,5385	10019,4615	17
44	9839,6683	9858,8769	9980,7914	10019,2086	16
45	9839,8003	9858,7560	9981,0443	10018,9557	15
46	9839,9323	9858,6351	9981,2972	10018,7027	14
47	9840,0641	9858,5140	9981,5501	10018,4498	13
48	9840,1959	9858,3929	9981,8030	10018,1970	12
49	9840,3276	9858,2717	9982,0558	10017,9441	11
50	9840,4592	9858,1505	9982,3087	10017,6912	10
51	9840,5908	9858,0291	9982,5616	10017,4383	9
52	9840,7222	9857,9077	9982,8145	10017,1855	8
53	9840,8536	9857,7863	9983,0673	10016,9326	7
54	9840,9849	9857,6648	9983,3202	10016,6798	6
55	9841,1162	9857,5432	9983,5730	10016,4269	5
56	9841,2473	9857,4215	9983,8258	10016,1741	4
57	9841,3784	9857,2997	9984,0787	10015,9212	3
58	9841,5094	9857,1779	9984,3315	10015,6684	2
59	9841,6404	9857,0560	9984,5843	10015,4156	1
60	9841,7712	9856,9341	9984,8371	10015,1628	0
		Sin. 46.		*Tan.* 46.	M

M	Sin. 44.		Tan. 44.		
0	9841,7712	9856,9341	9984.8371	10015,1628	60
1	9841,9020	9856,8120	9985.0900	10014,9100	59
2	9842,0327	9856,6899	9985.3428	10014,6572	58
3	9842,1634	9856,5677	9985.5956	10014,4043	57
4	9842,2939	9856,4455	9985.8484	10014,1515	56
5	9842,4244	9856,3232	9986.1012	10013,8987	55
6	9842,5548	9856,2008	9986.3540	10013,6460	54
7	9842,6851	9856,0783	9986.6067	10013,3932	53
8	9842,8154	9855.9558	9986.8595	10013,1404	52
9	9842,9455	9855.8332	9987.1123	10012,8876	51
10	9843,0756	9855.7105	9987.3651	10012,6349	50
11	9843,2057	9855.5878	9987.6178	10012,3821	49
12	9843,3356	9855.4650	9987.8706	10012,1293	48
13	9843,4655	9855.3421	9988.1234	10011,8766	47
14	9843,5953	9855.2191	9988.3761	10011,6238	46
15	9843,7250	9855.0961	9988.6289	10011,3711	45
16	9843,8546	9854.9730	9988.8816	10011,1183	44
17	9843,9842	9854.8498	9989.1344	10010,8656	43
18	9844,1137	9854.7266	9989.3871	10010,6128	42
19	9844,2431	9854.6033	9989.6398	10010,3601	41
20	9844,3725	9854.4799	9989.8926	10010,1074	40
21	9844,5017	9854.3564	9990.1453	10009,8546	39
22	9844,6309	9854.2329	9990.3980	10009,6019	38
23	9844.7600	9854.1093	9990.6507	10009,3492	37
24	9844,8891	9853.9856	9990.9035	10009,0965	36
25	9845,0181	9853.8618	9991.1562	10008,8437	35
26	9845,1469	9853.7380	9991.4089	10008,5910	34
27	9845,2758	9853.6141	9991.6616	10008,3383	33
28	9845,4045	9853.4902	9991.9143	10008,0856	32
29	9845,5332	9853.3661	9992.1670	10007,8329	31
30	9845,6618	9853.2420	9992.4197	10007,5802	30
		Sin. 45.		Tan. 65.	M

M	Sin. 44.		Tan. 44.		
30	9845,6618	9853,2420	9992,4197	10007,5802	30
31	9845,7903	9853,1178	9992,6724	10007,3275	29
32	9845,9187	9852,9936	9992,9251	10007,0748	28
33	9846,0471	9852,8693	9993,1778	10006,8221	27
34	9846,1754	9852,7449	9993,4305	10006,5694	26
35	9846,3036	9852,6204	9993,6832	10006,3167	25
36	9846,4317	9852,4958	9993,9359	10006,0641	24
37	9846,5598	9852,3712	9994,1886	10005,8114	23
38	9846,6878	9852,2465	9994,4412	10005,5587	22
39	9846,8157	9852,1218	9994,6939	10005,3060	21
40	9846,9436	9851,9969	9994,9466	10005,0533	20
41	9847,0713	9851,8720	9995,1993	10004,8066	19
42	9847,1991	9851,7471	9995,4520	10004,5480	18
43	9847,3267	9851,6220	9995,7047	10004,2953	17
44	9847,4542	9851,4969	9995,9573	10004,0426	16
45	9847,5817	9851,3717	9996,2100	10003,7899	15
46	9847,7091	9851,2464	9996,4627	10003,5373	14
47	9847,8364	9851,1211	9996,7153	10003,2846	13
48	9847,9637	9850,9957	9996,9680	10003,0319	12
49	9848,0909	9850,8702	9997,2297	10002,7793	11
50	9848,2180	9850,7446	9997,4733	10002,5260	10
51	9848,3450	9850,6190	9997,7260	10002,2739	9
52	9848,4720	9850,4933	9997,9787	10002,0212	8
53	9848,5988	9850,3675	9998,2313	10001,7686	7
54	9848,7257	9850,2416	9998,4840	10001,5159	6
55	9848,8524	9850,1157	9998,7367	10001,2633	5
56	9848,9790	9849,9897	9998,9893	10001,0106	4
57	9849,1056	9849,8636	9999,2420	10000,7579	3
58	9849,2321	9849,7375	9999,4946	10000,5053	2
59	9849,3586	9849,6113	9999,7473	10000,2526	1
60	9849,4850	9849,4850	10000,0000	10000,0000	0
		Sin. 45.		Tan. 45.	M.

Lectori practicæ Matheseos studioso, S. P.

CANON noster vsum habet, in Triangulorum sphæricorum solutione, eundem quem tabulæ *Sinuum* rectorum & *Tangentium* ab alijs editæ, sed praxin paulo faciliorem. Nam eorum multiplicationem per additionem, & divisionem per subtractionem, & extractionem radicis quadratæ per bipartitionem evitamus.

Vt si datis tribus lateribus quæratur angulus, erit

Vt rectangulum sub *Sinibus* crurum,
ad quadratum *Radij*:
Ita rectangulum sub *Sinibus* semisummæ trium laterum,
& differentiæ inter hanc semisummam & basin,
ad quadratum *Co-sinus* semianguli quæsiti.

Et in triangulo primæ paginæ *P Z S*, (referente Polum, Zenith, & Solem) datis lateribus, *P S Gr.* 70, & *Z P Gr.* 38 *M.* 30, & *Z S Gr.* 40, si quæratur angulus *P Z S*, cuius basis est *P S*: summa laterum erit *Gr.* 148 *M.* 30, semisumma *Gr.* 74 *M.* 15, differentia inter semisummam & basin *Gr.* 4. *M.* 15.

Hic nos pro quadrato *Radij* ponimus 20000,0000 *Radij* duplum, cui addimus 9983,3805 *Sinum Gr.* 74, *M.* 15, 8869.8679 *Sinum Gr.* 4, *M.* 15, fient 38853,2484. Deinde pro rectangulo divisore addentes 9794, 1495 *Sinum Gr.* 38 *M.* 30. & 9808,0675 *Sinum Gr.* 40, facimus 19602,2170, & auferimus è 38853, 2484, ita restant 19251 0314. Horum semissis est 9625,5157 *Sinus* semianguli externi *Gr.* 24, *M.* 58 *S.* 24: & *Co-sinus* simianguli interni *Gr.* 65, *M.* 1, *S.* 36, & proinde totus angulus quæsitus est *Gr.* 130, *M.* 3, *S.* 12.

Quod

Quod si quis pro *Sinibus* auferendis addat eorum complementa ad *Radium*, non alia indigebit subtractione. Vt patere potest ex collatione vtriusque praxeos.

Gr. *M.*			
70 0			
38 30	9794,1495		205,8505
40 0	9808,0675		191,9325
148 30	19602,2170		
74 15	9983,3805		9983,3805
4 15	8869,8679		8869,8679
	20000,0000		
	38853,2484		
Gr. *M.* S.	19251,0314	*Gr.* *M.* S.	19251,0314
24 58 24	9625,5157	65 1 36	9625,5157
49 56 48		130 3 12	

Eadem ratione, sed maiori compendio, solvuntur cætera quæ quæri solent in triangulis sphæricis, sine ope *Secantium* aut *Sinuum versorum*, vt pluribus non sit opus aut præceptis aut exemplis.

Idem si desideres in triangulis rectilineis, adiunge nostris, Amici & Collegæ *Henrici Briggij* Logarithmos. Nam eo nitimur fundamento, eodem vtimur operandi modo.

Vale, & si hæc tibi gratia fuerint, plura à nobis in hoc genere expecta.

FINIS.

The firſt thouſand Logarithmes now againe ſet forth by the Authour *Henrie Briggs* profeſſor of Geometrie in the Vniverſitie of Oxford, who undertooke this worke at the entreatie, and with the approbation of the firſt Inventer of Logarithmes, worthy of all honor, *Iohn Nepeir* Baron of *Merchiſton.*

THe Reader hath here a ſhort view of thoſe 30000. Logarithmes, which are now coming forth in Latin, and hereafter in Engliſh, which will affoord us,

The Quinteſſence of the Golden rule.

The valuation of Annuities, and the ſolution of all ordinary difficult queſtions of that kind.

The quantitie of any plaine Triangle, whoſe ſides are given, together with the altitude thereof: the Diameters of the Circles inſcribed and circumſcribed; and the quantitie of any of the Angles.

The Diameter being givē, the circumference & Area of a Circle, and the Superficies and Soliditie of a Globe.

The quantitie of auy round Caske.

And ſo neare as may be, the ſquaring of a Circle, the cubing of a Globe, the doubling or tribling of a Cube.

And in generall, The enlarging or diminiſhing of any plaine or ſolid figure, keeping the ſame forme; or the transforming it in any proportion aſſigned.

The alteration of the ſides of any given plaine Triangle, keeping the ſame Area, and the ſame Perimeter.

The deſcription of a Peripherie, every point whereof ſhall frō the three angles of any givē Triangle, keep the diſtances according to any poſſible proportiōs aſſigned.

Having two ſides of a right angled Triangle given, to find the third: and generally all that may bee found in all right lined Triangles whatſoever.

In tenui ſed non tenuis fructuſve laborve.

Nū	Logarithm.	Nū	Logarithm.	Nū	Logarithm.
1	0	34	1531,47892	67	1826,07480
2	301,02999	35	1544,06804	68	1832,50891
3	477,12125	36	1556,30250	69	1838,84909
4	602,05999	37	1568,20172	70	1845,09804
5	698,97000	38	1579,78360	71	1851,25835
6	778,15125	39	1591,06461	72	1857,33250
7	845,09804	40	1602,05999	73	1863,32286
8	903,08999	41	1612,78386	74	1869,23172
9	954,24251	42	1623,24929	75	1875,06126
10	1000,00000	43	1633,46846	76	1880,81359
11	1041,39269	44	1643,45268	77	1886,49073
12	1079,18125	45	1653,21251	78	1892,09460
13	1113,94335	46	1662,75783	79	1897,62709
14	1146,12804	47	1672,09786	80	1903,08999
15	1176,09126	48	1681,24124	81	1908,48502
16	1204,11998	49	1690,19608	82	1913,81385
17	1230,44892	50	1698,97000	83	1919,07809
18	1255,27251	51	1707,57018	84	1924,27929
19	1278,75360	52	1716,00334	85	1929,41893
20	1301,02999	53	1724,27587	86	1934,49845
21	1322,21929	54	1732,39376	87	1939,51925
22	1342,42268	55	1740,36269	88	1944,48267
23	1361,72784	56	1748,18803	89	1949,39001
24	1380,21124	57	1755,87486	90	1954,24251
25	1397,94001	58	1763,42799	91	1959,04139
26	1414,97335	59	1770,85201	92	1963,78783
27	1431,36376	60	1778,15125	93	1968,48295
28	1447,15803	61	1785,32984	94	1973,12785
29	1462,39800	62	1792,39169	95	1977,72361
30	1477,12125	63	1799,34055	96	1982,27123
31	1491,36169	64	1806,17997	97	1986,77173
32	1505,14998	65	1812,91336	98	1991,22608
33	1518,51394	66	1819,54394	99	1995,63519
34	1531,47892	67	1826,07480	100	2000,00000

Nū.	Logarithm.	Differ.	Nū.	Logarithm.	Differ.
101	2004,32137	427880	134	2127,10480	322897
102	2008,60017	423705	135	2130,33377	320514
103	2012,83722	419612	136	2133,53891	318166
104	2017,03334	415596	137	2136,72057	315852
105	2021,18930	411657	138	2139,87909	313571
106	2025,30587	407791	139	2143,01480	311324
107	2029,38378	403998	140	2146,12804	309107
108	2033,42376	400274	141	2149,21911	306923
109	2037,42650	396619	142	2152,28834	304770
110	2041,39269	393029	143	2155,33604	302645
111	2045,32298	389504	144	2158,36249	300551
112	2049,21802	386042	145	2161,36800	298486
113	2053,07844	382641	146	2164,35286	296447
114	2056,90485	379299	147	2167,31733	294439
115	2060,69784	376015	148	2170,26172	292455
116	2064,45799	372787	149	2173,18627	290499
117	2068,18586	369615	150	2176,09126	288569
118	2071,88201	366495	151	2178,97695	286664
119	2075,54696	363429	152	2181,84359	284784
120	2079,18125	360412	153	2184,69143	282929
121	2082,78537	357446	154	2187,52072	281098
122	2086,35983	354528	155	2190,33170	279290
123	2089,90511	351658	156	2193,12460	277505
124	2093,42169	348832	157	2195,89965	275744
125	2096,91001	346054	158	2198,65709	274003
126	2100,37055	343317	159	2201,39712	272286
127	2103,80372	340625	160	2204,11998	270590
128	2107,20997	337974	161	2206,82588	268913
129	2110,58971	335364	162	2209,51501	267259
130	2113,94335	332795	163	2212,18760	265625
131	2117,27130	330263	164	2214,84385	264009
132	2120,57393	327771	165	2217,48394	262415
133	2123,85164	325316	166	2220,10809	260838
134	2127,10480	322897	167	2222,71647	259281

Nũ	Logarithm.	Differ.	Nũ	Logarithm.	Differ.
167	2222,71647	259281	201	2303,19606	215531
168	2225,30928	257742	202	2305,35137	214467
169	2227,88670	256222	203	2307,49604	213413
170	2230,44892	254719	204	2309,63017	212369
171	2232,99611	253234	205	2311,75386	211336
172	2235,52845	251765	206	2313,86722	210313
173	2238,04610	250315	207	2315,97035	209298
174	2240,54925	248880	208	2318,06333	208296
175	2243,03805	247462	209	2320,14629	207300
176	2245,51267	246060	210	2322,21929	206317
177	2247,97327	244673	211	2324,28246	205340
178	2250,42000	243303	212	2326,33586	204374
179	2252,85303	241948	213	2328,37960	203417
180	2255,27251	240606	214	2330,41377	202469
181	2257,67857	239282	215	2332,43846	201529
182	2260,07139	237970	216	2334,45375	200598
183	2262,45109	236673	217	2336,45973	199676
184	2264,81782	235391	218	2338,45649	198762
185	2267,17173	234121	219	2340,44411	197857
186	2269,51294	232867	220	2342,42268	196959
187	2271,84161	231624	221	2344,39227	196070
188	2274,15785	230395	222	2346,35297	195189
189	2276,46180	229180	223	2348,30486	194316
190	2278,75360	227977	224	2350,24802	193450
191	2281,03337	226786	225	2352,18252	192592
192	2283,30123	225608	226	2354,10844	191742
193	2285,55731	224442	227	2356,02586	190899
194	2287,80173	223288	228	2357,93485	190063
195	2290,03461	222146	229	2359,83548	189236
196	2292,25607	221016	230	2361,72784	188414
197	2294,46623	219897	231	2363,61198	187600
198	2296,66519	218789	232	2365,48798	186794
199	2298,85308	217691	233	2367,3459	185994
220	2301,02999	216606	234	2369,21586	185200

Nu.	Logarithm.	Differ.	Nu.	Logarithm.	Differ.
234	2369,21586	185200	267	2426,51126	162353
235	2371,06786	184414	268	2428,13479	161749
236	2372,91200	183635	269	2429,75228	161148
237	2374,74835	182861	270	2431,36376	160553
238	2376,57696	182094	271	2432,96929	159961
239	2378,39790	181334	272	2434,56890	159375
240	2380,21124	180580	273	2436,16265	158791
241	2382,01704	179833	274	2437,75056	158213
242	2383,81537	179090	275	2439,33269	157639
243	2385,60627	178356	276	2440,90908	157069
244	2387,38983	177625	277	2442,47977	156503
245	2389,16608	176903	78	2444,04480	155940
246	2390,93511	176184	79	2445,60420	155383
247	2392,69695	175473	280	2447,15803	154829
248	2394,45168	174767	281	2448,70632	154279
249	2396,19935	174066	282	2450,24911	153733
250	2397,94001	173371	283	2451,78644	153190
251	2399,67372	172682	284	2453,31834	152652
252	2401,40054	171998	285	2454,84486	152117
253	2403,12052	171320	286	2456,36603	151587
254	2404,83372	170646	287	2457,88190	151059
255	2406,54018	169979	288	2459,39249	150535
256	2408,23997	169315	289	2460,89784	150016
257	2409,93312	168659	290	2463,39800	149499
258	2411,61971	168005	291	2463,89299	148986
259	2413,29976	167359	292	2465,38285	148477
260	2414,97335	166716	293	2466,86762	147971
261	2416,64051	166078	294	2468,34733	147469
262	2418,30129	165446	295	2469,82202	146969
263	2419,95575	164818	296	2471,29171	146474
264	2421,60393	164194	297	2472,75645	145981
265	2423,24587	163577	298	2474,21626	145493
266	2424,88164	162962	299	2475,67119	145006
267	2426,51126	162353	300	3477,12125	144524

Nũ.	Logarithm.	Differ.	Nũ	Logarithm.	Differ.
301	2478,56650	144044	334	2523,74647	129834
302	2480,00694	143569	335	2525,04481	129447
303	2481,44263	143095	336	2526,33928	129062
304	2482,87358	142626	337	2527,62990	128680
305	2484,29984	142159	338	2528,91670	128300
306	2485,72143	141695	339	2530,19970	127922
307	2487,13838	141234	340	2531,47892	127546
308	2488,55072	140776	341	2532,75438	127173
309	2489,95848	140321	342	2534,02611	126801
310	2491,36169	139470	343	2535,29412	126432
311	2492,76039	139320	344	2536,55844	126066
312	2494,15459	138975	345	2537,81910	125700
313	2495,54434	138531	346	2539,07610	125337
314	2496,92965	138090	347	2540,32947	124977
315	2498,31055	137653	348	2541,57924	124619
316	2499,68708	137218	349	2542,82543	124261
317	2501,05926	136786	350	2544,06804	123908
318	2502,42712	136356	351	2545,30712	123554
319	2503,79068	135930	352	2546,54266	123205
320	2505,14998	135505	353	2547,77471	122855
321	2506,50503	135084	354	2549,00326	122509
322	2507,85587	134665	355	2550,22835	122165
323	2509,20252	134249	356	2551,45000	121822
324	2510,54501	133835	357	2552,66822	121481
325	2511,88336	133424	358	2553,88303	121142
326	2513,21760	133015	359	2555,09445	120805
327	2514,54775	132609	360	2556,30250	120470
328	2515,87384	132206	361	2557,50720	120137
329	2517,19590	131804	362	2558,70857	119806
330	2518,51394	131405	363	2559,90663	119475
331	2519,82799	131009	364	2561,10138	119148
332	2521,13808	130615	365	2562,29286	118823
333	2522,44423	130224	366	2563,48109	118497
334	2523,74647	129834	367	2564,66606	118176

Nũ.	Logarithm.	Differ.	Nũ.	Logarithm.	Differ.
367	2564,66606	118176	401	2603,14437	108168
368	2565,84782	117855	402	2604,22605	107900
369	2567,02637	117535	403	2605,30505	107632
370	2568,20172	117219	404	2606,38137	107365
371	2569,37391	116903	405	2607,45502	107101
372	2570,54294	116589	406	2608,52603	106838
373	2571,70883	116277	407	2609,59441	106575
374	2572,87160	115967	408	2610,66016	106315
375	2574,03127	115657	409	2611,72331	106055
376	2575,18784	115351	410	2612,78386	105796
377	2576,34135	115045	411	2613,84182	105540
378	2577,49180	114741	412	2614,89722	105283
379	2578,63921	114439	413	2615,95005	105029
380	2579,78360	114138	414	2617,00034	104776
381	2580,92498	113838	415	2618,04810	104523
382	2582,06336	113541	416	2619,09333	104272
383	2583,19877	113245	417	2620,13605	104023
384	2584,33122	112951	418	2621,17628	103774
385	2585,46073	112657	419	2622,21402	103527
386	2586,58730	112367	420	2623,24929	103281
387	2587,71097	112076	421	2624,28210	103035
388	2588,83173	111787	422	2625,31245	102792
389	2589,94960	111501	423	2626,34037	102549
390	2591,06461	111215	424	2627,36586	102307
391	2592,17676	110931	425	2628,38893	102067
392	2593,28607	110648	426	2629,40960	101828
393	2594,39255	110367	427	2630,42788	101589
394	2595,49622	110088	428	2631,44377	101352
395	2596,59710	109809	429	2632,45729	101117
396	2597,69519	109532	430	2633,46846	100881
397	2598,79051	109256	431	2634,47727	100648
398	2599,88307	108983	432	2635,48375	100415
399	2600,97190	108709	433	3636,48790	100183
400	2602,05999	108438	434	2637,48973	99953

Nū.	Logarithm.	Differ.	Nū.	Logarithm.	Differ.
434	2637,48973	99953	467	2669,31688	92897
435	2638,48926	99723	468	2670,24585	92699
436	2639,48649	99495	469	2671,17284	92502
437	2640,48144	99267	470	2672,09786	92305
438	2641,47411	99041	471	2673,02091	92109
439	2642,46452	98816	472	2673,94200	91914
440	2643,45268	98591	473	2674,86114	91720
441	2644,43859	98368	474	2675,77834	91527
442	2645,42227	98146	475	2676,69361	91334
443	2646,40373	97924	476	2677,60695	91143
444	2647,38297	97704	477	2678,51838	90952
445	2648,36001	97485	478	2679,42790	90761
446	2649,33486	97266	479	2680,33551	90573
447	2650,30752	97049	480	2681,24124	90384
448	2651,27801	96833	481	2682,14508	90196
449	2652,24634	96617	482	2683,04704	90009
450	2653,21251	96403	483	2683,94713	89823
451	2654,17654	96189	484	2684,84536	89638
452	2655,13843	95977	485	2685,74174	89453
453	2656,09820	95765	486	2686,63627	89269
454	2657,05585	95555	487	2687,52896	89086
455	2658,01140	95344	488	2688,41982	88904
456	2658,96484	95136	489	2689,30886	88722
457	2659,91620	94928	490	2690,19608	88541
458	2660,86548	94721	491	2691,08149	88361
459	2661,81269	94514	492	2691,96510	88182
460	2662,75783	94310	493	2692,84692	88003
461	2663,70093	94105	494	2693,72695	87825
462	2664,64198	93901	495	2694,60520	87648
463	2665,58099	93699	496	2695,48168	87471
464	2666,51798	93497	497	2696,35639	87295
465	2667,45295	93297	498	2697,22934	87121
466	2668,38592	93096	499	2698,10055	86945
467	2669,31688	92897	500	2698,97000	86772

Nũ	Logarithm.	Differ.	Nũ	Logarithm.	Differ
501	2699,83773	86599	534	2727,54126	81252
502	2700,70372	86427	535	2728,35378	81101
503	2701,56799	86255	536	2729,16479	80950
504	2702,43054	86084	537	2729,97429	80799
505	2703,29138	85914	538	2730,78228	80649
506	2704,15052	85744	539	2731,58877	80499
507	2705,00796	85575	540	2732,39376	80351
508	2705,86371	85407	541	2733,19727	80202
509	2706,71778	85240	542	2733,99929	80054
510	2707,57018	85072	543	2734,79983	79907
511	2708,42090	84906	544	2735,59890	79760
512	2709.26996	84741	545	2736,39650	79614
513	2710,11737	84575	546	2737,19264	79469
514	2710,96312	84411	547	2737,98733	79323
515	2711,80723	84247	548	2738,78056	79178
516	2712,64970	84084	549	2739,57234	79035
517	2713,49054	83922	550	2740,36269	78891
518	2714,32976	83760	551	2741,15160	78748
519	2715,16736	83598	552	2741,93908	78605
520	2716,00334	83438	553	2742,72513	78463
521	2716.83772	83278	554	2743,50976	78322
522	2717,67050	83119	555	2744,29298	78181
523	2718,50169	82960	556	2745,07479	78041
524	2719,33129	82801	557	2745,85520	77900
525	2720,15930	82644	558	2746,63420	77761
526	2720,98574	82488	559	2747,41181	77622
527	2721,81062	82330	560	2748,18803	77483
528	2722,63392	82175	561	2748,96286	77346
529	2723,45567	82020	562	2749,73632	77207
530	2724,27587	81865	563	2750,50839	77071
531	2725,09452	81711	564	2751,27910	76925
532	2725,91163	81558	565	2752,04845	76798
533	2726,72721	81405	566	2752,81643	76663
534	2727.54126	81252	567	2753,58306	76528

Nū	Logarithm.	Differ	Nū	Logarithm.	Differ
567	2753,58306	76528	601	2778,87447	72202
568	2754,34834	76393	602	2779,59649	72082
569	2755,11227	76259	603	2780,31731	71963
570	2755,87486	76125	604	2781,03694	71843
571	2756,63611	75992	605	2781,75537	71725
572	2757,39603	75859	606	2782,47262	71607
573	2758.15462	75727	607	2783,18869	71489
574	2758,91189	75595	608	2783,90358	71371
575	2759,66784	75464	609	2784,61729	71255
576	2760,42248	75333	610	2785.32984	71137
577	2761,17581	75203	611	2786,04121	71021
578	2761,92784	75072	612	2786,75142	70905
579	2762,67856	74943	613	2787,46047	70790
580	2763,42799	74814	614	2788,16837	70675
581	2764,17613	74685	615	2788,87512	70559
582	2764,92298	74557	616	2789,58071	70445
583	2765,66855	74430	617	2790,28516	70332
584	2766,41285	74302	618	2790,98848	70217
585	2767,15587	74175	619	2791,69065	70104
586	2767,89762	74048	620	2792,39169	69991
587	2768,63810	73923	621	2793,09160	69878
588	2769,37733	73796	622	2793,79038	69767
589	2770,11529	73672	623	2794,48805	69654
590	2770,85201	73547	624	2795,18459	69543
591	2771,58748	73423	625	2795,88002	69431
592	2772,32171	73298	626	2796,57433	69321
593	2773,05469	73175	627	2797,26754	69210
594	2773,78644	73053	628	2797.95964	69101
595	2774,51697	72929	629	2798,65065	68990
596	2775,24626	72807	630	2799,34055	68881
597	2775,97433	72685	631	2800,02936	68772
598	2776,70118	72564	632	2800,71708	68663
599	2777,42682	72443	633	2801,40371	68555
600	2778,15125	72322	634	2802,08926	68447

Nu.	Logarithm.	Differ.	Nu.	Logarithm.	Differ.
634	2802,08926	68447	667	2824,12583	65063
635	2802,77373	68339	668	2824,77646	64966
636	2803,45712	68231	669	2825,42612	64868
637	2804,13943	68125	670	2826,07480	64772
638	2804,82068	68018	671	2826,72252	64675
639	2805,50086	67911	672	2827,36927	64579
640	2806,17997	67806	673	2828,01506	64484
641	2806,85803	67700	674	2828,65990	64387
642	2807,53503	67594	675	2829,30377	64294
643	2808,21097	67490	676	2829,94670	64197
644	2808,88587	67384	677	2830,58867	64102
645	2809,55971	67281	678	2831,22969	64008
646	2810,23252	67176	679	2831,86977	63914
647	2810,90428	67073	680	2832,50891	63820
648	2811,57501	66969	681	2833,14711	63726
649	2812,24470	66866	682	2833,78437	63633
650	2812,91336	66763	683	2834,42070	63540
651	2813,58099	66661	684	2835,05610	63447
652	2814,24760	66558	685	2835,69057	63355
653	2814,91318	66457	686	2836,32412	63262
654	2815,57775	66355	687	2836,95674	63170
655	2816,24130	66254	688	2837,58844	63078
656	2816,90384	66153	689	2838,21922	62987
657	2817,56537	66052	690	2838,84909	62895
658	2818,22589	65952	691	2839,47805	62804
659	2818,88541	65853	692	2840,10609	62714
660	2819,54394	65752	693	2840,73323	62624
661	2820,20146	65653	694	2841,35947	62533
662	2820,85799	65554	695	2841,98480	62444
663	2821,51353	65455	696	2842,60924	62354
664	2822,16808	65357	697	2843,23278	62264
665	2822,82165	65258	698	2843,85542	62176
666	2823,47423	65160	699	2844,47718	62086
667	2824,12583	65063	700	2845,09804	61998

Nū	Logarithm.	Differ.	Nū	Logarithm.	Differ.
701	2845,71802	61909	734	2865,69606	59128
702	2846,33711	61822	735	2866,28734	59047
703	2846,95533	61733	736	2866,87781	58968
704	2847,57266	61646	737	2867,46749	58887
705	2848,18912	61558	738	2868,05636	58808
706	2848,80470	61471	739	2868,64444	58728
707	2849,41941	61385	740	2869,23172	58649
708	2850,03326	61298	741	2869,81821	58570
709	2850,64624	61211	742	2870.40391	58490
710	2851,25835	61125	743	2870,98881	58413
711	2851,86960	61039	744	2871,57294	58333
712	2852,47999	60954	745	2872,15627	58256
713	2853,08953	60868	746	2872,73883	58177
714	2853,69821	60783	747	2873,32060	58100
715	2854,30604	70698	748	2873,90160	58022
716	2854,91302	60614	749	2874,48182	57944
717	2855,51916	60528	750	2875,06126	57808
718	2856,12444	60445	751	2875,63994	57790
719	2856,72889	60361	752	2876,21784	57714
720	2857,33250	60276	753	2876,79498	57637
721	2857,93526	60194	754	2877,37135	57560
722	2858,53720	60110	755	2877,94695	57485
723	2859,13830	60027	756	2878,52180	57408
724	2859,73857	59944	757	2879,09588	57333
725	2860,33801	59861	758	2879,66921	57257
726	2860,93662	59779	759	2880,24178	57181
727	2861,53441	59697	760	2880,81359	57107
728	2862,13138	59615	761	2881,38466	57031
729	2862,72753	59533	762	2881,95497	56957
730	2863,32286	59452	763	2882,52454	56882
731	2863,91738	59370	764	2883,09336	56808
732	2864,51108	59289	765	2883,66144	56733
733	2865,10397	59209	766	2884,22877	56659
734	2865,69606	59128	767	2884,79536	56586

Nū	Logarithm.	Differ	Nū	Logarithme	Differ
767	2884,79536	56586	801	2903,63252	54185
768	2885,36122	56512	802	2904,17437	54118
769	2885,92634	56439	803	2904,71555	54050
770	2886,49073	56365	804	2905,25605	53983
771	2887,05438	56292	805	2905,79588	53916
772	2887,61730	56219	806	2906,33504	53849
773	2888,17949	56147	807	2906,87353	53783
774	2888,74096	56074	808	2907,41136	53716
775	2889,30170	56002	809	2907,94852	53650
776	2889,86172	55930	810	2908,48502	53583
777	2890,42102	55858	811	2909,02085	53518
778	2890,97960	55786	812	2909,55603	53452
779	2891,53746	55714	813	2910,09055	53385
780	2892,09460	55643	814	2910,62440	53321
781	2892,65103	55572	815	2911,15761	53255
782	2893,20675	55501	816	2911,69016	53190
783	2893,76176	55430	817	2912,22206	53124
784	2894,31606	55360	818	2912,75330	53060
785	2894,86966	55289	819	2913,28390	52995
786	2895,42255	55218	820	2913,81385	52931
787	2895,97473	55149	821	2914,34316	52866
788	2896,52622	55078	822	2914,87182	52802
789	2897,07700	55009	823	2915,39984	52737
790	2897,62709	54939	824	2915,92721	52674
791	2898,17648	54870	825	2916,45395	52610
792	2898,72518	54801	826	2916,98005	52546
793	2899,27319	54731	827	2917,50551	52483
794	2899,82050	54663	828	2918,03034	524[illegible]9
795	2900,36713	54594	829	2918,55453	52356
796	2900,91307	54525	830	2919,07809	52293
797	2901,45832	54457	831	2919,60102	52231
798	2902,00289	54389	832	2920,12333	52167
799	2902,54678	54321	833	2920,64500	52105
800	2903,08999	54253	834	2921,16605	52043

Nũ	Logarithm.	Differ
834	2921,16605	52043
835	2921,68648	51980
836	2922,20627	51918
837	2922,72546	51856
838	2923,24402	51794
839	2923,76196	51733
840	2924,27929	51671
841	2924,79600	51609
842	2925,31209	51548
843	2925,82757	51488
844	2926,34245	51426
845	2926,85671	51365
846	2927,37036	51305
847	2927,88341	51244
848	2928,39585	51184
849	2928,90769	51124
850	2929,41893	51063
851	2929,92956	51003
852	2930,43959	50944
853	2930,94903	50884
854	2931,45787	50824
855	2931,96611	50765
856	2932,47376	50706
857	2932,98082	50647
858	2933,48729	50587
859	2933,99316	50529
860	2934,49845	50470
861	2935,00315	50412
862	2935,50727	50353
863	2936,01080	50294
864	2936,51374	50237
865	2937,01611	50178
866	2937,51789	50121
867	2938.01010	50063
867	2938,01910	50063
868	2938,51973	50005
869	2939,01978	49947
870	2939,51925	49891
871	2940,01816	49832
872	2940,51648	49776
873	2941,01424	49719
874	2941,51143	49662
875	2942.00805	49606
876	2942,50411	49548
877	2942,99959	49493
878	2943,49452	49436
879	2943,98888	49379
880	2944,48267	49324
881	2944,97591	49268
882	2945,46859	49211
883	2945.96070	49157
884	2946,45227	49100
885	2946,94327	49045
886	2947,43372	48990
887	2947,92362	48935
888	2948,41297	48879
889	2948,90176	48825
890	2949,39001	48769
891	2949,87770	48715
892	2950,36485	48661
893	2950,85146	48606
894	2951,33752	48552
895	2951,82304	48497
896	2952,30801	48443
897	2952,79244	48390
898	2953,27634	48335
899	2953,75969	48282
900	2954,24251	48228

Nū	Logarithm,	Differ	Nū	Logarithm,	Differ
901	2954,72479	48175	934	2970,34688	46473
902	2955,20654	48121	935	2970,81161	46424
903	2955,68775	48068	936	2971,27585	46374
904	2956,16843	48015	937	2971,73959	46325
905	2956,64858	47962	938	2972,20284	46275
906	2957,12820	47909	939	2972,66559	46226
907	2957,60729	47856	940	2973,12785	46177
908	2958,08585	47803	941	2973,58962	46128
909	2958,56388	47751	942	2974,05090	46079
910	2959,04139	47699	943	2974,51169	46030
911	2959.51838	47646	944	2974,97199	45982
912	2959,99484	47594	945	2975 43181	45933
913	2960,47078	47542	946	2975,89114	45884
914	2960,94620	47489	947	2976,34998	45836
915	2961,42106	47438	948	2976,80834	45787
916	2961,89547	47387	949	2977,26621	45740
917	2962,36934	47334	950	2977,72361	45691
918	2962,84268	47283	951	2978,18052	45643
919	2963,31551	47232	952	2978,63695	45595
920	2963,78783	47180	993	2979,09290	45547
921	2964,25963	47129	954	2979,54837	45500
922	2964,73092	47078	955	2980,00337	45452
923	2965,20170	47027	956	2980,45789	45405
924	2965,67197	46976	957	2980,91194	45357
925	2966,14173	46926	958	2981,36551	45310
926	2966,61099	46874	959	2981,81862	45262
927	2967,07973	46825	960	2982,27123	45216
928	2967,54798	46773	961	2982,72339	45168
929	2968,01571	46724	962	2983,17507	45122
930	2968,48295	46673	963	2983,62629	45074
931	296894968	46623	964	2984,07703	45028
932	2969,41591	46573	965	2984,52731	44982
933	2969,88164	46524	966	2984,97713	44934
934	29703 4688	46473	967	2985,42647	44889

Nu	Logarithm.	Differ.	Nū	Logarithm.	Differ
967	2955,42647	44889	984	2992,99510	44113
968	2985,87536	44842	985	2993,43623	44068
969	2986,32378	44795	986	2993,87691	44024
970	2986,77173	44750	987	2994,31715	43979
971	2987.21923	44703	988	2994.75694	43935
972	2987,66626	44658	989	2995,19629	43890
973	2988,11284	44612	990	2995,63519	43846
974	2988,55896	44566	991	2996,07365	43802
975	2989,00462	44520	992	2996,51167	43758
976	2989,44982	44474	993	2996,94925	43713
977	2989,89456	44429	994	2997,38638	43670
978	2990,33885	44384	995	2997,82308	43626
979	2990,78269	44339	996	2998,25934	43582
980	2991,22608	44293	997	2998,69516	43538
981	2991,66901	44248	998	2999,13054	43495
982	2992,11149	44203	999	2999.56549	43451
983	2992,55352	44158	1000	3000,00000	

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